Circles

CHAPTER 10

TRY YOURSELF SOLUTIONS

1. (i) True. Radius of circle always lies inside the circle.
(ii) True, An arc is the connected section of circumference of the circle.

2. (i) Diameter = 2 × radius of a circle

(ii) Circle having the same centre and different radii are called concentric circles.
(iii) Point A is neither interior point nor exterior point of the circle. It lies on the circle.

3. Let r be the radius of circle.
In ΔAOC,

$OA = OC = r$ [Radii of same circle]

$\therefore \angle OAC = \angle OCA = x$ ...(i) [$\because$ Angles opposite to equal sides of a triangle are equal]

$\because$ BOC is a straight line.

$\therefore \angle AOC + \angle AOB = 180°$

$\Rightarrow \angle AOC = 180° - 70°$

$\Rightarrow \angle AOC = 110°$

Thus, angle subtended by chord AC at centre O, $\angle AOC = 110°$

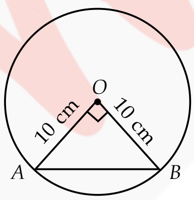
Now, in ΔAOC, $\angle OAC + \angle OCA + \angle AOC = 180°$

$x + x + \angle AOC = 180°$ [Using (i)]

$\Rightarrow 2x = 180° - 110°$

$\Rightarrow 2x = 70° \Rightarrow x = 35°$

4. Let AB be the chord of a circle which makes a right angle at centre O.



Radius of circle = 10 cm [Given]

$\therefore OA = OB = 10$ cm ...(i)

Now, in right ΔOAB, we have

$AB^2 = OA^2 + OB^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = (10)^2 + (10)^2$ [Using (i)]

$\Rightarrow AB^2 = 100 + 100$

$\Rightarrow AB = \sqrt{200} \Rightarrow AB = 10\sqrt{2}$ cm

Hence, length of chord of circle is $10\sqrt{2}$ cm.

5. Given, $AB = BC = CA$
We know that, equal chords of a circle subtend equal angles at the centre.

$\therefore \angle AOB = \angle BOC = \angle AOC$ ...(i)

Now, $\angle AOB + \angle BOC + \angle AOC = 360°$ [$\because$ Sum of angles at a point is 360°]

$\Rightarrow 3\angle AOB = 360°$ [Using (i)]

$\Rightarrow \angle AOB = \frac{360°}{3} = 120°$

Hence, angle subtended by the chords AB, BC and CA at the centre O is 120°.

6. Given, $CD = DE = EF = FG$
We know that, equal chords of a circle subtend equal angles at the centre.

$\therefore \angle COD = \angle DOE = \angle EOF = \angle FOG = 40°$ ...(i)

Now, $\angle COG = \angle COD + \angle DOE + \angle EOF + \angle FOG$

$\Rightarrow \angle COG = 4 \times \angle COD = 4 \times 40°$

$\Rightarrow \angle COG = 160°$

$\therefore$ Reflex $\angle COG = 360° - 160° = 200°$

7. Given, radius $(OB) = 5$ cm, $OC = 3$ cm and $OC \perp AB$.
Now, in right angled ΔOCB,

$OB^2 = OC^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + BC^2$

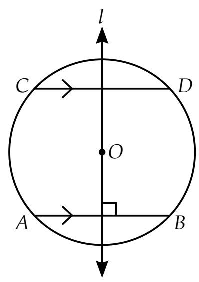
$\Rightarrow BC^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow BC = 4$ cm [$\because BC \neq -4$, as length can't be negative]

We know that, the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore AB = 2BC = 2 \times 4 = 8$ cm

8. We know that the perpendicular bisector of any chord of a circle always passes through the centre of the circle. Since, l is the perpendicular bisector of AB. Therefore, l passes through the centre, O of the circle.
But, $l \perp AB$ and $AB \parallel CD \Rightarrow l \perp CD$.
Thus, $l \perp CD$ and passes through the centre, O of the circle. So, l is the perpendicular bisector of CD also.

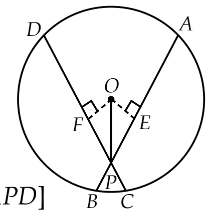


9. Let there be two circles which intersect at three points say at A, B and C. Clearly, A, B and C are not collinear. We know that through three non-collinear points A, B and C one and only one circle can pass. Therefore, there cannot be two circles passing through A, B and C. In other words, the two circles cannot intersect at more than two points.

10. Given, PQ and RS are two chords of a circle having centre at O and $ON = 4$ cm.
Since, equal chords of a circle are equidistant from the centre.

$\therefore OM = ON = 4$ cm

11. Draw $OE \perp AB$ and $OF \perp CD$.



In ΔOEP and ΔOFP, we have

$\angle OEP = \angle OFP$ [Each equal 90°]

$OP = OP$ [Common]

and $\angle OPE = \angle OPF$ [$\because OP$ bisects $\angle APD$]

$\therefore \Delta OEP \cong \Delta OFP$ [By AAS congruency criteria]

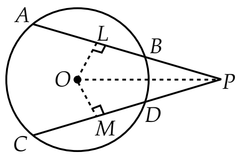
$\Rightarrow\quad OE = OF$ [By C.P.C.T.]

Thus, chords AB and CD are equidistant from the centre O of the circle.

But, chords of a circle which are equidistant from the centre are equal.

$\therefore\quad AB = CD$

12. **Given :** AB and CD are two equal chords of a circle intersecting at a point P.



To prove : $PB = PD$

Construction : Join OP, draw $OL \perp AB$ and $OM \perp CD$

Proof : We have, $AB = CD$

$\Rightarrow\quad OL = OM$ ...(i)

[$\because$ Equal chords of a circle are equidistant from the centre]

Now, in ΔOLP and ΔOMP,

$OL = OM$ [From (i)]

$\angle OLP = \angle OMP$ [Each equal to 90°]

$OP = OP$ [Common]

$\therefore\ \Delta OLP \cong \Delta OMP$ [By RHS congruency criteria]

$\Rightarrow\quad LP = MP$ [By C.P.C.T.]... (ii)

Also, $AB = CD$ [Given]

$\Rightarrow \frac{1}{2}(AB) = \frac{1}{2}(CD)$

$\Rightarrow\quad BL = DM$ ... (iii)

[$\because$ The perpendicular drawn from the centre of a circle bisects the chord.]

On subtracting (iii) from (ii), we get

$LP - BL = MP - DM$

$\Rightarrow\quad PB = PD$

13. In ΔOAB, $OA = OB$ [Radii of same circle]

$\therefore\quad \angle OBA = \angle OAB = 40°$

[$\because$ Angles opposite to equal sides of a triangle are equal]

Also, $\angle AOB + \angle OBA + \angle OAB = 180°$

[$\because$ Sum of angles of a triangle is 180°]

$\therefore\quad \angle AOB + 40° + 40° = 180°$

$\Rightarrow\quad \angle AOB = 180° - 80° = 100°$

Since, the angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of the circle.

$\therefore\quad \angle AOB = 2\angle ACB \Rightarrow 100° = 2\angle ACB$

$\therefore\quad \angle ACB = 50°$

14. Given, $\angle AOC = 130°$

Reflex $\angle AOC = 360° - \angle AOC$

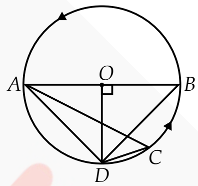
$\Rightarrow$ Reflex $\angle AOC = 360° - 130° = 230°$

We know that, the angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of the circle.

$\therefore\quad \angle ABC = \frac{1}{2}$ (Reflex $\angle AOC$)

$= \frac{1}{2} \times 230° = 115°$

15. Given, circle $C(O, r)$ and $OD \perp AB$.



$\therefore\quad \angle AOD = \angle BOD = 90°$

We know, angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of circle.

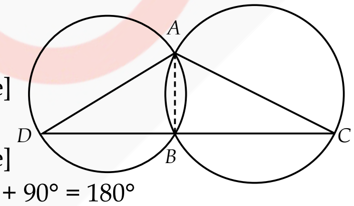
So, $\angle BOD = 2\angle BAD$

$\Rightarrow\quad \angle BAD = \frac{1}{2}\angle BOD = \frac{1}{2} \times 90° = 45°$

Similarly, $\angle AOD = 2\angle ACD$

$\Rightarrow\quad \angle ACD = \frac{1}{2}\angle AOD = \frac{1}{2} \times 90° = 45°$

16. Join AB.



$\angle ABD = 90°$

[Angle in a semi-circle]

$\angle ABC = 90°$

[Angle in a semi-circle]

So, $\angle ABD + \angle ABC = 90° + 90° = 180°$

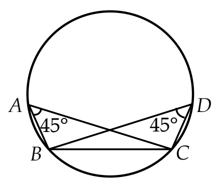
Therefore, DBC is a straight line. Thus, B lies on the line segment DC.

17. $\angle ACB = \angle BDA$ [$\because$ Angles in the same segment.]

But, $\angle ACB = 40°$ [Given]

$\Rightarrow\quad y = 40°$

18. True



Given, $\angle BAC = 45°$ and $\angle BDC = 45°$, which shows that angles in the same segment of a circle are equal. Thus, A, B, C and D are concyclic.

19. Given, $ED \parallel AC$, $\angle CBE = 50°$

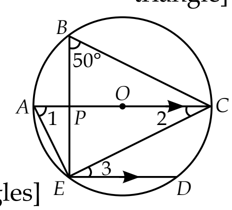
$\angle CBE = \angle 1$ [Angles in the same segment]

$\therefore\quad \angle 1 = 50°$ ...(i) ($\because\ \angle CBE = 50°$)

$\angle AEC = 90°$ ...(ii) [Angle in a semi-circle]

Now, in ΔAEC,

$\angle 1 + \angle AEC + \angle 2 = 180°$ [By Angle sum property of a triangle]



$\Rightarrow\quad 50° + 90° + \angle 2 = 180°$

[From (i) and (ii)]

$\Rightarrow\quad \angle 2 = 180° - 140°$

$\Rightarrow\quad \angle 2 = 40°$ ...(iii)

Now, $ED \parallel AC$ [Given]

$\therefore\quad \angle 2 = \angle 3$ [Alternate interior angles]

$\therefore\quad \angle 3 = 40°$ *i.e.*, $\angle CED = 40°$

20. Since, MAB is a straight line.

$\therefore\quad \angle MAD + \angle DAB = 180°$

$\Rightarrow\quad \angle DAB = 180° - \angle MAD = 180° - 110°$

$\therefore\quad \angle DAB = 70°$

Since, $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle BAD + \angle BCD = 180°$

$\Rightarrow \quad \angle BCD = 180° - \angle BAD = 180° - 70° \Rightarrow \angle BCD = 110°$

Now, DCN is a straight line.

$\therefore \quad \angle DCB + \angle BCN = 180°$

$\Rightarrow \quad \angle BCN = 180° - \angle DCB = 180° - 110°$

$\therefore \quad \angle BCN = 70°$

21. Since, PSY is a straight line.

$\therefore \quad \angle PSR + \angle RSY = 180°$

$\Rightarrow \quad \angle PSR = 180° - \angle RSY = 180° - 74°$

$\therefore \quad \angle PSR = 106°$ ...(i)

Now, Reflex $\angle POR = 2 \times \angle PSR$

[$\because$ Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$\therefore$ Reflex $\angle POR = 2 \times 106°$ [Using (i)]

$= 212°$

22. Given : $ABCD$ is a parallelogram. A circle, whose centre O, passes through A, B is so drawn that it intersects AD at P and BC at Q.

To prove : P, Q, C and D are concyclic.

Construction : Join PQ.

Proof : $\because$ A, P, Q and B are four points lying on a circle.

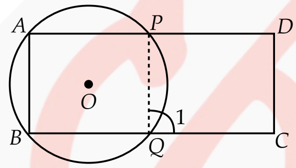
$\therefore \quad APQB$ is a cyclic quadrilateral.

$\angle 1 = \angle A$ [Exterior angle property of a cyclic quadrilateral]

But $\angle A = \angle C$ [Opposite angles of parallelogram $ABCD$]

$\therefore \quad \angle 1 = \angle C$ ... (i)

But $\angle C + \angle D = 180°$ [Sum of co-interior angles is 180°]



$\Rightarrow \quad \angle 1 + \angle D = 180°$ [From (i)]

Thus, the quadrilateral $QCDP$ is cyclic.

So, the points P, Q, C and D are concyclic.