CHAPTER 2

Polynomials

NCERT FOCUS

SOLUTIONS

EXERCISE - 2.1

1. (i) Given polynomial can be written as
$4x^2 - 3x + 7x^0$
Since exponent of variable in each term is a whole number.
$\therefore$ $4x^2 - 3x + 7$ is a polynomial in one variable.
(ii) Given polynomial can be written as $y^2 + \sqrt{2}y^0$
Since exponent of variable in each term is a whole number.
$\therefore$ $y^2 + \sqrt{2}$ is a polynomial in one variable.
(iii) Given polynomial can be written as $3t^{1/2} + \sqrt{2}t$
Now, exponent of variable in first term is $\frac{1}{2}$ which is not a whole number.
$\therefore$ $3t^{1/2} + \sqrt{2}t$ is not a polynomial.
(iv) Given polynomial can be written as $y + 2 \cdot y^{-1}$. Now, exponent of variable in second term is –1 which is not a whole number.
$\therefore$ $y + \frac{2}{y}$ is not a polynomial.
(v) $x^{10} + y^3 + t^{50}$
Here, exponent of every variable is a whole number, but $x^{10} + y^3 + t^{50}$ is a polynomial in x, y and t, *i.e.*, in three variables. So, it is not a polynomial in one variable.

2. (i) In the given polynomial $2 + x^2 + x$, the coefficient of x^2 is 1.
(ii) In the given polynomial $2 - x^2 + x^3$, the coefficient of x^2 is (–1).
(iii) In the given polynomial $\frac{\pi}{2}x^2 + x$, the coefficient of x^2 is $\pi/2$.
(iv) In the given polynomial $\sqrt{2}x - 1$, the coefficient of x^2 is 0.

3. (i) A binomial of degree 35 can be $3x^{35} - 4$.
(ii) A monomial of degree 100 can be $\sqrt{2}y^{100}$.

4. (i) The given polynomial is $5x^3 + 4x^2 + 7x$. The highest power of the variable x is 3. So, the degree of the polynomial is 3.
(ii) The given polynomial is $4 - y^2$. The highest power of the variable y is 2. So, the degree of the polynomial is 2.
(iii) The given polynomial is $5t - \sqrt{7}$. The highest power of variable t is 1. So, the degree of the polynomial is 1.
(iv) Since, $3 = 3x^0$ [$\because x^0 = 1$]
So, the degree of the polynomial is 0.

5. (i) The degree of polynomial $x^2 + x$ is 2. So, it is a quadratic polynomial.
(ii) The degree of polynomial $x - x^3$ is 3. So, it is a cubic polynomial.
(iii) The degree of polynomial $y + y^2 + 4$ is 2. So, it is a quadratic polynomial.
(iv) The degree of polynomial $1 + x$ is 1. So, it is a linear polynomial.
(v) The degree of polynomial $3t$ is 1. So, it is a linear polynomial.
(vi) The degree of polynomial r^2 is 2. So, it is a quadratic polynomial.
(vii) The degree of polynomial $7x^3$ is 3. So, it is a cubic polynomial.

EXERCISE - 2.2

1. Let $p(x) = 5x - 4x^2 + 3$
(i) $p(0) = 5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = 3$
Thus, the value of $5x - 4x^2 + 3$ at $x = 0$ is 3.
(ii) $p(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -9 + 3 = -6$
Thus, the value of $5x - 4x^2 + 3$ at $x = -1$ is –6.
(iii) $p(2) = 5(2) - 4(2)^2 + 3 = 10 - 4(4) + 3$
$= 10 - 16 + 3 = -3$
Thus, the value of $5x - 4x^2 + 3$ at $x = 2$ is – 3.

2. (i) We have, $p(y) = y^2 - y + 1$.
$\therefore$ $p(0) = (0)^2 - 0 + 1 = 0 - 0 + 1 = 1$,
$p(1) = (1)^2 - 1 + 1 = 1 - 1 + 1 = 1$,
$p(2) = (2)^2 - 2 + 1 = 4 - 2 + 1 = 3$
(ii) We have, $p(t) = 2 + t + 2t^2 - t^3$
$\therefore$ $p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2 + 0 + 0 - 0 = 2$,
$p(1) = 2 + 1 + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$,
$p(2) = 2 + 2 + 2(2)^2 - (2)^3 = 2 + 2 + 8 - 8 = 4$
(iii) We have, $p(x) = x^3$
$\therefore$ $p(0) = (0)^3 = 0$, $p(1) = (1)^3 = 1$, $p(2) = (2)^3 = 8$
(iv) We have, $p(x) = (x - 1)(x + 1)$
$p(0) = (0 - 1)(0 + 1) = -1 \times 1 = -1$,
$p(1) = (1 - 1)(1 + 1) = (0)(2) = 0$,
$p(2) = (2 - 1)(2 + 1) = (1)(3) = 3$

3. (i) We have, $p(x) = 3x + 1$
$\therefore$ $p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0.$
So, $x = -\frac{1}{3}$ is a zero of $3x + 1$.
(ii) We have, $p(x) = 5x - \pi$
$\therefore$ $p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi \neq 0$
So, $x = \frac{4}{5}$ is not a zero of $5x - \pi$.

(iii) We have, $p(x) = x^2 - 1$,
$p(1) = (1)^2 - 1 = 1 - 1 = 0$
So, $x = 1$ is a zero of $x^2 - 1$.
Also, $p(-1) = (-1)^2 - 1 = 1 - 1 = 0$
So, $x = -1$ is also a zero of $x^2 - 1$.
(iv) We have, $p(x) = (x + 1)(x - 2)$
$p(-1) = (-1 + 1)(-1 - 2) = (0)(-3) = 0$
So, $x = -1$ is a zero of $(x + 1)(x - 2)$.
Also, $p(2) = (2 + 1)(2 - 2) = (3)(0) = 0$
So, $x = 2$ is also a zero of $(x + 1)(x - 2)$.
(v) We have, $p(x) = x^2 \Rightarrow p(0) = (0)^2 = 0$.
So, $x = 0$ is a zero of x^2.
(vi) We have, $p(x) = lx + m$

$$\therefore\ p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = -m + m = 0.$$

So, $x = \left(-\frac{m}{l}\right)$ is a zero of $lx + m$.

(vii) We have, $p(x) = 3x^2 - 1$

$$\therefore\ p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$$

So, $x = \left(\frac{-1}{\sqrt{3}}\right)$ is a zero of $3x^2 - 1$.

$$\text{Also, } p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3 \neq 0$$

So, $\frac{2}{\sqrt{3}}$ is not a zero of $3x^2 - 1$.

(viii) We have, $p(x) = 2x + 1$

$$\therefore\ p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \neq 0$$

So, $x = \frac{1}{2}$ is not a zero of $2x + 1$.

4. Finding zero of polynomial $p(x)$, is same as solving the polynomial equation $p(x) = 0$.
(i) We have, $p(x) = x + 5$.
Put $p(x) = 0 \Rightarrow x + 5 = 0 \Rightarrow x = -5$
Thus, zero of $x + 5$ is -5.
(ii) We have, $p(x) = x - 5$.
Put $p(x) = 0 \Rightarrow x - 5 = 0 \Rightarrow x = 5$
Thus, zero of $x - 5$ is 5.
(iii) We have, $p(x) = 2x + 5$.
Put $p(x) = 0 \Rightarrow 2x + 5 = 0 \Rightarrow 2x = -5 \Rightarrow x = \frac{-5}{2}$
Thus, zero of $2x + 5$ is $-\frac{5}{2}$.
(iv) We have, $p(x) = 3x - 2$.
Put $p(x) = 0 \Rightarrow 3x = 2 \Rightarrow x = 2/3$
Thus, zero of $3x - 2$ is $\frac{2}{3}$.
(v) We have, $p(x) = 3x$.
Put $p(x) = 0 \Rightarrow 3x = 0 \Rightarrow x = 0$
Thus, zero of $3x$ is 0.
(vi) We have, $p(x) = ax$, $a \neq 0$.
Put $p(x) = 0 \Rightarrow ax = 0 \Rightarrow x = 0$
Thus, zero of ax is 0.
(vii) We have, $p(x) = cx + d$, $c \neq 0$
Put $p(x) = 0 \Rightarrow cx + d = 0 \Rightarrow cx = -d \Rightarrow x = -\frac{d}{c}$
Thus, zero of $cx + d$ is $-\frac{d}{c}$.

EXERCISE - 2.3

1. Let $p(x) = x^3 + 3x^2 + 3x + 1$
(i) The zero of $(x + 1)$ is -1. So, by remainder theorem, $p(-1)$ is the remainder when $p(x)$ is divided by $x + 1$.
$\therefore\ p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$
$= -1 + 3 - 3 + 1 = 0$
Thus, the required remainder = 0
(ii) The zero of $x - \frac{1}{2}$ is $\frac{1}{2}$.

$$\therefore\ p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{1 + 6 + 12 + 8}{8} = \frac{27}{8}$$

Thus, the required remainder = 27/8.
(iii) The zero of x is 0.
$\therefore\ p(0) = (0)^3 + 3(0)^2 + 3(0) + 1 = 0 + 0 + 0 + 1 = 1$
Thus, the required remainder = 1.
(iv) The zero of $x + \pi$ is $(-\pi)$.
$\therefore\ p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$
$= -\pi^3 + 3\pi^2 - 3\pi + 1$
Thus, the required remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1$.
(v) The zero of $5 + 2x$ is $\left(-\frac{5}{2}\right)$.

$$\therefore\ p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$$

$$= \frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = \frac{-27}{8}$$

Thus, the required remainder is $\left(-\frac{27}{8}\right)$.

2. We have, $p(x) = x^3 - ax^2 + 6x - a$ and zero of $x - a$ is a.
$\therefore\ p(a) = (a)^3 - a(a)^2 + 6(a) - a$
$= a^3 - a^3 + 6a - a = 5a$
Thus, the required remainder = $5a$

3. We have, $p(x) = 3x^3 + 7x$ and zero of $7 + 3x$ is $\frac{-7}{3}$.

$$\therefore\ p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$$

$$= 3\left(\frac{-343}{27}\right) + \left(\frac{-49}{3}\right) = -\frac{343}{9} - \frac{49}{3} = -\frac{490}{9}$$

Since, $\left(\frac{-490}{9}\right) \neq 0$ *i.e.*, the remainder is not 0.
$\therefore\ 3x^3 + 7x$ is not divisible by $7 + 3x$.
Thus, $(7 + 3x)$ is not a factor of $3x^3 + 7x$.

EXERCISE - 2.4

1. The zero of $x + 1$ is -1.

(i) Let $p(x) = x^3 + x^2 + x + 1$

$\therefore$ $p(-1) = (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

$\because$ $p(-1) = 0$, so by factor theorem, $(x + 1)$ is a factor of $x^3 + x^2 + x + 1$.

(ii) Let $p(x) = x^4 + x^3 + x^2 + x + 1$

$\therefore$ $p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$
$= 1 - 1 + 1 - 1 + 1 = 1$

$\because$ $p(-1) \neq 0$, so by factor theorem, $(x + 1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$\therefore$ $p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$
$= 1 - 3 + 3 - 1 + 1 = 1$

$\because$ $p(-1) \neq 0$, so by factor theorem, $(x + 1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) Let $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

$\therefore$ $p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = -2 + 2 + 2\sqrt{2} = 2\sqrt{2}$

$\because$ $p(-1) \neq 0$, so by factor theorem, $(x + 1)$ is not a factor of $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$.

2. (i) We have, $p(x) = 2x^3 + x^2 - 2x - 1$ and $g(x) = x + 1$.
Since zero of $x + 1$ is -1.

$\therefore$ $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0$

$\because$ $p(-1) = 0$, so by factor theorem, $g(x)$ is a factor of $p(x)$.

(ii) We have, $p(x) = x^3 + 3x^2 + 3x + 1$ and $g(x) = x + 2$.
Since zero of $x + 2$ is -2.

$\therefore$ $p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$
$= -8 + 12 - 6 + 1 = -14 + 13 = -1$

$\because$ $p(-2) \neq 0$, so by factor theorem, $g(x)$ is not a factor of $p(x)$.

(iii) We have, $p(x) = x^3 - 4x^2 + x + 6$ and $g(x) = x - 3$.
Since zero of $x - 3$ is 3.

$\therefore$ $p(3) = (3)^3 - 4(3)^2 + 3 + 6$
$= 27 - 36 + 3 + 6 = 0$

$\because$ $p(3) = 0$, so by factor theorem, $g(x)$ is a factor of $p(x)$.

3. Since $(x - 1)$ is a factor of $p(x)$.

$\therefore$ $p(1)$ should be equal to 0. [By factor theorem]

(i) Here, $p(x) = x^2 + x + k$

$\therefore$ $p(1) = (1)^2 + 1 + k = 0 \Rightarrow k + 2 = 0 \Rightarrow k = -2$.

(ii) Here, $p(x) = 2x^2 + kx + \sqrt{2}$

$\therefore$ $p(1) = 2(1)^2 + k(1) + \sqrt{2} = 0 \Rightarrow 2 + k + \sqrt{2} = 0$

$\Rightarrow$ $k = -2 - \sqrt{2} = -(2+\sqrt{2})$

(iii) Here, $p(x) = kx^2 - \sqrt{2}x + 1$

$\therefore$ $p(1) = k(1)^2 - \sqrt{2}(1) + 1 = 0 \Rightarrow k - \sqrt{2} + 1 = 0$

$\Rightarrow$ $k = \sqrt{2} - 1$

(iv) Here, $p(x) = kx^2 - 3x + k$

$\therefore$ $p(1) = k(1)^2 - 3(1) + k = 0 \Rightarrow k - 3 + k = 0$

$\Rightarrow$ $2k - 3 = 0 \Rightarrow k = 3/2$.

4. (i) We have,
$12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$
$= 4x(3x - 1) - 1(3x - 1) = (3x - 1)(4x - 1)$
Thus, $12x^2 - 7x + 1 = (3x - 1)(4x - 1)$

(ii) We have, $2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$
$= x(2x + 1) + 3(2x + 1) = (2x + 1)(x + 3)$
Thus, $2x^2 + 7x + 3 = (2x + 1)(x + 3)$

(iii) We have, $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$
$= 3x(2x + 3) - 2(2x + 3) = (2x + 3)(3x - 2)$
Thus, $6x^2 + 5x - 6 = (2x + 3)(3x - 2)$

(iv) We have, $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$
Thus, $3x^2 - x - 4 = (3x - 4)(x + 1)$

5. (i) We have, $x^3 - 2x^2 - x + 2$
Rearranging the terms, we have
$x^3 - 2x^2 - x + 2 = x^3 - x - 2x^2 + 2$
$= x(x^2 - 1) - 2(x^2 - 1) = (x^2 - 1)(x - 2)$
$= [(x)^2 - (1)^2](x - 2)$
$= (x - 1)(x + 1)(x - 2)$
Thus, $x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2)$

(ii) We have, $x^3 - 3x^2 - 9x - 5$
$= x^3 + x^2 - 4x^2 - 4x - 5x - 5$
$= x^2(x + 1) - 4x(x + 1) - 5(x + 1)$
$= (x + 1)(x^2 - 4x - 5) = (x + 1)(x^2 - 5x + x - 5)$
$= (x + 1)[x(x - 5) + 1(x - 5)] = (x + 1)(x - 5)(x + 1)$
Thus, $x^3 - 3x^2 - 9x - 5 = (x + 1)(x - 5)(x + 1)$

(iii) We have, $x^3 + 13x^2 + 32x + 20$
$= x^3 + x^2 + 12x^2 + 12x + 20x + 20$
$= x^2(x + 1) + 12x(x + 1) + 20(x + 1)$
$= (x + 1)(x^2 + 12x + 20) = (x + 1)(x^2 + 2x + 10x + 20)$
$= (x + 1)[x(x + 2) + 10(x + 2)] = (x + 1)(x + 2)(x + 10)$
Thus, $x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$

(iv) We have, $2y^3 + y^2 - 2y - 1$
$= 2y^3 - 2y^2 + 3y^2 - 3y + y - 1$
$= 2y^2(y - 1) + 3y(y - 1) + 1(y - 1)$
$= (y - 1)(2y^2 + 3y + 1) = (y - 1)(2y^2 + 2y + y + 1)$
$= (y - 1)[2y(y + 1) + 1(y + 1)] = (y - 1)(y + 1)(2y + 1)$
Thus, $2y^3 + y^2 - 2y - 1 = (y - 1)(y + 1)(2y + 1)$

Note : We can also solve it by long division method also.

EXERCISE - 2.5

1. (i) We have, $(x + 4)(x + 10)$
Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$, we have
$(x + 4)(x + 10) = x^2 + (4 + 10)x + (4 \times 10) = x^2 + 14x + 40$

(ii) We have, $(x + 8)(x - 10)$.
Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$, we have
$(x + 8)(x - 10) = x^2 + [8 + (-10)]x + [8 \times (-10)]$
$= x^2 + (-2)x + (-80) = x^2 - 2x - 80$

(iii) We have, $(3x + 4)(3x - 5)$
Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$, we have
$(3x + 4)(3x - 5) = (3x)^2 + [4 + (-5)]3x + [4 \times (-5)]$
$= 9x^2 + (-1)3x + (-20) = 9x^2 - 3x - 20$

(iv) We have, $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

Using the identity $(a + b)(a - b) = a^2 - b^2$, we have

$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=(y^2)^2-\left(\frac{3}{2}\right)^2=y^4-\frac{9}{4}$

(v) We have, $(3 - 2x)(3 + 2x)$

Using the identity, $(a + b)(a - b) = a^2 - b^2$, we have

$(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2 = 9 - 4x^2$

2. (i) We have, $103 \times 107 = (100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7) \times 100 + (3 \times 7)$

[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 10000 + (10) \times 100 + 21$

$= 10000 + 1000 + 21 = 11021$

(ii) We have, $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 + [(-5) + (-4)] \times 100 + [(-5) \times (-4)]$

[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 10000 + (-9) \times 100 + 20$

$= 10000 + (-900) + 20 = 9120$

(iii) We have, $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$ [Using $(x + y)(x - y) = x^2 - y^2$]

$= 10000 - 16 = 9984$

3. (i) We have, $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2 = (3x + y)(3x + y)$

[Using $a^2 + 2ab + b^2 = (a + b)^2$]

(ii) We have, $4y^2 - 4y + 1$

$= (2y)^2 - 2(2y)(1) + (1)^2 = (2y - 1)^2 = (2y - 1)(2y - 1)$

[Using $a^2 - 2ab + b^2 = (a - b)^2$]

(iii) We have, $x^2-\frac{y^2}{100}=(x)^2-\left(\frac{y}{10}\right)^2$

$=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$ [Using $a^2 - b^2 = (a + b)(a - b)$]

4. We know that

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) $(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(v) $(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2=\left(\frac{1}{4}a\right)^2+\left(-\frac{1}{2}b\right)^2+(1)^2$

$+2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)+2\left(-\frac{1}{2}b\right)(1)+2(1)\left(\frac{1}{4}a\right)$

$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$

5. We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) Now, $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$=(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+2(-\sqrt{2}x)(y)$

$+2(2\sqrt{2}z)(y)+2(2\sqrt{2}z)(-\sqrt{2}x)$

$=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

6. We know that, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ ... (1)

and $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ ... (2)

(i) $(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)$ [By (1)]

$= 8x^3 + 1 + 6x(2x + 1) = 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$ [By (2)]

$= 8a^3 - 27b^3 - 18ab(2a - 3b) = 8a^3 - 27b^3 - (36a^2b - 54ab^2)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left(\frac{3}{2}x+1\right)^3=\left(\frac{3}{2}x\right)^3+(1)^3+3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$ [By (1)]

$=\frac{27}{8}x^3+1+\frac{9}{2}x\left[\frac{3}{2}x+1\right]$

$=\frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x=\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$

(iv) $\left(x-\frac{2}{3}y\right)^3=x^3-\left(\frac{2}{3}y\right)^3-3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$ [By (2)]

$=x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$

$=x^3-\frac{8}{27}y^3-\left(2x^2y-\frac{4}{3}xy^2\right)$

$=x^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$

7. (i) We have,

$99^3 = (100 - 1)^3 = (100)^3 - 1^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300 = 970299$

(ii) We have, $102^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200 = 1061208$

(iii) We have, $(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$

$= 1000000000 - 8 - 6000(1000 - 2)$

$= 1000000000 - 8 - 6000000 + 12000 = 994011992$

8. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 6ab(2a + b)$

$= (2a)^3 + (b)^3 + 3(2a)(b)(2a + b)$

$= (2a + b)^3 = (2a + b)(2a + b)(2a + b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - (b)^3 - 6ab\,(2a - b)$

$= (2a)^3 - (b)^3 - 3(2a)(b)(2a - b)$

$= (2a - b)^3 = (2a - b)(2a - b)(2a - b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$

$= (3 - 5a)^3 = (3 - 5a)(3 - 5a)(3 - 5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$
$= (4a - 3b)^3 = (4a - 3b)(4a - 3b)(4a - 3b)$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3 = \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

9. (i) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
$\Rightarrow$ $(x + y)^3 - 3(x + y)(xy) = x^3 + y^3$
$\Rightarrow$ $x^3 + y^3 = (x + y)\,[(x + y)^2 - 3xy]$
$= (x + y)\,(x^2 + y^2 + 2xy - 3xy)$
$\Rightarrow$ $(x + y)(x^2 + y^2 - xy) = x^3 + y^3$
(ii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
$\Rightarrow$ $(x - y)^3 + 3xy(x - y) = x^3 - y^3$
$\Rightarrow$ $(x - y)[(x - y)^2 + 3xy] = x^3 - y^3$
$\Rightarrow$ $(x - y)\,(x^2 + y^2 - 2xy + 3xy) = x^3 - y^3$
$\Rightarrow$ $(x - y)(x^2 + y^2 + xy) = x^3 - y^3$

10. (i) We know that
$(x^3 + y^3) = (x + y)(x^2 - xy + y^2)$
We have, $27y^3 + 125z^3 = (3y)^3 + (5z)^3$
$= (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$
$= (3y + 5z)(9y^2 - 15yz + 25z^2)$
(ii) We know that
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
We have, $64m^3 - 343n^3 = (4m)^3 - (7n)^3$
$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$
$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

11. We have, $27x^3 + y^3 + z^3 - 9xyz$
$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$
Using the identity, $x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
We have, $(3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$
$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x \times y) - (y \times z) - (z \times 3x)]$
$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

12. R.H.S. $= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (z^2 + x^2 - 2xz)]$

$= \frac{1}{2}(x + y + z)[2(x^2 + y^2 + z^2 - xy - yz - zx)]$

$= 2 \times \frac{1}{2} \times (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$= x^3 + y^3 + z^3 - 3xyz$ = L.H.S.

13. Since, $x + y + z = 0$
$\Rightarrow$ $x + y = -z \Rightarrow (x + y)^3 = (-z)^3$
$\Rightarrow$ $x^3 + y^3 + 3xy(x + y) = -z^3$
$\Rightarrow$ $x^3 + y^3 + 3xy(-z) = -z^3$ $[\because x + y = -z]$
$\Rightarrow$ $x^3 + y^3 - 3xyz = -z^3 \Rightarrow x^3 + y^3 + z^3 = 3xyz$
Hence, if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

14. (i) We have, $(-12)^3 + (7)^3 + (5)^3$
Let $x = -12$, $y = 7$ and $z = 5$.
Then, $x + y + z = -12 + 7 + 5 = 0$
We know that if $x + y + z = 0$, then, $x^3 + y^3 + z^3 = 3xyz$
$\therefore$ $(-12)^3 + (7)^3 + (5)^3 = 3[(-12)(7)(5)] = 3[-420] = -1260$
(ii) $(28)^3 + (-15)^3 + (-13)^3$
Let $x = 28$, $y = -15$ and $z = -13$. Then,
$x + y + z = 28 - 15 - 13 = 0$
We know that if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$
$\therefore$ $(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$
$= 3(5460) = 16380$

15. Area of a rectangle = (Length) × (Breadth)
(i) $25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12$
$= 5a(5a - 4) - 3(5a - 4) = (5a - 4)(5a - 3)$
Thus, the possible length and breadth are $(5a - 3)$ and $(5a - 4)$ respectively.
(ii) $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$
$= 7y(5y + 4) - 3(5y + 4) = (5y + 4)(7y - 3)$
Thus, the possible length and breadth are $(7y - 3)$ and $(5y + 4)$.

16. Volume of a cuboid = (Length) × (Breadth) × (Height)
(i) Volume $= 3x^2 - 12x$
We have, $3x^2 - 12x = 3x\,(x - 4) = 3 \times x \times (x - 4)$
$\therefore$ The possible dimensions of the cuboid are 3, x and $(x - 4)$.
(ii) Volume $= 12ky^2 + 8ky - 20k$
We have, $12ky^2 + 8ky - 20k$
$= 4 \times k \times (3y^2 + 2y - 5) = 4k[3y^2 - 3y + 5y - 5]$
$= 4k[3y(y - 1) + 5(y - 1)] = 4k[(3y + 5) \times (y - 1)]$
$= 4k \times (3y + 5) \times (y - 1)$
Thus, the possible dimensions of the cuboid are $4k$, $(3y + 5)$ and $(y - 1)$.