Number Systems

CHAPTER 1

TRY YOURSELF

SOLUTIONS

1. A rational number lying between $\frac{1}{3}$ and $\frac{1}{5}$

$=\left(\frac{1}{3}+\frac{1}{5}\right)\div 2=\left(\frac{5+3}{15}\right)\times\frac{1}{2}=\frac{8}{15}\times\frac{1}{2}=\frac{4}{15}$

Thus, the required rational number between $\frac{1}{3}$ and $\frac{1}{5}$ is $\frac{4}{15}$ *i.e.,* $\frac{1}{3}>\frac{4}{15}>\frac{1}{5}$.

2. Let $x = 4$, $y = 5$ and $n = 5$

Since $x < y$

$\therefore \quad d=\frac{y-x}{n+1}=\frac{(5-4)}{(5+1)}=\frac{1}{6}$

Thus, five rational numbers between 4 and 5 are $(x + d)$, $(x + 2d)$, $(x + 3d)$, $(x + 4d)$ and $(x + 5d)$.

i.e., $\left(4+\frac{1}{6}\right),\left(4+\frac{2}{6}\right),\left(4+\frac{3}{6}\right),\left(4+\frac{4}{6}\right)$ and $\left(4+\frac{5}{6}\right)$

i.e., $\frac{25}{6},\frac{26}{6},\frac{27}{6},\frac{28}{6}$ and $\frac{29}{6}$

i.e., $4<\frac{25}{6}<\frac{26}{6}<\frac{27}{6}<\frac{28}{6}<\frac{29}{6}<5$.

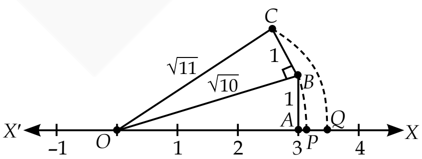
3. Here, $\frac{-1}{6}<\frac{-1}{7}$ or $\frac{-7}{42}<\frac{-6}{42}$ and $n = 3$.

Since, we need to find three rational numbers between $\frac{-7}{42}$ and $\frac{-6}{42}$, so multiply the numerator and denominator by $(3 + 1) = 4$ of $\frac{-7}{42}$ and $\frac{-6}{42}$ *i.e.,* $\frac{-7}{42}=\frac{-7\times 4}{42\times 4}=\frac{-28}{168}$ and $\frac{-6}{42}=\frac{-6\times 4}{42\times 4}=\frac{-24}{168}$

Thus, three rational number between $\frac{-1}{6}$ and $\frac{-1}{7}$ or $\frac{-7}{42}$ and $\frac{-6}{42}$ are $\frac{-25}{168},\frac{-26}{168}$ and $\frac{-27}{168}$ such that $\frac{-6}{42}>\frac{-25}{168}>\frac{-26}{168}>\frac{-27}{168}>\frac{-7}{42}$.

4. Draw a number line $X'OX$ and let O be the origin. Take $OA = 3$ units and draw $AB = 1$ unit such that $AB \perp OA$. Join OB. We get,

$OB=\sqrt{OA^2+AB^2}=\sqrt{3^2+1^2}=\sqrt{9+1}=\sqrt{10}$ units



With O as centre and radius $OB = \sqrt{10}$ units draw on arc which intersects OX at P. Then, $OP = OB = \sqrt{10}$ units

Thus, the point P represents $\sqrt{10}$ on the number line.

Now, draw $BC = 1$ unit such that $BC \perp OB$. Join OC. We get,

$OC=\sqrt{OB^2+BC^2}=\sqrt{(\sqrt{10})^2+1^2}=\sqrt{11}$ units

With O as centre and radius $OC = \sqrt{11}$ units draw an arc which intersects OX at Q. Then, $OQ = OC = \sqrt{11}$ units

Thus, the point Q represents $\sqrt{11}$ on the number line.

5. Draw a number line $X'OX$ and let O be the origin. Take $OA = 2$ units and draw $AB = 1$ unit such that $AB \perp OA$. Join OB. We get,

$OB=\sqrt{OA^2+AB^2}=\sqrt{2^2+1^2}=\sqrt{5}$ units

With O as centre and radius, $OB = \sqrt{5}$ units draw an arc which intersects OX at P.

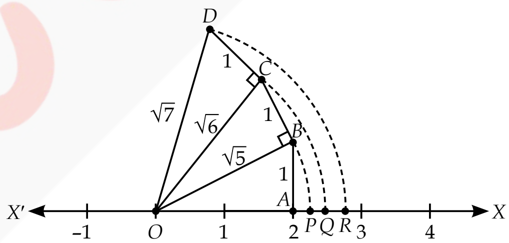
Then, $OP = OB = \sqrt{5}$ units

Thus, the point P represents $\sqrt{5}$ on the number line.

Now, draw $BC = 1$ unit such that $BC \perp OB$. Join OC.

We get, $OC=\sqrt{OB^2+BC^2}=\sqrt{(\sqrt{5})^2+1^2}$

$=\sqrt{5+1}=\sqrt{6}$ units



With O as centre and radius $OC = \sqrt{6}$ units draw an arc which intersects OX at Q. Then, $OQ = OC = \sqrt{6}$ units.

Thus, the point Q represents $\sqrt{6}$ on the number line.

Now, draw $CD = 1$ unit such that $CD \perp OC$. Join OD.

We get, $OD=\sqrt{OC^2+CD^2}=\sqrt{(\sqrt{6})^2+1^2}=\sqrt{7}$ units

With O as centre and radius $OD = \sqrt{7}$ units draw an arc which intersects OX at R. Then, $OR = OD = \sqrt{7}$ units

Thus, the point R represents $\sqrt{7}$ on the number line.

6. We have, 16) 35.0000 (2.1875

$$\begin{array}{r} 35.0000 \\ -32 \\ \hline 30 \\ -16 \\ \hline 140 \\ -128 \\ \hline 120 \\ -112 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

$\therefore \quad \frac{35}{16} = 2.1875$

7. Let $x = 0.1\overline{63}$. Then,

$x = 0.1636363...$ ...(i)

Multiplying (i) by 10 and 1000, we get

$10x = 1.636363...$ ...(ii)

$1000x = 163.636363...$ ...(iii)

Subtracting (ii) and from (iii), we get

$(1000x - 10x) = (163.636363\ ...) - (1.636363\ ...)$

$\Rightarrow \quad 990x = 162 \Rightarrow x = \frac{162}{990} \Rightarrow x = \frac{9}{55}$

Thus, $0.1\overline{63} = \frac{9}{55}$

8. (i) $\frac{96}{300} = \frac{32}{100} = \frac{8}{25} = \frac{2^3}{5^2}$

Since, denominator of $\frac{96}{300}$ is of the form $2^m \times 5^n$. Thus, the given number is terminating decimal.

Verification : On dividing 96 by 300, we get

$$\begin{array}{r|l} 300 & 96.00 \;(0.32 \\ & -900 \\ \hline & 600 \\ & -600 \\ \hline & 0 \end{array}$$

$\frac{96}{300} = 0.32$ is terminating decimal.

(ii) $\frac{169}{91} = \frac{13}{7}$

Since, denominator of $\frac{169}{91}$ is not in the form of $2^m \times 5^n$.

Thus, the given number is non-terminating recurring decimal.

Verification : On dividing 13 by 7, we get

$$\begin{array}{r|l} 7 & 13.0000000 \;(1.8571428... \\ & -7 \\ \hline & 60 \\ & -56 \\ \hline & 40 \\ & -35 \\ \hline & 50 \\ & -49 \\ \hline & 10 \\ & -7 \\ \hline & 30 \\ & -28 \\ \hline & 20 \\ & -14 \\ \hline & 60 \\ & -56 \\ \hline & 4 \end{array}$$

$\therefore \quad \frac{13}{7} = 1.\overline{857142}$

9. We have, $8.0025 = \frac{80025}{10000} = \frac{80025 \div 25}{10000 \div 25} = \frac{3201}{400}$

10. (i) Let $x = 0.\overline{585}$. Then,

$x = 0.585585585\ ...$ ...(i)

Multiplying (i) by 1000, we get

$1000\,x = 585.585585\ ...$ ...(ii)

Subtracting (i) from (ii), we get

$1000x - x = (585.585585\ ...) - (0.585585585\ ...)$

$\Rightarrow \quad 999x = 585 \Rightarrow x = \frac{585}{999} \Rightarrow x = \frac{65}{111}$

Thus, $0.\overline{585} = \frac{65}{111}$

(ii) Let $x = 0.\overline{35}$. Then,

$x = 0.353535\ ...$ ...(i)

Multiplying (i) by 100, we get

$100x = 35.3535\ ...$ ...(ii)

Subtracting (i) from (ii), we get

$100x - x = (35.3535\ ...) - (0.3535\ ...)$

$\Rightarrow \quad 99x = 35 \Rightarrow x = \frac{35}{99}$

Thus, $0.\overline{35} = \frac{35}{99}$

11. Given numbers are 0.60 and 0.66. Thus, the required irrational numbers will lie between 0.60 and 0.66.

Also, the irrational numbers have non-terminating non-repeating decimals. Hence, the irrational numbers between 0.60 and 0.66 are 0.61010010001... and 0.62020020002... .

12. To find an irrational number, firstly we will divide 1 by 7 and 1 by 3.

Now,

$$\begin{array}{r|l} 7 & 1.000000 \;(0.142857... \\ & -7 \\ \hline & 30 \\ & -28 \\ \hline & 20 \\ & -14 \\ \hline & 60 \\ & -56 \\ \hline & 40 \\ & -35 \\ \hline & 50 \\ & -49 \\ \hline & 1 \end{array}$$

$\therefore \quad \frac{1}{7} = 0.142857... = 0.\overline{142857}$

Now,

$$\begin{array}{r|l} 3 & 1.00 \;(0.33... \\ & -9 \\ \hline & 10 \\ & -9 \\ \hline & 1 \end{array}$$

$\therefore \quad \frac{1}{3} = 0.333... = 0.\overline{3}$

Thus, the required irrational number will lie between $0.\overline{142857}$ and $0.\overline{3}$. Also, the irrational numbers have non-terminating non-repeating decimals. Hence, the required irrational number between 1/7 and 1/3 is 0.2101001000... .

13. To find irrational numbers, firstly we will divide 2 by 3 and 8 by 9.

Now, $3\overline{)2.000}(0.666\ldots$

$$\begin{array}{r} -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

$\therefore \quad \frac{2}{3} = 0.666\ldots = 0.\overline{6}$

Now, $9\overline{)8.000}(0.888\ldots$

$$\begin{array}{r} -72 \\ \hline 80 \\ -72 \\ \hline 80 \\ -72 \\ \hline 8 \end{array}$$

$\therefore \quad \frac{8}{9} = 0.888\ldots = 0.\overline{8}$

Thus, the required irrational numbers will lie between $0.\overline{6}$ and $0.\overline{8}$. Also, the irrational numbers have non-terminating non-repeating decimals. Hence, the required irrational numbers between $\frac{2}{3}$ and $\frac{8}{9}$ are $0.6101001000\ldots$ and $0.7101001000\ldots$.

14. 3.671 lies between 3 and 4.

(i) Divide the distance between 3 and 4 into 10 equal parts.

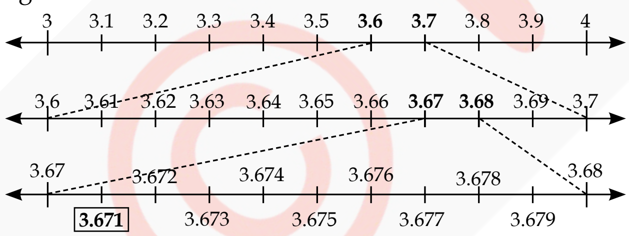
(ii) Mark the point 3.6 which is fourth from the left of 4 and sixth from the right of 3.

(iii) 3.67 lies between 3.6 and 3.7. Now, divide the distance between 3.6 and 3.7 into 10 equal parts.

(iv) Mark the point 3.67 which is seventh from the right of 3.6 and third from the left of the 3.7.

(v) The point 3.671 lies between 3.67 and 3.68.

(vi) Divide the distance between 3.67 and 3.68 into 10 equal parts, mark the point 3.671 which is first from the right of 3.67 and ninth from the left of 3.68.



15. We have, $1.\overline{3} = 1.3333\ldots$ which lies between 1 and 2.

(i) Divide the distance between 1 and 2 into 10 equal parts.

(ii) Mark the point 1.3, which is third from right of 1 and seventh from left of 2.

(iii) 1.33 lies between 1.3 and 1.4. Now, divide the distance between 1.3 and 1.4 into 10 equal parts.

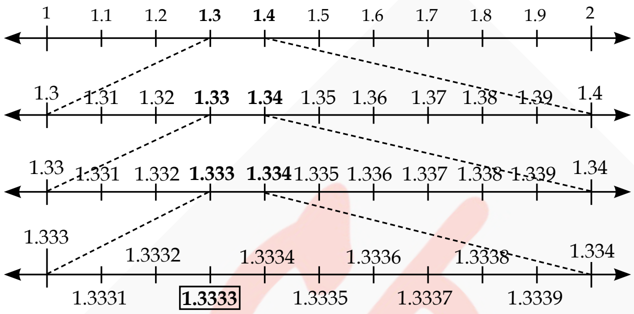
(iv) Mark the point 1.33, which is third from right of 1.3 and seventh from left of 1.4.

(v) 1.333 lies between 1.33 and 1.34. Now, divide the distance between 1.33 and 1.34 into 10 equal parts.

(vi) Mark the point 1.333, which is third from right of 1.33 and seventh from left of 1.34.

(vii) 1.3333 lies between 1.333 and 1.334. Now, divide the distance between 1.333 and 1.334 into 10 equal parts.

(viii) Mark the point 1.3333 which is third from right of 1.333 and seventh from left of 1.334.



16. Number 5.37777 lies between 5 and 6.

(i) Divide the distance between 5 and 6 into 10 equal parts.

(ii) Mark the point 5.3, which is third from the right of 5 and seventh from the left of 6.

(iii) 5.37 lies between 5.3 and 5.4. Now, divide the distance between 5.3 and 5.4 into 10 equal parts.

(iv) Mark the point 5.37, which is seventh from the right of 5.3 and third from the left of 5.4.

(v) 5.377 lies between 5.37 and 5.38. Now, divide the distance between 5.37 and 5.38 into 10 equal parts.

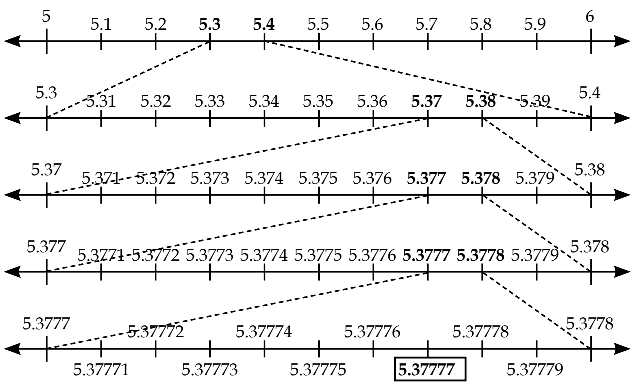
(vi) Mark the point 5.377, which is seventh from the right of 5.37 and third from the left of 5.38.

(vii) 5.3777 lies between 5.377 and 5.378. Now, divide the distance between 5.377 and 5.378 into 10 equal parts.

(viii) Mark the point 5.3777, which is seventh from the right of 5.377 and third from left of 5.378.

(ix) 5.37777 lies between 5.3777 and 5.3778. Now, divide the distance between 5.3777 and 5.3778 into 10 equal parts.

(x) Mark the point 5.37777, which is seventh from right of 5.3777 and third from left of 5.3778.



17. (i) We have, $\frac{8\sqrt{20}}{3\sqrt{20}} = \frac{8}{3}$, which is a rational number.

(ii) Here, 6 is a rational number and $\sqrt{3}$ is an irrational number. Since, we know that division of a rational number and an irrational number always gives an irrational number as quotient.

Hence, $\frac{6}{\sqrt{3}}$ is an irrational number.

(iii) We have, $(3+2\sqrt{7})-(-3+2\sqrt{7})$

$= 3+2\sqrt{7}+3-2\sqrt{7}=6$, which is a rational number.

18. We have, $(2+\sqrt{3})+(2-\sqrt{3})$

$=2+\sqrt{3}+2-\sqrt{3}=4$

19. We have, $\left(2\sqrt{2}+5\sqrt{3}\right)+\left(\sqrt{2}-3\sqrt{3}\right)$

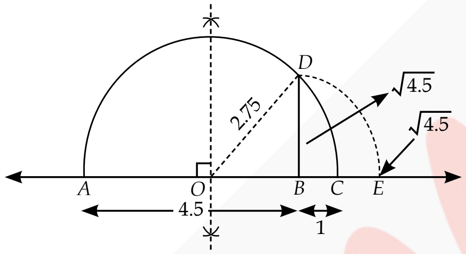
$=\left(2\sqrt{2}+\sqrt{2}\right)+\left(5\sqrt{3}-3\sqrt{3}\right)$

$=(2+1)\sqrt{2}+(5-3)\sqrt{3}=3\sqrt{2}+2\sqrt{3}$

20. We have, $(9\sqrt{7}+72)\div(3\sqrt{7}+24)$

$=\frac{9(\sqrt{7}+8)}{3(\sqrt{7}+8)}=3$

21. Draw a line segment, AB = 4.5 units and extend it to C such that BC = 1 unit and AC = 5.5 units. Mark O as mid-point of AC. Draw a semicircle with centre O and radius $OC = \frac{AC}{2}$ = 2.75 units. Draw $BD \perp AC$ and intersecting the semicircle at D.



In ΔOBD, $BD^2 = OD^2 - OB^2$

$\Rightarrow$ $BD^2 = (2.75)^2 - (1.75)^2 = (2.75 + 1.75)(2.75 - 1.75)$

$\Rightarrow$ $BD=\sqrt{4.5}$ units.

To represent $\sqrt{4.5}$ on the number line, let us treat the line BC as the number line, with B as zero, C as 1, and so on.

Draw an arc with centre B and radius BD = $\sqrt{4.5}$ units, which intersects the number line BC (produced) at E.

$BD = BE = \sqrt{4.5}$ units

$\therefore$ E represents $\sqrt{4.5}$.

22. We have, $(5+\sqrt{7})(2+\sqrt{5})$

$=5\times2+5\times\sqrt{5}+\sqrt{7}\times2+\sqrt{7}\times\sqrt{5}$

$=10+5\sqrt{5}+2\sqrt{7}+\sqrt{7\times5}=10+5\sqrt{5}+2\sqrt{7}+\sqrt{35}$

$[\because \sqrt{x}\cdot\sqrt{y}=\sqrt{xy}]$

23. We have, $(\sqrt{11}-\sqrt{5})^2 = 11+5-2\sqrt{11\times5}$

$\left[\because \left(\sqrt{x}-\sqrt{y}\right)^2 = x+y-2\sqrt{xy}\right]$

$=16-2\sqrt{55}$

24. We have, $\left(9+\sqrt{\frac{3}{2}}\right)\left(9-\sqrt{\frac{3}{2}}\right)$

$=81-\frac{3}{2}$ $\{\because (x+\sqrt{y})(x-\sqrt{y})=x^2-y\}$

$=\frac{162-3}{2}=\frac{159}{2}$

25. Given, $x=2\sqrt{5}+\sqrt{3}$ and $y=2\sqrt{5}-\sqrt{3}$

$\therefore$ $x+y=2\sqrt{5}+\sqrt{3}+2\sqrt{5}-\sqrt{3}=4\sqrt{5}$

Now, $(x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow$ $x^2 + y^2 = (x + y)^2 - 2xy$

$\Rightarrow$ $x^2+y^2=(4\sqrt{5})^2-2\times(2\sqrt{5}+\sqrt{3})(2\sqrt{5}-\sqrt{3})$

$\Rightarrow$ $x^2+y^2=80-2[(2\sqrt{5})^2-(\sqrt{3})^2]$

$[\because (x-y)(x+y)=x^2-y^2]$

$\Rightarrow$ $x^2 + y^2 = 80 - 2[20 - 3] = 80 - 2 \times 17$

$\Rightarrow$ $x^2 + y^2 = 80 - 34 = 46$

26. We have, $\frac{5}{\sqrt{3}-\sqrt{5}}$

$=\frac{5}{\sqrt{3}-\sqrt{5}}\times\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}=\frac{5(\sqrt{3}+\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2}$

$=\frac{5(\sqrt{3}+\sqrt{5})}{3-5}=-\frac{5}{2}(\sqrt{3}+\sqrt{5})$

27. We have, $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

On rationalising, we get

$\frac{(5+2\sqrt{3})}{(7+4\sqrt{3})}\times\frac{(7-4\sqrt{3})}{(7-4\sqrt{3})}=\frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$

$=\frac{35-20\sqrt{3}+14\sqrt{3}-2\sqrt{3}\times4\sqrt{3}}{(7)^2-(4\sqrt{3})^2}$

$=\frac{35-6\sqrt{3}-24}{49-48}=\frac{11-6\sqrt{3}}{1}$

$\therefore$ $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=11-6\sqrt{3}$ ...(i)

Given, $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$

$\Rightarrow$ $11-6\sqrt{3}=a+b\sqrt{3}$ [Using (i)]

On comparing, we get

a = 11 and b = –6

28. Given, $x=(4-\sqrt{15})$

Squaring both sides, we get

$x^2=(4-\sqrt{15})^2 \Rightarrow x^2=(4)^2+(\sqrt{15})^2-2\times4\times\sqrt{15}$

$\Rightarrow$ $x^2 = 16 + 15 - 8\sqrt{15}$

$\Rightarrow$ $x^2 = 31 - 8\sqrt{15}$ ...(i)

$\therefore$ $\frac{1}{x^2}=\frac{1}{31-8\sqrt{15}}$

On rationalising, we get

$\frac{1}{x^2}=\frac{1}{31-8\sqrt{15}}\times\frac{31+8\sqrt{15}}{31+8\sqrt{15}}\Rightarrow\frac{1}{x^2}=\frac{31+8\sqrt{15}}{(31)^2-(8\sqrt{15})^2}$

$\Rightarrow$ $\frac{1}{x^2}=\frac{31+8\sqrt{15}}{961-960}=\frac{31+8\sqrt{15}}{1}=(31+8\sqrt{15})$ ...(ii)

Now, $x^2+\frac{1}{x^2}=(31-8\sqrt{15})+(31+8\sqrt{15})$

[Using (i) and (ii)]

$\Rightarrow$ $x^2+\frac{1}{x^2}=31-8\sqrt{15}+31+8\sqrt{15}$ $\Rightarrow$ $x^2+\frac{1}{x^2}=62$

29. We have, $x = 1 - \sqrt{2}$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{1}{1-\sqrt{2}}$

On rationalising, we get

$$\frac{1}{x} = \frac{1\times(1+\sqrt{2})}{(1-\sqrt{2})\times(1+\sqrt{2})}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1+\sqrt{2}}{(1)^2-(\sqrt{2})^2} = \frac{(1+\sqrt{2})}{1-2} = -(1+\sqrt{2})$$

(i) Now, $x + \dfrac{1}{x} = (1-\sqrt{2}) - (1+\sqrt{2})$

$= 1-\sqrt{2}-1-\sqrt{2} = -2\sqrt{2}$

(ii) Now, $x - \dfrac{1}{x} = (1-\sqrt{2}) + (1+\sqrt{2}) = 2$

(iii) $\left(x+\dfrac{1}{x}\right)^2 = (-2\sqrt{2})^2$ [Using part (i)]

$\Rightarrow \quad x^2 + \dfrac{1}{x^2} + 2\times x\times\dfrac{1}{x} = 8$

$\Rightarrow \quad x^2 + \dfrac{1}{x^2} + 2 = 8 \;\Rightarrow\; x^2 + \dfrac{1}{x^2} = 6$

30. We have, $\dfrac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$

$$= \frac{(5^2)^{3/2}\times(3^5)^{3/5}}{(2^4)^{5/4}\times(2^3)^{4/3}} = \frac{5^{2\times\frac{3}{2}}\times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times 2^{3\times\frac{4}{3}}} \quad [\because\ (x^m)^n = x^{m\times n}]$$

$$= \frac{5^3\times 3^3}{2^5\times 2^4} = \frac{(5\times 3)^3}{2^{5+4}} \quad [\because\ x^m\times y^m = (x\times y)^m\,;\ x^m\times x^n = x^{m+n}]$$

$$= \frac{(15)^3}{2^9} = (15)^3\times 2^{-9}$$

31. We have, L.H.S. $= \dfrac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times 4^{\frac{1}{4}}}{10^{\frac{-1}{5}}\times 5^{\frac{3}{5}}} \div \dfrac{3^{\frac{4}{3}}\times 5^{\frac{-7}{5}}}{4^{\frac{-3}{5}}\times 6}$

$$= \left[\frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times(2^2)^{\frac{1}{4}}}{10^{\frac{-1}{5}}\times 5^{\frac{3}{5}}}\right] \div \left[\frac{3^{\frac{4}{3}}\times 5^{\frac{-7}{5}}}{(2^2)^{\frac{-3}{5}}\times(2\times 3)}\right]$$

$$= \left[\frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times 2^{2\times\frac{1}{4}}}{(2\times 5)^{\frac{-1}{5}}\times 5^{\frac{3}{5}}}\right] \div \left[\frac{3^{\frac{4}{3}}\times 5^{\frac{-7}{5}}}{2^{\frac{-3}{5}\times 2}\times 2\times 3}\right] \quad [\because\ (x^m)^n = x^{m\times n}]$$

$$= \left[\frac{2^{\frac{1}{2}+\frac{1}{2}}\times 3^{\frac{1}{3}}}{2^{\frac{-1}{5}}\times 5^{\frac{-1}{5}}\times 5^{\frac{3}{5}}}\right] \div \left[\frac{3^{\frac{4}{3}-1}\times 5^{\frac{-7}{5}}}{2^{\frac{-6}{5}+1}}\right] \quad \begin{bmatrix}\because x^m\times x^n = x^{m+n}\\ x^m\div x^n = x^{m-n}\end{bmatrix}$$

$$= \left[\frac{2^{1-\left(\frac{-1}{5}\right)}\times 3^{\frac{1}{3}}}{5^{\frac{-1}{5}+\frac{3}{5}}}\right] \div \left[\frac{3^{\frac{1}{3}}\times 5^{\frac{-7}{5}}}{2^{\frac{-1}{5}}}\right] = \left[\frac{2^{\frac{6}{5}}\times 3^{\frac{1}{3}}}{5^{\frac{2}{5}}}\right] \div \left[\frac{3^{\frac{1}{3}}\times 5^{\frac{-7}{5}}}{2^{\frac{-1}{5}}}\right]$$

$$= \frac{2^{\frac{6}{5}}\times 3^{\frac{1}{3}}}{5^{\frac{2}{5}}} \times \frac{2^{\frac{-1}{5}}}{3^{\frac{1}{3}}\times 5^{\frac{-7}{5}}} = \frac{2^{\frac{6}{5}-\frac{1}{5}}\times 3^{\frac{1}{3}-\frac{1}{3}}}{5^{\frac{2}{5}-\frac{7}{5}}}$$

$$= \frac{2^1\times 3^0}{5^{-1}} = 2\times 5 = 10 = \text{R.H.S.}$$

32. Given, $a - b = 0 \Rightarrow \dfrac{2^{x-1}}{2^{x-2}} - \dfrac{2^{-x}}{2^{x+1}} = 0$

$\Rightarrow \quad 2^{x-1-x+2} - 2^{-x-x-1} = 0$ $[\because\ x^m \div x^n = x^{m-n}]$

$\Rightarrow \quad 2^1 - 2^{-2x-1} = 0 \;\Rightarrow\; 2^{-2x-1} = 2^1$

On comparing power of 2, we have

$\Rightarrow \quad -2x - 1 = 1 \Rightarrow x = -1$

33. We have, $\dfrac{6^{2/3}\times\sqrt[3]{6^7}}{\sqrt[3]{6^6}}$

$$= \frac{6^{2/3}\times(6^7)^{1/3}}{(6^6)^{1/3}} = \frac{6^{2/3}\times 6^{7\times 1/3}}{6^{6\times 1/3}} \quad [\because (x^m)^n = x^{m\times n}]$$

$$= \frac{6^{2/3}\times 6^{7/3}}{6^2} = \frac{6^{\frac{2}{3}+\frac{7}{3}}}{6^2} \quad [\because\ x^m\times x^n = x^{m+n}]$$

$$= \frac{6^3}{6^2} = 6^{3-2} \quad \left[\because x^m \div x^n = x^{m-n}\right]$$

$= 6$

34. We have,

$$\text{L.H.S.} = \left(\frac{1}{x^{a-b}}\right)^{\frac{1}{(a-c)}} \cdot \left(\frac{1}{x^{b-c}}\right)^{\frac{1}{(b-a)}} \cdot \left(\frac{1}{x^{c-a}}\right)^{\frac{1}{(c-b)}}$$

$$= (x)^{\frac{1}{(a-b)}\times\frac{1}{(a-c)}} \cdot (x)^{\frac{1}{(b-c)}\times\frac{1}{(b-a)}} \cdot (x)^{\frac{1}{(c-a)}\times\frac{1}{(c-b)}} \quad [\because\ (x^m)^n = x^{m\times n}]$$

$$= (x)^{\frac{1}{-(a-b)(c-a)}} \cdot (x)^{\frac{1}{-(b-c)(a-b)}} \cdot (x)^{\frac{1}{-(c-a)(b-c)}}$$

$$= (x)^{\frac{1}{-(a-b)(c-a)}+\frac{1}{-(b-c)(a-b)}+\frac{1}{-(c-a)(b-c)}} \quad [\because\ x^m\times x^n = x^{m+n}]$$

$$= (x)^{\frac{-(b-c)-(c-a)-(a-b)}{(a-b)(b-c)(c-a)}} = (x)^{\frac{-b+c-c+a-a+b}{(a-b)(b-c)(c-a)}} = x^0 = 1 \quad [\because\ x^0 = 1]$$

= R.H.S.