EXAM DRILL

Lines and Angles

CHAPTER 6

SOLUTIONS

1. (b) : Let x be the measure of angle which is 2° more than its supplement. Then, we have
$x = 2° + (180° - x)$
$\Rightarrow 2x = 182° \Rightarrow x = 91°$

2. (a) : $l \parallel m$ only when
$(2x - 30)° = (x + 20)°$ [Corresponding angles]
$\Rightarrow 2x - 30 = x + 20$
$\Rightarrow x = 50.$

3. (b) : Clearly, $\angle 2 + \angle 5 = 180°$ [Co-interior angles]
$\angle 2 + \angle 3 = 180°$ [By linear pair axiom]
and $\angle 1 + \angle 2 = 180°$ [By linear pair axiom]
But, $\angle 2$ and $\angle 8$ are alternate interior angles.

4. (b) : By angle sum property of a triangle, sum of angles of a triangle is 180°, which is equal to two right angles.

5. (a) : A triangle can have at most 1 right angle, otherwise, sum of its three angles cannot be 180°.

6. (c) : Clearly, by exterior angle property of a triangle
$\angle A + \angle B = \angle ACD \Rightarrow \angle A + 40° = 120° \Rightarrow \angle A = 80°$

7. (d) : Clearly, $\angle ABC = 180° - x$
[By linear pair axiom]
and $\angle BAF = \angle ABC + \angle ACB$
[By exterior angle property of a triangle]
$\Rightarrow 100° = 180° - x + 60°$
$\Rightarrow x = 180° - 100° + 60° = 140°$

8. (b) : Let the measure of other interior opposite angle be x. Then,
$45° + x = 100° \Rightarrow x = 100° - 45° = 55°.$

9. Clearly, $\frac{5}{3}$ of right angle $= \frac{5}{3} \times 90° = 5 \times 30° = 150°$
Now, supplement of 150° = 180° – 150° = 30°

10. If two parallel lines are intersected by a transversal, then alternate interior angles are equal.

11. Two lines parallel to the same line are parallel to each other.

12. Since, sum of the three angles of a triangle is 180° and one angle of a triangle is 90°.
Sum of remaining two angles = 180° – 90° = 90°.

13. For ABC to be a line, the sum of the two adjacent angles must be 180° *i.e.*, $x + y = 180°$.

14. Clearly, $l \parallel m$ as sum of a pair of co-interior angles is 180°. But p is not parallel to q as sum of none of the pair of the co-interior angle is 180°.

15. Since, the sum of given angles = 45° + 64° + 72°
= 181° ≠ 180°.
So, no triangle can be drawn with the given angles.

16. Since, $AB \parallel CD$ therefore
$\angle 1 = \angle 5$ [Corresponding angles]
$\Rightarrow 120° - x° = 5x° \Rightarrow 6x° = 120° \Rightarrow x = 20$
Thus, $\angle 1 = 120° - 20° = 100°$ and $\angle 5 = 5x° = 5 \times 20° = 100°$

17. Since, $BD \parallel CE$ and BC is a transversal.
$\therefore x = 70°$ [Alternate interior angles]
Also, $y = 180° - 120°$ [Corresponding angles]
$\Rightarrow y = 60°$

18. Since, PR is a bisector of $\angle APQ$.
$\therefore \angle APQ = 72°$
Also, $\angle PQD = 180° - 108° = 72°$ (By linear pair axiom)
Now, as $\angle APQ = \angle PQD$ [Alternate interior angles]
So, $AB \parallel CD$

19. Clearly, $\angle BCE = 60°$ [Corresponding angles]
$\Rightarrow y + 25° = 60°$ [Exterior angle property of a triangle]
$\Rightarrow y = 35°$

20. Given, $z° = \frac{x° + y°}{2}$
$\Rightarrow 2z° = x° + y°$
Also, $x° + y° + z° = 180°$ (By angle sum property of a triangle)
$\Rightarrow 2z° + z° = 180° \Rightarrow z° = 60°.$

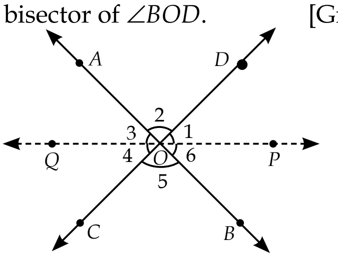
21. Using linear pair axiom, we have,
$\angle 1 + \angle BAE = 180°$ ...(i)
$\angle 2 + \angle CBF = 180°$ ...(ii)
and $\angle 3 + \angle ACD = 180°$ ...(iii)
Adding (i), (ii) and (iii), we have
$(\angle 1 + \angle 2 + \angle 3) + (\angle BAE + \angle CBF + \angle ACD) = 540°$
$\Rightarrow 180° + (\angle BAE + \angle CBF + \angle ACD) = 540°$
$\Rightarrow \angle ACD + \angle BAE + \angle CBF = 360°.$

22. Clearly, $\angle ACB = \angle APQ = 40°$
(Alternate interior angles)
Now, in ΔABC, we have
$\angle BAC = 180° - \angle B - \angle C$
[By angle sum property of a triangle]
$= 180° - 35° - 40° = 105°$
$\therefore x = 180° - 105° = 75°$ [By linear pair axiom]

23. In ΔAEB, $\angle AEB = 180° - 70° - 30° = 80°$
[By angle sum property of a triangle]
$\Rightarrow \angle DEC = \angle AEB = 80°$ [Vertically opposite angles]
Now, in ΔDEC, $\angle EDC = 180° - \angle DEC - \angle DCE$
[By angle sum property of a triangle]
$= 180° - 80° - 90° = 10°$

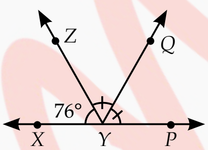
24. In ΔABC, $\angle ABC = 180° - 40° - 90° = 50°$
[By angle sum property of a triangle]
Now, in ΔDEB, $x° = 180° - \angle EBD - \angle BED$
[By angle sum property of a triangle]
$= 180° - 50° - 100° = 30°$
[$\because \angle EBD = \angle ABC = 50°$ and $\angle BED = 100°$ (Given)]
$\Rightarrow \quad x = 30$

25. In order to prove that OP and OQ are in the same line, it is sufficient to prove that $\angle POQ = 180°$.
Now, OP is the bisector of $\angle BOD$. [Given]



$\therefore \quad \angle 1 = \angle 6$ ...(i)
Also, OQ is the bisector of $\angle AOC$. [Given]
$\therefore \quad \angle 3 = \angle 4$ ...(ii)
Also, $\angle 2 = \angle 5$ ...(iii) [Vertically opposite angles]
Now, we know that sum of the angles formed at a point is 360°.
$\therefore \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$
$\Rightarrow \quad (\angle 1 + \angle 6) + (\angle 3 + \angle 4) + (\angle 2 + \angle 5) = 360°$
$\Rightarrow \quad 2\angle 1 + 2\angle 3 + 2\angle 2 = 360° \Rightarrow 2(\angle 1 + \angle 2 + \angle 3) = 360°$
$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = 180° \Rightarrow \angle POQ = 180°$
Hence, rays OP and OQ are on the same straight line.

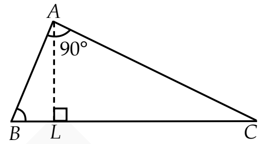
26. Since, YQ bisects $\angle ZYP$



$\therefore \quad \angle ZYQ = \angle QYP = \frac{1}{2}\angle ZYP$ ...(i)
Also, $\angle XYZ = 76°$ [Given] ...(ii)
and $\angle XYZ + \angle ZYQ + \angle QYP = 180°$
[$\because$ XP is a straight line]
$\Rightarrow \quad 76° + \angle ZYQ + \angle ZYQ = 180°$ [Using (i) and (ii)]
$\Rightarrow \quad 2\angle ZYQ = 180° - 76° = 104°$
$\Rightarrow \quad \angle ZYQ = \frac{1}{2} \times 104° = 52°$
$\therefore \quad \angle XYQ = \angle XYZ + \angle ZYQ = 76° + 52° = 128°$
Now, reflex $\angle QYP = 360° - \angle QYP = 360° - \angle ZYQ$
$= 360° - 52° = 308°$

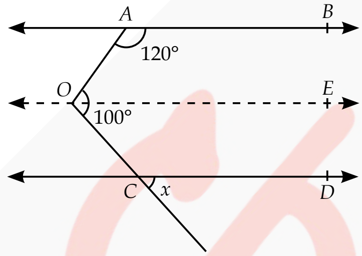
27. Since, $AC \parallel DE$
$\therefore \quad \angle ACB = \angle DEC$ [Corresponding angles]
Thus, $y = 55°$
Now, in $\angle ABC$, $\angle BAC + \angle ABC + \angle ACB = 180°$
[By angle sum property of triangle]
$\Rightarrow \quad \angle ABC = 180° - (70° + 55°) = 55°$
and $x = \angle ABC = 55°$ [Corresponding angles]

28. In ΔABL, we have
$\angle BAL + \angle ALB + \angle B = 180°$
$\Rightarrow \quad \angle BAL + 90° + \angle B = 180°$
[$\because AL \perp BC \Rightarrow \angle ALB = 90°$]
$\Rightarrow \quad \angle BAL + \angle B = 90°$
$\Rightarrow \quad \angle BAL = 90° - \angle B$ ...(i)
In ΔABC, we have
$\angle A + \angle B + \angle C = 180°$ [By angle sum property of a triangle]



$\Rightarrow \quad 90° + \angle B + \angle C = 180°$
$\Rightarrow \quad \angle B + \angle C = 90°$
$\Rightarrow \quad \angle C = 90° - \angle B$
$\Rightarrow \quad \angle ACB = 90° - \angle B$ ...(ii)
From (i) and (ii), we get $\angle BAL = \angle ACB$.

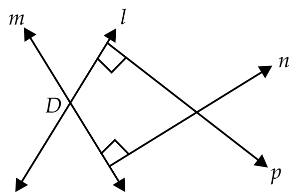
29. Let us draw a line OE parallel to AB. Then, clearly $OE \parallel CD$ also. [$\because AB \parallel CD$]



Now, as $AB \parallel OE$ and OA is a transversal.
$\therefore \quad \angle OAB + \angle AOE = 180°$ [Co-interior angles] ...(i)
Similarly $\angle COE + \angle OCD = 180°$
[Co-interior angles] ...(ii)
Adding (i) and (ii), we get
$\angle OAB + \angle AOE + \angle COE + \angle OCD = 360°$
$\Rightarrow \quad 120° + \angle AOC + 180° - x = 360°$
$\Rightarrow \quad 120° + 100° + 180° - x = 360°$
$\Rightarrow \quad x = 40°$

30. Given, $\angle AED = \angle BDC + \angle BAE$ ...(i) [Given]
Also, $\angle AED = \angle ABE + \angle BAE$ ...(ii)
[By exterior angle property of a triangle]
From (i) and (ii), we get
$\angle BDC + \angle BAE = \angle ABE + \angle BAE$
$\Rightarrow \quad \angle ABE = \angle BDC$
$\Rightarrow \quad \angle ABD = \angle BDC$ *i.e.*, a pair of alternate interior angles is equal.
So, $AB \parallel CD$.

31. Let lines l and m are two intersecting lines. Again, let n and p be another two lines which are perpendicular to m and l respectively.



Let us assume that lines n and p are not intersecting, that means they are parallel to each other *i.e.*, $n \parallel p$ ...(i)
Now, as $n \perp m$ and $n \parallel p$
$\therefore \quad p \perp m$
But $p \perp l$. $\therefore$ $m \parallel l$.
Which is not possible, because it is given that m and l are intersecting lines. So, our assumption is wrong.
Thus lines n and p intersect each other.

32. Since $AB \parallel CD$ and transversal DE intersects them at E and D respectively.
$\therefore \quad \angle AED = \angle CDP$ [Corresponding angles]
$\Rightarrow \quad \angle AED = 34°$ ...(i)
Now, ray EF stand on AB at E.
$\therefore \quad \angle AEF + \angle BEF = 180°$
$\Rightarrow \quad \angle AEP + \angle PEF + \angle BEF = 180°$
[$\because \angle AEF = \angle AEP + \angle PEF$]

$\Rightarrow$ $34° + \angle PEF + 78° = 180°$

[$\because \angle AEP = \angle AED = 34°$ from (i)]

$\Rightarrow$ $\angle PEF = 180° - 112°$

$\Rightarrow$ $\angle PEF = 68°$ ...(ii)

$\Rightarrow$ $\angle DEF = 68°$

Now, $EF \parallel DQ$ and transversal DE intersects them at E and D respectively.

$\therefore$ $\angle FED = \angle PDQ$ [Corresponding angles]

$\Rightarrow$ $\angle PDQ = 68°$ [$\because \angle FED = \angle PEF = 68°$ from (ii)]

Hence, $\angle PDQ = \angle DEF = 68°$ and $\angle AED = 34°$.

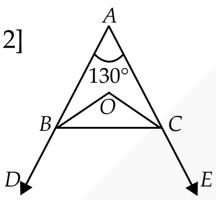
33. Let angles of a triangle be $\angle A$, $\angle B$ and $\angle C$, with $\angle A = 130°$

Clearly, in ΔABC, $\angle A + \angle B + \angle C = 180°$

[By angle sum property of a triangle]

$\Rightarrow$ $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180°}{2} = 90°$

[On dividing both sides by 2]



$\Rightarrow$ $\frac{1}{2}\angle B + \frac{1}{2}\angle C = 90° - \frac{1}{2}\angle A$

$\Rightarrow$ $\frac{1}{2}\angle B + \frac{1}{2}\angle C = 90° - \frac{130°}{2}$

$\Rightarrow$ $\frac{1}{2}\angle B + \frac{1}{2}\angle C = 25°$ ...(i)

Now, in ΔOBC

$\angle OBC + \angle OCB + \angle BOC = 180°$

[By angle sum property of a triangle]

$\Rightarrow$ $\frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BOC = 180°$ ...(ii)

[$\because$ BO and CO are bisectors of $\angle B$ and $\angle C$ respectively]

From (i) and (ii), we get

$\angle BOC = 180° - 25° = 155°$

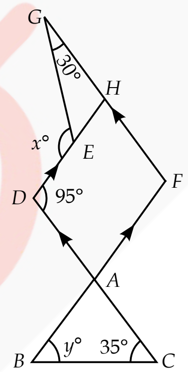
34. Since, $DE \parallel AF$ and AD is a transversal.

$\therefore$ $\angle EDA + \angle DAF = 180°$ [Co-interior angles]

$\Rightarrow$ $\angle DAF = 180° - 95° = 85°$ ...(i)

Clearly, $\angle BAC = \angle DAF$ [Vertically opposite angles]

$\Rightarrow$ $\angle BAC = 85°$ [Using (i)]



In ΔABC, we have

$85° + y° + 35° = 180°$

[By angle sum property of a triangle]

$\Rightarrow$ $y° = 180° - 120° = 60°$

$\Rightarrow$ $y = 60$

Now, let us extend DE such that it intersect FG at H as shown in the figure.

Since, $AD \parallel FG$ and DH is a transversal.

$\therefore$ $\angle GHE = 95°$

[Alternate interior angles]

Also, $x° = \angle EGH + \angle GHE$

[By exterior angle property of a triangle]

$\Rightarrow$ $x° = 30° + 95° = 125°$

$\Rightarrow$ $x = 125$

Thus, $x = 125$ and $y = 60$.