CHAPTER 9

Areas of Parallelograms and Triangles

EXAM DRILL

SOLUTIONS

1. **(b):** Clearly, in option (b), ΔPQR and ΔQRS are on the same base. But, they do not lie between the same parallels.

2. **(a) :** Since, ΔPXQ and parallelogram $PQRS$ are on the same base PQ and between the same parallels PQ and RS.

$\therefore \quad ar(\Delta PXQ) = \frac{1}{2}\, ar\,(\|^{gm}\, PQRS) = \frac{1}{2}(20 \times 8) = 80 \text{ cm}^2$

3. **(b):** Area of the rhombus $= \left(\frac{1}{2} \times d_1 \times d_2\right)$

$= \left(\frac{1}{2} \times 12 \times 16\right) \text{cm}^2 = 96 \text{ cm}^2$

4. **(c) :** Area of the trapezium

$= \frac{1}{2} \times$ sum of parallels sides × distance between them

$= \left\{\frac{1}{2} \times (12+8) \times \frac{13}{2}\right\} \text{cm}^2 = 65 \text{ cm}^2.$

5. **(d):** There are two polygons $PQRA$ and $BQRS$, both lie on the same base QR and between the same parallels PS and QR.

6. Let $2x$ and $11x$ be height and area of the parallelogram respectively.

$\therefore$ Area of parallelogram = Base × Height

$\Rightarrow \quad 11x = \text{Base} \times 2x \Rightarrow \text{Base} = \frac{11}{2}$ units

7. Since, parallelogram $ABCD$ and ΔAEB are on the same base AB and between same parallels PQ and RS.

$\therefore \quad ar\,(\Delta AEB) = \frac{1}{2}\, ar\,(\|^{gm} ABCD)$

$\Rightarrow \quad ar\,(\|^{gm}\, ABCD) = 2\, ar\,(AEB) = 2 \times 23 = 46 \text{ cm}^2$

8. Since, a diagonal of a parallelogram divides it into two congruent triangles. Also, diagonals of a parallelogram bisect each other and median of a triangle divides it into two triangles of equal area.

$\therefore$ The diagonals of a parallelogram divides it into four triangles of equal area.

9. Area of parallelogram $PQRS = PQ \times SM$

$\Rightarrow \quad 88 = K \times 8 \Rightarrow K = 11$ cm

10. $\because$ Area of parallelogram = Base × Height

$\therefore \quad PQ \times ST = SP \times QM$

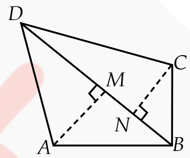
$\Rightarrow \quad 12 \times 9 = SP \times 6 \Rightarrow SP = \frac{12 \times 9}{6} = 18$ cm

11. Since R is the mid-pint of EF.

$\therefore$ AR is the Median of ΔAEF.

$\therefore \quad ar(\Delta AER) = ar(\Delta AFR)$ [$\because$ Median divides a triangle into two triangles of equal area]

12. $ar(\text{quad. } ABCD)$

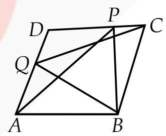


$= ar(\Delta ABD) + ar(\Delta BCD)$

$= \frac{1}{2}(BD \times AM) + \frac{1}{2}(BD \times CN)$

$= \frac{1}{2} BD(AM + CN)$

13. Since, ΔAPB and parallelogram $ABCD$ are on the same base AB and between same parallels AB and DC.



$\therefore \quad ar(\Delta APB) = \frac{1}{2} ar(\|^{gm}\, ABCD)$

$\Rightarrow \quad 13 = \frac{1}{2}\, ar(\|^{gm}\, ABCD)$

$\Rightarrow \quad ar(\|^{gm}\, ABCD) = 2 \times 13 = 26 \text{ cm}^2$

Also, ΔBQC and parallelogram $ABCD$ are on the same base BC and between same parallels BC and AD.

$\therefore \quad ar(\Delta BQC) = \frac{1}{2}\, ar(\|^{gm}\, ABCD) = \frac{1}{2} \times 26 = 13 \text{ cm}^2$

14. Since, ΔAEB and parallelogram $ABCD$ are on the same base AB and between same parallels AB and DC.

$\therefore \quad ar(\Delta AEB) = \frac{1}{2}\,(\|^{gm}\, ABCD) = \frac{1}{2} \times 30 = 15 \text{ cm}^2$

Now, $ar(\Delta ADE) + ar(\Delta BCE) + ar(\Delta AEB) = ar(\|^{gm}\, ABCD)$

$\Rightarrow \quad ar(\Delta ADE) + ar(\Delta BCE) = (30 - 15) \text{ cm}^2 = 15 \text{ cm}^2$

15. From figure, the transversal DB is intersecting a pair of lines DC and AB such that

$\angle CDB = \angle ABD = 90°$

Thus, these angles form a pair of alternate equal angles.

$\therefore \quad DC \parallel AB$

Also, $DC = AB = 2.5$ units

$\therefore$ Quadrilateral $ABCD$ is a parallelogram.

Now, area of parallelogram $ABCD$

= Base × Corresponding altitude

= 2.5 × 4 = 10 sq. units

16. Since, ΔCDN and parallelogram $ABCD$ are on the same base CD and between same parallels AB and DC.

$\therefore \quad ar(\Delta CDN) = \frac{1}{2} ar(\|^{gm}\, ABCD) = \frac{1}{2} \times 48 \text{ cm}^2 = 24 \text{ cm}^2$

Since, median MN of ΔCDN divides it into two triangles of equal area.

$\therefore \quad ar(\Delta NDM) = \frac{1}{2} ar(\Delta CDN) = \frac{1}{2} \times 24 = 12 \text{ cm}^2$

17. Given, $OD = 10$ cm and $OE = 2\sqrt{5}$ cm

$\therefore \quad OD^2 = OE^2 + DE^2$

$\Rightarrow \quad DE = \sqrt{OD^2 - OE^2} = \sqrt{(10)^2 - (2\sqrt{5})^2} = 4\sqrt{5}$ cm

Now, $ar(\text{rect.} OCDE) = OE \times DE = 2\sqrt{5} \times 4\sqrt{5} \text{ cm}^2 = 40 \text{ cm}^2$

18. Since, a median of triangle divides it into two triangles of equal area.

$\therefore$ $ar(\Delta AMC) = \frac{1}{2} ar(\Delta ACD)$ ...(i)

($\because$ AM is median of ΔACD)

Let h be the height of ΔABE from vertex A on opposite side BE.

Then, $ar(\Delta ABE) = ar(\Delta ABC) + ar(\Delta ACD) + ar(\Delta ADE)$

$= \frac{1}{2} \times BC \times h + \frac{1}{2} \times CD \times h + \frac{1}{2} \times DE \times h$

$= \frac{1}{2} \times h(BC + CD + DE) = \frac{1}{2} \times h(3CD)$ ($\because$ $BC = CD = DE$)

$= 3 \times \frac{1}{2} \times CD \times h = 3ar(\Delta ACD)$

$\Rightarrow$ $ar(\Delta ACD) = \frac{1}{3} ar(\Delta ABE)$ ...(ii)

From (i) and (ii), we get

$ar(\Delta AMC) = \frac{1}{2} \times \frac{1}{3} ar(\Delta ABE) = \frac{1}{6} ar(\Delta ABE)$

19. Since, ΔACD and ΔACP are on the same base AC and between the same parallels AC and DP.

$\therefore$ $ar(\Delta ACD) = ar(\Delta ACP)$

Adding $ar(\Delta ABC)$ on both sides, we get

$ar(\Delta ACD) + ar(\Delta ABC) = ar(\Delta ACP) + ar(\Delta ABC)$

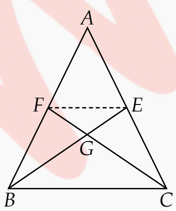
$\Rightarrow$ $ar(\text{quad. } ABCD) = ar(\Delta ABP)$ ...(i)

Now, as $ar(\Delta ABP) = \frac{1}{2} \times BP \times \text{height} = \frac{1}{2} \times 9 \times 12 = 54 \text{ cm}^2$

($\because$ Height of triangle is distance between parallels AD and BP)

$\therefore$ $ar(ABCD) = 54 \text{ cm}^2$ [Using (i)]

20. Join EF. Since the line segment joining the mid-points of two sides of a triangle is parallel to the third side. So, $FE \parallel BC$.



Clearly, ΔBEF and ΔCEF are on the same base EF and between the same parallels EF and BC.

$\therefore$ $ar(\Delta BEF) = ar(\Delta CEF)$

$\Rightarrow$ $ar(\Delta BEF) - ar(\Delta GEF) = ar(\Delta CEF) - ar(\Delta GEF)$

$\Rightarrow$ $ar(\Delta BFG) = ar(\Delta CEG)$ ...(i)

We know that a median of a triangle divides it into two triangles of equal areas.

$\therefore$ $ar(\Delta BEC) = ar(\Delta ABE)$

$\Rightarrow$ $ar(\Delta BGC) + ar(\Delta CEG) = ar(\text{quad. } AFGE) + ar(\Delta BFG)$

$\Rightarrow$ $ar(\Delta BGC) + ar(\Delta BFG) = ar(\text{quad. } AFGE) + ar(\Delta BFG)$ [Using (i)]

$\Rightarrow$ $ar(\Delta BGC) = ar(\text{quad. } AFGE) = 35 \text{ cm}^2$

21. Since, ΔADF and ΔDFB are on the same base DF and between the same parallels CD and AB.

$\therefore$ $ar(\Delta ADF) = ar(\Delta DFB) = 3 \text{ cm}^2$ ...(i)

Also, in ΔADF and ΔECF, we have

$\angle AFD = \angle EFC$ [Vertically opposite angles]

$AD = CE$ [$\because$ $AD = BC = CE$]

and $\angle DAF = \angle CEF$

[$\because$ $BE \parallel AD$ and AE is transversal, therefore alternate interior angles are equal]

$\therefore$ $\Delta ADF \cong \Delta ECF$ [By AAS congruence criterion]

$\Rightarrow$ $ar(\Delta ECF) = ar(\Delta ADF) = 3 \text{ cm}^2$ ...(ii) [From (i)]

Now, in ΔBFE, C is the mid-point of BE.

$\therefore$ FC is a median of ΔBFE.

So, $ar(\Delta CFE) = ar(\Delta BFC) = 3 \text{ cm}^2$ ...(iii)

Now, $ar(\Delta BDC) = ar(\Delta DFB) + ar(\Delta BFC)$

$= (3 + 3) \text{ cm}^2 = 6 \text{ cm}^2$ [From (i) and (ii)]

As we know that diagonal of a parallelogram divides it into two congruent triangles.

$\therefore$ Area of parallelogram $ABCD$

$= 2 \times \text{Area of } \Delta BDC = (2 \times 6) \text{ cm}^2 = 12 \text{ cm}^2$

Thus, the area of parallelogram $ABCD$ is 12 cm^2.

22. Given, $ar(\|^{gm}\ PQRS) = ar(\|^{gm}\ PABC)$

$\Rightarrow$ $ar(\|^{gm}\ PQRS) - ar(\|^{gm}\ QOCP)$

$= ar(\|^{gm}\ PABC) - ar(\|^{gm}\ QOCP)$

$\Rightarrow$ $ar(\|^{gm}\ ORSC) = ar(\|^{gm}\ ABOQ)$

$\Rightarrow$ $2ar(\Delta ORC) = 2 \times ar(\Delta OBQ)$ [$\because$ Diagonal of a parallelogram divides it into two congruent triangles]

$\Rightarrow$ $ar(\Delta ORC) = ar(\Delta OBQ)$

$\Rightarrow$ $ar(\Delta ORC) + ar(\Delta OBR) = ar(\Delta OBQ) + ar(\Delta OBR)$

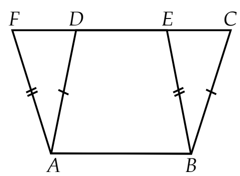
$\Rightarrow$ $ar(\Delta BRC) = ar(\Delta BRQ)$.

Now, ΔBRC and ΔBRQ lie on the same base BR and equal in area, so they must be between the same parallels.

$\therefore$ $QC \parallel BR$.

Hence proved.

23. Given: Two parallelograms $ABCD$ and $ABEF$, which have the same base AB and are between the same parallel lines AB and FC.



To prove : $ar(\|^{gm}\ ABCD) = ar(\|^{gm}\ ABEF)$

Proof : In ΔADF and ΔBCE, we have

$\angle ADF = \angle BCE$ (Corresponding angles from $AD \parallel BC$ and transversal FC)

$\angle AFD = \angle BEC$ (Corresponding angles from $AF \parallel BE$ and transversal FC)

$\Rightarrow$ $\angle DAF = \angle CBE$ (By angle sum property of triangle) ...(i)

Also, we have $AD = BC$ [$\because$ $ABCD$ is a $\|^{gm}$ and opposite sides of a parallelogram are equal] ...(ii)

and $AF = BE$ [$\because$ $ABEF$ is a $\|^{gm}$ and opposite sides of a parallelogram are equal] ...(iii)

Now, from (i), (ii) and (iii), we get

$\Delta ADF \cong \Delta BCE$ (By SAS congruence criteria)

$\Rightarrow$ $ar(\Delta ADF) = ar(\Delta BCE)$ ...(iv)

[Areas of congruent figures are equal]

Now, $ar(\|^{gm}\ ABCD) = ar(\text{quad. } ABED) + ar(\Delta BCE)$

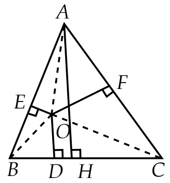
$= ar(\text{quad. } ABED) + ar(\Delta ADF)$ [Using (iv)]

$= ar(\|^{gm}\ ABEF)$

Thus, $ar(\|^{gm}\ ABCD) = ar(\|^{gm}\ ABEF)$

Hence proved.

24. Let ABC is an equilateral triangle. O is any point in the interior of the triangle and perpendiculars OD, OE and OF are drawn from O to the sides BC, BA and AC respectively of the ΔABC.



Join OA, OB, OC and draw $AH \perp BC$.

Now, $ar(\Delta AOB) = \frac{1}{2}(AB)(OE)$

$= \frac{1}{2}(BC)(OE)$ $\quad (\because AB = BC)$ $\quad$ ...(i)

$ar(\Delta BOC) = \frac{1}{2}(BC)(OD)$ $\quad$ ...(ii)

and $ar(\Delta AOC) = \frac{1}{2}(AC)(OF) = \frac{1}{2}(BC)(OF)$ $\quad$ ...(iii)

$(\because AC = BC)$

Adding (i), (ii) and (iii) we get

$ar(\Delta AOB) + ar(BOC) + ar(\Delta AOC) = \frac{1}{2}BC(OE + OD + OF)$

$\Rightarrow \quad ar(\Delta ABC) = \frac{1}{2}BC\,(OE + OD + OF)$

$\Rightarrow \quad \frac{1}{2}(BC)(AH) = \frac{1}{2}(BC)(OE + OD + OF)$

$\Rightarrow \quad AH = OE + OD + OF$

$\Rightarrow \quad OD + OE + OF = AH$, which is constant for a given ΔABC.

Hence proved.