Heron's Formula

CHAPTER 12

TRY YOURSELF

SOLUTIONS

1. Let the sides of the triangle be $3x$, $5x$ and $7x$.
We are given that
Perimeter of the triangle = 300 m

$\Rightarrow$ $3x + 5x + 7x = 300 \Rightarrow 15x = 300 \Rightarrow x = 20$

$\therefore$ The lengths of the three sides are 3 × 20 m, 5 × 20 m and 7 × 20 m *i.e.*, 60 m, 100 m, 140 m.

$\therefore$ Semi-perimeter, $s = \frac{300}{2}\text{ m} = 150\text{ m}$

Area of the triangle

$= \sqrt{150 \times (150-60) \times (150-100) \times (150-140)}$

$= \sqrt{150 \times 90 \times 50 \times 10} = \sqrt{15 \times 9 \times 5 \times 10000}$

$= 15 \times 100 \times \sqrt{3} = 1500\sqrt{3}\text{ m}^2$

2. Let each side of the triangle be a.

$\therefore$ Perimeter = $3a$ = 60 cm $\Rightarrow$ a = 20 cm

$\therefore$ Semi-perimeter, $s = \frac{60}{2} = 30\text{ cm}$

Area of triangle $= \sqrt{s(s-a)(s-a)(s-a)}$

$= \sqrt{30(30-20)(30-20)(30-20)}$

$= \sqrt{30 \times 10 \times 10 \times 10} = 100\sqrt{3}\text{ cm}^2$

3. Sides of triangle are a = 15 cm, b = 15 cm and c = 12 cm

$\therefore$ Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{15+15+12}{2} = 21\text{ cm}$

$\therefore$ Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-15)(21-12)}$

$= \sqrt{21 \times 6 \times 6 \times 9} = 18\sqrt{21}\text{ cm}^2$

4. Let $AB = c$ = 60 m, $BC = a$ = 56 m and $AC = b$ = 52 m.

$\therefore$ Semi-perimeter, $s = \frac{a+b+c}{2}$

$= \frac{60+56+52}{2} = \frac{168}{2} = 84\text{ m}$

Area of triangular park $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{84(84-56)(84-52)(84-60)}$

$= \sqrt{84 \times 28 \times 32 \times 24} = \sqrt{12 \times 7 \times 12 \times 2 \times 7 \times 4 \times 16 \times 2}$

$= 12 \times 7 \times 4 \times 4 = 1344\text{ m}^2$

Also, area of park $= \frac{1}{2} \times BC \times AP$

$\Rightarrow$ $1344 = \frac{1}{2} \times 56 \times AP$ $\Rightarrow$ $AP = \frac{1344}{28} = 48\text{ m}.$

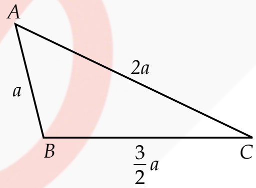
$\therefore$ The distance between the lamp posts at A and P is 48 m.

5. Let AB be the shortest side.

$\therefore$ $AB = a$ units, $BC = \frac{3}{2}a$ units and $AC = 2a$ units

$\therefore$ Semi-perimeter, $s = \frac{AB+BC+CA}{2}$

$= \frac{a + \frac{3}{2}a + 2a}{2} = \frac{9}{4}a$ units



Now, $s-(AB) = \frac{9}{4}a - a = \frac{5}{4}a$

$s-(BC) = \frac{9}{4}a - \frac{3}{2}a = \frac{3}{4}a$

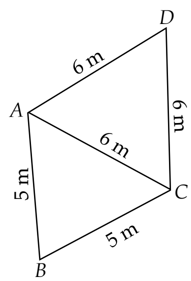
$s-(CA) = \frac{9}{4}a - 2a = \frac{1}{4}a$

$\therefore$ Area of $\Delta ABC = \sqrt{\frac{9}{4}a \times \frac{5}{4}a \times \frac{3}{4}a \times \frac{1}{4}a}$

$= \sqrt{\frac{9 \times 3 \times 5}{4 \times 4 \times 4 \times 4}a^4} = \frac{3\sqrt{15}}{4 \times 4}a^2$

$= \frac{3\sqrt{15}}{16}a^2$ sq. units

6. Diagonal AC divides the quadrilateral $ABCD$ into two triangles ΔACD and ΔABC.



For ΔACD, a = 6 m, b = 6 m, c = 6 m.

Semi-perimeter, $s = \frac{6+6+6}{2} = 9\text{ m}$

Area of ΔACD

$= \sqrt{9 \times (9-6) \times (9-6) \times (9-6)} = 9\sqrt{3}\text{ m}^2$

For ΔABC, a = 5 m, b = 5 m and c = 6 m.

Semi-perimeter, $s = \frac{5+5+6}{2} = 8\text{ m}$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

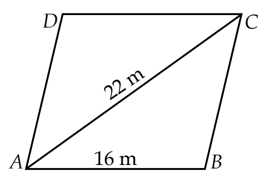
$= \sqrt{8(8-5)(8-5)(8-6)}$

$= \sqrt{8 \times 3 \times 3 \times 2} = \sqrt{16 \times 9} = 12\text{ m}^2$

Thus, the area of the quadrilateral $ABCD$
= Area of ΔABC + Area of ΔACD

$= (12 + 9\sqrt{3})\text{m}^2 = 3(4 + 3\sqrt{3})\text{m}^2$

7. Let $ABCD$ be the rhombus having sides



$AB = BC = CD = DA = a$

Perimeter of rhombus = 64 m

$\Rightarrow$ $4a = 64\text{ m} \Rightarrow a = 16\text{ m}$

In ΔABC,

a = 16 m, b = 16 m, c = 22 m

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{16+16+22}{2} = \frac{54}{2} = 27\text{m}$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{27(27-16)(27-16)(27-22)}$

$= \sqrt{27\times 11\times 11\times 5} = 33\sqrt{15}\,\text{m}^2$

Since diagonal divides the rhombus into two equal triangles.

$\therefore$ Area of rhombus $ABCD$ = 2 × Area of ΔABC

$= 2\times 33\sqrt{15} = 66\sqrt{15}\,\text{m}^2$

8. In a conical tent, pieces are in the form of isosceles triangle, whose sides are 6 m, 13 m, 13 m.

Semi-perimeter of triangle, $s = \frac{6+13+13}{2} = \frac{32}{2} = 16$ m

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

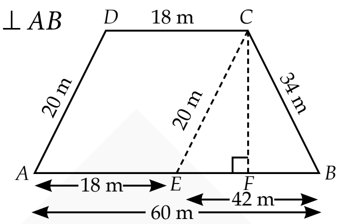
$= \sqrt{16(16-6)(16-13)(16-13)}$

$= \sqrt{16\times 10\times 3\times 3} = 12\sqrt{10}\ \text{m}^2$

Thus, area of one red colour triangular piece is $12\sqrt{10}\ \text{m}^2$. Since, tent is made of 14 pieces of two colours. So, number of triangular pieces of red colour is 7.

Hence, cloth of red colour required to make the conical tent = $7\times 12\sqrt{10} = 84\sqrt{10}\ \text{m}^2$.

9. Through C, Draw $CF \perp AB$ and $CE \parallel AD$



$\therefore$ AE = 18 m

and CE = 20 m

and BE = 42 m

In ΔBCE, a = 20 m, b = 34 m, c = 42 m

semi-perimeter $s = \frac{20+34+42}{2} = \frac{96}{2} = 48$ m

Now, area of $\Delta EBC = \sqrt{48(48-20)(48-34)(48-42)}$

$= \sqrt{48\times 28\times 14\times 6} = 336\ \text{m}^2$

Also, area of $\Delta BEC = \frac{1}{2}$ (base × height)

$\Rightarrow \frac{1}{2} \times 42 \times CF = 336 \Rightarrow 21 \times CF = 336$

$\Rightarrow CF = \frac{336}{21} = 16$ m

Now, area of parallelogram $AECD = AE \times CF$

= 18 × 16 = 288 m^2

$\therefore$ Area of ground = Area of ΔBCE + Area of parallelogram $AECD$ = 336 + 288 = 624 m^2

$\therefore$ Cost of cementing the ground at ₹ 15 per m^2

= ₹ 15 × 624

= ₹ 9360