

POLYNOMIALS



TRY YOURSELF

SOLUTIONS

1. (i) $2\sqrt{2}y^5 + 3y^4$ is a polynomial in y , since exponent of y in each term is a whole number.

(ii) $2\sqrt{x} + x\sqrt{2}$ can be written as $2x^{1/2} + \sqrt{2}x$. Here, the exponent of variable in $2x^{1/2}$ is $1/2$, which is not a whole number. Therefore, the given expression is not a polynomial.

(iii) $x^2 + \frac{3}{x^2} + 4$ can be written as $x^2 + 3x^{-2} + 4$. Here, the exponent of variable in term $3x^{-2}$ is -2 , which is not a whole number. Therefore, the given expression is not a polynomial.

2. There are three terms in the given polynomial, namely, x^2 , $\frac{\pi}{2}x$ and -7 .

3. The given polynomial, $p(x) = 13x^{15} + 12x^{14} + 11x^{13} + 10x^{12} + 9x^{11}$ can be written as $13x^{15} + 12x^{14} + 11x^{13} + 10x^{12} + 9x^{11} + 0x^{10}$.

\therefore Coefficient of x^{10} is 0.

4. Monomial = $4x^8$ and binomial = $2x + 7$

5. We have, $(x^3 + 5)(4 - x^5) = 4x^3 - x^8 + 20 - 5x^5$
Clearly, the highest power of the variable is 8. Therefore, the degree of the given polynomial is 8.

6. The only term here is 100, which can be written as $100x^0$. And exponent of x is 0. Therefore, the degree of the given polynomial is 0.

7. (i) $5x^{28} + 3$ (ii) $2x^9 + 6x + 7$

8. (i) Clearly, $5x^2 + 8x$ is a polynomial of degree 2. So, it is a quadratic polynomial.

(ii) Clearly, $2x - x^3$ is a polynomial of degree 3. So, it is a cubic polynomial.

(iii) Clearly, $3 + 2x$ is a polynomial of degree 1. So, it is a linear polynomial.

(iv) Clearly, $5x^3$ is a polynomial of degree 3. So, it is a cubic polynomial.

9. (i) Clearly, $5 - x - x^2$ is a polynomial of degree 2. So, it is a quadratic polynomial.

(ii) Clearly, p^4 is a polynomial of degree 4. So, it is a biquadratic polynomial.

10. We have, $p(x) = \sqrt{2}x^2 + \sqrt{2}x + 6$

$$p(\sqrt{2}) = \sqrt{2}(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) + 6$$

$$= 2\sqrt{2} + 2 + 6 = 2\sqrt{2} + 8$$

11. We have, $p(y) = 3y^4 - 2y^3 + 15y + k$

$$p(1) = -1 \Rightarrow 3(1)^4 - 2(1)^3 + 15(1) + k = -1$$

$$\Rightarrow 3 - 2 + 15 + k = -1 \Rightarrow 16 + k = -1 \Rightarrow k = -17$$

12. We have, $p(t) = 3t^4 + 1$

$$\therefore p(0) = 3(0)^4 + 1 = 3 \times 0 + 1 = 1,$$

$$p(1) = 3(1)^4 + 1 = 3 \times 1 + 1 = 4,$$

$$p(-1) = 3(-1)^4 + 1 = 3 \times 1 + 1 = 4,$$

$$p(3) = 3(3)^4 + 1 = 3 \times 81 + 1 = 244 \text{ and}$$

$$p(-3) = 3(-3)^4 + 1 = 3 \times 81 + 1 = 244$$

13. Let $p(x) = x + 2$. Then $p(2) = 2 + 2 = 4$, $p(-2) = -2 + 2 = 0$. Therefore, -2 is a zero of the polynomial $x + 2$, but 2 is not.

14. Let $p(x) = x^2 - 2x - 3$

$$\text{Now, } p(3) = 3^2 - 2(3) - 3 = 9 - 6 - 3 = 0$$

Since, $p(3) = 0$, therefore $x = 3$ is a root of the polynomial equation $x^2 - 2x - 3 = 0$.

15. We have, $q(x) = 2x - 7$. To find its zero, put $q(x) = 0$.

$$\therefore 2x - 7 = 0 \Rightarrow 2x = 7 \Rightarrow x = \frac{7}{2}.$$

Thus, zero of polynomial $q(x)$ is $\frac{7}{2}$.

16. We have, $p(y) = ly - m$; $l \neq 0$. To find its zero, put $p(y) = 0$.

$$\therefore ly - m = 0 \Rightarrow ly = m \Rightarrow y = \frac{m}{l}.$$

Thus, $y = \frac{m}{l}$ is the zero of the polynomial $p(y) = ly - m$.

17. Here, degree of $p(x) = 4$ and degree of $g(x) = 1$. So degree of $g(x) <$ degree of $p(x)$. By long division method, we get

$$\begin{array}{r} x + 3 \overline{) 7x^4 + 3x^3 - 2x^2 + x - 4} \\ \underline{7x^4 + 21x^3} \\ -18x^3 - 2x^2 \\ \underline{-18x^3 - 54x^2} \\ 52x^2 + x \\ \underline{52x^2 + 156x} \\ -155x - 4 \\ \underline{-155x - 465} \\ 461 \end{array}$$

Thus, quotient is $7x^3 - 18x^2 + 52x - 155$ and remainder is 461.

18. To check if $x + 1$ is a factor of $x^3 + 1$, we divide $x^3 + 1$ by $x + 1$. Therefore, by long division method, we get

$$\begin{array}{r}
 x+1 \overline{) x^3+1} \quad (x^2-x+1 \\
 \underline{-x^3+x^2} \\
 -x^2+1 \\
 \underline{-x^2-x} \\
 + \quad + \\
 x+1 \\
 \underline{x+1} \\
 - \quad - \\
 0
 \end{array}$$

So, we find that the remainder is 0. Therefore, $x+1$ is a factor of x^3+1 .

$$\begin{array}{r}
 19. \quad x+1 \overline{) 5x^2+x-1} \quad (5x-4 \\
 \underline{5x^2+5x} \\
 -4x-1 \\
 \underline{-4x-4} \\
 + \quad + \\
 3
 \end{array}$$

Here, Dividend $p(x) = 5x^2 + x - 1$, Divisor $g(x) = x + 1$, Quotient $q(x) = 5x - 4$, Remainder $r(x) = 3$

$$\therefore \text{R.H.S.} = g(x)q(x) + r(x) = (x+1)(5x-4) + 3 = 5x^2 - 4x + 5x - 4 + 3 = 5x^2 + x - 1 = \text{L.H.S.}$$

20. Let $p(x) = ax^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + a$ be the given polynomials.

Since zero of $(x-3)$ is 3, therefore remainders when $p(x)$ and $q(x)$ are divided by $(x-3)$ are given by $p(3)$ and $q(3)$ respectively.

By the given condition, we have $p(3) = q(3)$

$$\Rightarrow a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4 = 3^3 - 4 \times 3 + a$$

$$\Rightarrow 27a + 36 + 9 - 4 = 27 - 12 + a$$

$$\Rightarrow 26a + 26 = 0 \Rightarrow 26a = -26 \Rightarrow a = -1.$$

21. Clearly, $q(t)$ will be a multiple of $2t-1$ only if $2t-1$ divides $q(t)$ leaving remainder zero.

Now, zero of $2t-1$ is $\frac{1}{2}$.

$$\begin{aligned}
 \therefore \text{Remainder} &= q\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) - 2 \\
 &= \frac{1}{2} + \frac{3}{4} + \frac{1}{2} - 2 = \frac{-1}{4} \neq 0
 \end{aligned}$$

Since the remainder obtained on dividing $q(t)$ by $2t-1$ is not 0. Therefore, $q(t)$ is not a multiple of $2t-1$.

22. Let $p(x) = x^4 - k^2x^2 + 2x - k$

\therefore Zero of the polynomial $x - k$ is k , therefore by remainder theorem, remainder when $p(x)$ is divided by $(x - k)$ is given by $p(k)$.

$$\therefore p(k) = k^4 - k^2(k)^2 + 2k - k = k^4 - k^4 + k = k$$

So, the remainder is k .

23. Let $p(x) = 3x^2 + kx + 6$ be the given polynomial. As, $(x+3)$ is a factor of $p(x)$, therefore $p(-3) = 0$.

$$\Rightarrow 3(-3)^2 + k(-3) + 6 = 0 \Rightarrow 27 - 3k + 6 = 0$$

$$\Rightarrow 33 - 3k = 0 \Rightarrow k = 11$$

24. Let $p(x) = 4x^2 - bx - ca$.

As $(x-a)$ is a factor of $p(x)$, therefore $p(a) = 0$.

$$\therefore 4a^2 - ba - ca = 0$$

$$\Rightarrow a(4a - b - c) = 0$$

$$\Rightarrow 4a - b - c = 0 \quad [\because a \neq 0]$$

$$\Rightarrow 4a = b + c$$

$$\Rightarrow a = \frac{b+c}{4}$$

25. Let $p(x) = 2x^3 + 6x + 8$.

In order to prove that $(x+1)$ is a factor of $p(x)$, it is sufficient to show that $p(-1) = 0$.

$$\text{Now, } p(-1) = 2(-1)^3 + 6(-1) + 8 = -2 - 6 + 8 = 0$$

Hence, $(x+1)$ is a factor of the given polynomial.

26. Let $p(x) = x^3 - 3x^2 - 10x + 24$

In order to prove that $(x-2)$, $(x+3)$ and $(x-4)$ are factors of $p(x)$, it is sufficient to show that $p(2)$, $p(-3)$ and $p(4)$ are equal to zero.

$$\text{Now, } p(2) = 2^3 - 3(2)^2 - 10(2) + 24 = 8 - 12 - 20 + 24 = 0,$$

$$p(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24$$

$$= -27 - 27 + 30 + 24 = 0$$

$$\text{And } p(4) = 4^3 - 3(4)^2 - 10(4) + 24 = 64 - 48 - 40 + 24 = 0$$

Hence, $(x-2)$, $(x+3)$ and $(x-4)$ are factors of the given polynomial.

27. Given polynomial is $2r^2 + 5r - 3$

On comparing it with $ax^2 + bx + c$, we get

$$a = 2, b = 5 \text{ and } c = -3$$

Here, $ac = -6$

So, we need to find two numbers whose sum is 5 and product is -6 . One such pair is 6 and (-1) .

$$\text{So, } 2r^2 + 5r - 3 = 2r^2 + (6-1)r - 3 = 2r^2 + 6r - r - 3$$

$$= 2r(r+3) - 1(r+3) = (2r-1)(r+3)$$

28. Given polynomial is $x^2 + 3\sqrt{3}x + 6$

On comparing it with $ax^2 + bx + c$, we get

$$a = 1, b = 3\sqrt{3} \text{ and } c = 6$$

Here, $ac = 6$.

So, we need to find two numbers whose sum is $3\sqrt{3}$ and product is 6. One such pair is $\sqrt{3}$ and $2\sqrt{3}$.

$$\text{So, } x^2 + 3\sqrt{3}x + 6 = x^2 + (\sqrt{3} + 2\sqrt{3})x + 6$$

$$= x^2 + \sqrt{3}x + 2\sqrt{3}x + 6 = x(x + \sqrt{3}) + 2\sqrt{3}(x + \sqrt{3})$$

$$= (x + 2\sqrt{3})(x + \sqrt{3})$$

So, factors of $x^2 + 3\sqrt{3}x + 6$ are $(x + 2\sqrt{3})$ and $(x + \sqrt{3})$.

29. Area = $16a^2 - 32a + 15$

$$= 16a^2 - 20a - 12a + 15 \quad (\text{By splitting the middle term})$$

$$= 4a(4a-5) - 3(4a-5) = (4a-3)(4a-5)$$

$$[\text{Here, } 4a-3 > 0 \text{ and } 4a-5 > 0 \text{ because } a > \frac{5}{4}]$$

= (Length) \times (Breadth)

$$\therefore \text{Length} = 4a - 3$$

$$\text{and Breadth} = 4a - 5$$

$$(\because 4a - 3 > 4a - 5).$$

30. Let $p(y) = 15y^2 - 8y + 1$

$$= 15\left(y^2 - \frac{8}{15}y + \frac{1}{15}\right) = 15q(y), \text{ (say)}$$

$$\text{where } q(y) = y^2 - \frac{8}{15}y + \frac{1}{15}$$

Now, factors of 1 are ± 1 and factors of 15 are $\pm 1, \pm 3, \pm 5, \pm 15$. So, some possibilities for the zeroes of $q(y)$ are $\pm 1, \pm \frac{1}{3}, \pm \frac{1}{5}, \pm \frac{1}{15}$.

Now, we find that

$$q\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^2 - \frac{8}{15}\left(\frac{1}{3}\right) + \frac{1}{15} = \frac{1}{9} - \frac{8}{45} + \frac{1}{15} = \frac{5-8+3}{45} = 0$$

$$\text{and } q\left(\frac{1}{5}\right) = \left(\frac{1}{5}\right)^2 - \frac{8}{15}\left(\frac{1}{5}\right) + \frac{1}{15} \\ = \frac{1}{25} - \frac{8}{75} + \frac{1}{15} = \frac{3-8+5}{75} = 0$$

\therefore By factor theorem, $\left(y - \frac{1}{3}\right)$ and $\left(y - \frac{1}{5}\right)$ are the factors of $q(y)$

$$\text{Hence, } p(y) = 15\left(y - \frac{1}{3}\right)\left(y - \frac{1}{5}\right) \\ = 15\left(\frac{3y-1}{3}\right)\left(\frac{5y-1}{5}\right) = (3y-1)(5y-1)$$

31. Let $p(x) = x^2 - 22x + 120$

Now, if $p(x) = (x - \alpha)(x - \beta)$, we know that constant term will be $\alpha\beta = 120$. So, look for the factors of 120.

Some of these are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 20$

$$\text{Now, } p(10) = (10)^2 - 22(10) + 120 = 0$$

$\therefore (x - 10)$ is a factor of $p(x)$.

Now to find, the other factor, divide $p(x)$ by $(x - 10)$.

$$\begin{array}{r} x-10 \overline{) x^2 - 22x + 120} \\ \underline{x^2 - 10x} \\ -12x + 120 \\ \underline{-12x + 120} \\ + - \\ 0 \end{array}$$

So, other factor of $p(x) = (x - 12)$

$$\text{Hence } p(x) = (x - 12)(x - 10).$$

32. Since, $(2x + 3)$ is a factor of the given polynomial, therefore let us divide $4x^3 + 12x^2 + 5x - 6$ by $2x + 3$ to get the other factors.

$$\begin{array}{r} 2x+3 \overline{) 4x^3 + 12x^2 + 5x - 6} \\ \underline{4x^3 + 6x^2} \\ 6x^2 + 5x - 6 \\ \underline{6x^2 + 9x} \\ -4x - 6 \\ \underline{-4x - 6} \\ + + \\ 0 \end{array}$$

$$\therefore 4x^3 + 12x^2 + 5x - 6 = (2x + 3)(2x^2 + 3x - 2)$$

Now, we will factorise $2x^2 + 3x - 2$ to find the other two factors, by splitting its middle term.

$$\therefore 2x^2 + 3x - 2 = 2x^2 + 4x - x - 2 \\ = 2x(x + 2) - 1(x + 2) = (2x - 1)(x + 2)$$

$$\text{Hence, } 4x^3 + 12x^2 + 5x - 6 = (2x + 3)(2x - 1)(x + 2)$$

33. Let $p(x) = x^3 - 6x^2 + 3x + 10$

All possible factors of 10 are $\pm 1, \pm 2, \pm 5$ and ± 10 .

$$\text{Now, we find that } p(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10 \\ = -1 - 6 - 3 + 10 = 0,$$

$$p(2) = 2^3 - 6(2)^2 + 3(2) + 10 = 8 - 24 + 6 + 10 = 0 \text{ and}$$

$$p(5) = 5^3 - 6(5)^2 + 3(5) + 10 = 125 - 150 + 15 + 10 = 0$$

So, by factor theorem, $(x + 1), (x - 2)$ and $(x - 5)$ are the factors of $p(x)$.

34. Let $p(t) = 2t^3 - 5t^2 - 19t + 42$

$$= 2\left[t^3 - \frac{5}{2}t^2 - \frac{19}{2}t + 21\right] = 2q(t), \text{ (say)}$$

$$\text{where } q(t) = t^3 - \frac{5}{2}t^2 - \frac{19}{2}t + 21$$

Factors of 21 are $\pm 1, \pm 3, \pm 7$ and ± 21

$$\text{Now, we can find that } q(-3) = (-3)^3 - \frac{5}{2}(-3)^2 - \frac{19}{2}(-3) + 21 \\ = -27 - \frac{45}{2} + \frac{57}{2} + 21 = 0$$

So, $(t + 3)$ is one of the factor of $q(t)$. Let us divide $q(t)$ by $(t + 3)$ to find other factors.

$$\begin{array}{r} t+3 \overline{) t^3 - \frac{5}{2}t^2 - \frac{19}{2}t + 21} \phantom{(t^2 - \frac{11}{2}t + 7)} \\ \underline{t^3 + 3t^2} \\ -\frac{11}{2}t^2 - \frac{19}{2}t + 21 \\ \underline{-\frac{11}{2}t^2 - \frac{33}{2}t} \\ + + \\ 7t + 21 \\ \underline{7t + 21} \\ 0 \end{array}$$

$$\text{So, } q(t) = (t + 3)\left(t^2 - \frac{11}{2}t + 7\right)$$

$$\text{Now, } t^2 - \frac{11}{2}t + 7 = t^2 - \frac{7}{2}t - 2t + 7$$

$$= t\left(t - \frac{7}{2}\right) - 2\left(t - \frac{7}{2}\right) = \left(t - \frac{7}{2}\right)(t - 2)$$

$$\text{So, } p(t) = 2(t + 3)\left(t - \frac{7}{2}\right)(t - 2)$$

$$= (t + 3)(2t - 7)(t - 2)$$

Short cut method : We have $p(t) = 2t^3 - 5t^2 - 19t + 42$

By hit and trial method, we have

$$p(2) = 2(2)^3 - 5(2)^2 - 19(2) + 42 = 16 - 20 - 38 + 42 = 0$$

$\therefore (t - 2)$ is one of the factors of $p(t)$.

$$\text{So, } 2t^3 - 5t^2 - 19t + 42 = 2t^2(t - 2) - t(t - 2) - 21(t - 2)$$

$$= (t - 2)[2t^2 - t - 21] = (t - 2)(2t^2 - 7t + 6t - 21)$$

$$= (t - 2)[t(2t - 7) + 3(2t - 7)] = (t - 2)(2t - 7)(t + 3)$$

$$\begin{aligned} \mathbf{35.} \quad (2x - 3y)(2x - 3y) &= (2x - 3y)^2 \\ &= (2x)^2 - 2(2x)(3y) + (3y)^2 \quad [\because (x - y)^2 = x^2 - 2xy + y^2] \\ &= 4x^2 - 12xy + 9y^2 \end{aligned}$$

36. We have, $101 \times 103 = (100 + 1) \times (100 + 3)$

$$= (100)^2 + (1 + 3)(100) + (1)(3)$$

$$[\because (x + a)(x + b) = x^2 + (a + b)x + ab]$$

$$= 10000 + 400 + 3 = 10403$$

37. (i) We have, $0.54 \times 0.54 - 0.46 \times 0.46$
 $= (0.54)^2 - (0.46)^2 = (0.54 + 0.46)(0.54 - 0.46) = 1 \times 0.08 = 0.08.$

$[\because x^2 - y^2 = (x - y)(x + y)]$

(ii) We have, $(0.99)^2 = (1 - 0.01)^2$
 $= (1)^2 - 2 \times 1 \times 0.01 + (0.01)^2 = 1 - 0.02 + 0.0001$

$[\because (x - y)^2 = x^2 - 2xy + y^2]$

$= 1.0001 - 0.02 = 0.9801$

38. We have, $(3x + 2y)^2 = (3x)^2 + (2y)^2 + 2 \times 3x \times 2y$

$[\because (x + y)^2 = x^2 + 2xy + y^2]$

$\Rightarrow (3x + 2y)^2 = 9x^2 + 4y^2 + 12xy$

$\Rightarrow 12^2 = 9x^2 + 4y^2 + 12 \times 6$

$[\because 3x + 2y = 12 \text{ and } xy = 6 \text{ (Given)}]$

$\Rightarrow 144 = 9x^2 + 4y^2 + 72$

$\Rightarrow 144 - 72 = 9x^2 + 4y^2 \Rightarrow 9x^2 + 4y^2 = 72$

39. We have, $x - \frac{1}{x} = -1$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = (-1)^2$ [On squaring both sides]

$\Rightarrow x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 1$ $[\because (x - y)^2 = x^2 - 2xy + y^2]$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 1 \Rightarrow x^2 + \frac{1}{x^2} = 1 + 2 = 3$

40. $49a^2 + 70ab + 25b^2 = (7a)^2 + 2(7a)(5b) + (5b)^2$

$= (7a + 5b)^2$ $[\because x^2 + 2xy + y^2 = (x + y)^2]$

$= (7a + 5b)(7a + 5b)$

41. (i) We have, $(x - 2y - 3z)^2$

$= \{x + (-2y) + (-3z)\}^2$

$= x^2 + (-2y)^2 + (-3z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-3z)$
 $+ 2 \times (-3z) \times x$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx]$

$= x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6zx$

(ii) We have, $(-x + 2y + z)^2 = \{(-x) + 2y + z\}^2$

$= (-x)^2 + (2y)^2 + z^2 + 2 \times (-x)(2y) + 2 \times 2y \times z + 2 \times (-x) \times z$

$= x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx$

42. We have, $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

$= (3x)^2 + (2y)^2 + (-4z)^2 + 2(3x)(2y) + 2(2y)(-4z) + 2(-4z)(3x)$

$= (3x + 2y - 4z)^2 = (3x + 2y - 4z)(3x + 2y - 4z)$

43. (i) We have, $21^3 - 15^3$

$= (21 - 15)\{(21)^2 + 21 \times 15 + (15)^2\}$

$[\because x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$

$= 6 \times (441 + 315 + 225) = 6 \times 981 = 5886$

(ii) We have, $(999)^3 = (1000 - 1)^3$

$= (1000)^3 - (1)^3 - 3(1000)(1)(1000 - 1)$

$[\because (x - y)^3 = x^3 - y^3 - 3xy(x - y)]$

$= 1000000000 - 1 - 3000000 + 3000 = 997002999$

44. We have, $8x^3 + y^3 + 27z^3 - 18xyz$

$= (2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$

$= (2x + y + 3z)\{(2x)^2 + (y)^2 + (3z)^2 - (2x)(y) - (y)(3z) - (3z)(2x)\}$

$[\because x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)]$

$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6zx)$

45. We have, $8x^3 - (2x - y)^3 = (2x)^3 - (2x - y)^3$

$= [2x - (2x - y)][(2x)^2 + 2x \times (2x - y) + (2x - y)^2]$

$= y[4x^2 + 4x^2 - 2xy + 4x^2 + y^2 - 4xy]$

$= y[12x^2 - 6xy + y^2]$

