Coordinate Geometry

TRY YOURSELF SOLUTIONS

1. We have, $P(-6, 3)$ and $Q(-1, -4)$
Using distance formula, we get

$$PQ = \sqrt{(-1+6)^2 + (-4-3)^2} = \sqrt{25+49} = \sqrt{74} \text{ units}$$

2. We have, $P(a + b, a - b)$ and $Q(a - b, -a - b)$

$$\therefore \quad PQ = \sqrt{[(a-b)-(a+b)]^2 + [(-a-b)-(a-b)]^2}$$
$$= \sqrt{(a-b-a-b)^2 + (-a-b-a+b)^2}$$
$$= \sqrt{4b^2 + 4a^2} = 2\sqrt{a^2+b^2} \text{ units}$$

3. Let $P(x, -2)$ and $Q(3, 2)$ be the given points.
Then, we have $PQ = 5$ [Given]

$$\therefore \quad \sqrt{(3-x)^2 + (2+2)^2} = 5$$

$\Rightarrow$ $(3 - x)^2 + 16 = 25$ [Squaring both sides]
$\Rightarrow$ $x^2 + 9 - 6x + 16 = 25$
$\Rightarrow$ $x^2 - 6x = 0 \Rightarrow x(x - 6) = 0$
$\Rightarrow$ $x = 0$ or $x = 6$

4. Using distance formula, we have

$$AB = \sqrt{(6-3)^2 + (0-5)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$
$$BC = \sqrt{(1-6)^2 + (-3-0)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$
$$CD = \sqrt{(-2-1)^2 + (2+3)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$
$$DA = \sqrt{(3+2)^2 + (5-2)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$
$$\text{Diagonal, } AC = \sqrt{(1-3)^2 + (-3-5)^2}$$
$$= \sqrt{4+64} = \sqrt{68} \text{ units}$$
$$\text{and Diagonal, } BD = \sqrt{(-2-6)^2 + (2-0)^2}$$
$$= \sqrt{64+4} = \sqrt{68} \text{ units}$$

Since, $AB = BC = CD = DA$ *i.e.*, all the sides of given quadrilateral are equal in length and also, $AC = BD$ *i.e.*, diagonal are of same length.
Therefore, the given points are the vertices of a square $ABCD$.
Hence proved.

5. Using distance formula, we have

$$AB = \sqrt{(2+2)^2 + (-1-3)^2} = \sqrt{16+16} = 4\sqrt{2} \text{ units}$$
$$BC = \sqrt{(4-2)^2 + (-3+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$
$$\text{and } CA = \sqrt{(-2-4)^2 + (3+3)^2} = \sqrt{36+36} = 6\sqrt{2} \text{ units}$$

Now, as $AB + BC = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$ units
i.e., $AB + BC = CA$
$\therefore$ A, B and C are collinear points.

6. Let $A(3, 2)$, $B\left(4, \frac{5}{2}\right)$ and $C(5, 3)$ are given points.
Using distance formula, we have

$$AB = \sqrt{(4-3)^2 + \left(\frac{5}{2}-2\right)^2} = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2} \text{ units}$$
$$BC = \sqrt{(5-4)^2 + \left(3-\frac{5}{2}\right)^2} = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2} \text{ units}$$
$$\text{and } AC = \sqrt{(5-3)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$
$$\text{Now, as } AB + BC = \frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2} = \sqrt{5} \text{ units} = AC$$

$\therefore$ The given points are collinear.
Hence proved.

7. Given, point $P(x, y)$ is equidistant from the points $A(6, 1)$ and $B(1, 6)$
$\therefore$ $AP = PB \Rightarrow AP^2 = PB^2$
$\Rightarrow$ $(x - 6)^2 + (y - 1)^2 = (x - 1)^2 + (y - 6)^2$
$\Rightarrow$ $x^2 + 36 - 12x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 36 - 12y$
$\Rightarrow$ $-10x = -10y \Rightarrow x = y$
Hence proved.

8. Given, $A(a + b, b - a)$ and $B(a - b, a + b)$ are equidistant from $P(x, y)$.
$\therefore$ $AP = PB \Rightarrow (AP)^2 = (PB)^2$
$\Rightarrow$ $[x - (a + b)]^2 + [y - (b - a)]^2 = [x - (a - b)]^2 + [y - (a + b)]^2$
$\Rightarrow$ $x^2 + (a + b)^2 - 2(a + b)x + y^2 + (b - a)^2 - 2(b - a)y$
$= x^2 + (a - b)^2 - 2(a - b)x + y^2 + (a + b)^2 - 2(a + b)y$
$\Rightarrow$ $-2[(a + b)x + (b - a)y] = -2[(a - b)x + (a + b)y]$
$\Rightarrow$ $(a + b)x + (b - a)y = (a - b)x + (a + b)y$
$\Rightarrow$ $(a + b - a + b)x = (a + b - b + a)y$
$\Rightarrow$ $2bx = 2ay$ or $bx = ay$
Hence proved.

9. Let P be $(0, y)$, which is equidistant from $A(4, 8)$ and $B(-6, 6)$.
$\therefore$ $AP = PB \Rightarrow AP^2 = PB^2$
$\Rightarrow$ $(4 - 0)^2 + (8 - y)^2 = (0 + 6)^2 + (y - 6)^2$
$\Rightarrow$ $16 + 64 + y^2 - 16y = 36 + y^2 + 36 - 12y$
$\Rightarrow$ $80 - 16y = 72 - 12y$

$\Rightarrow\ 4y = 8 \Rightarrow y = 2$

$\therefore$ Coordinates of P are (0, 2).

Now, $AP = \sqrt{(4-0)^2+(8-2)^2}$

$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$ units

10. Let the point $P(0, y)$ divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$ in the ratio $k : 1$.

Using section formula, we have coordinates of P are $\left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$.

Since, x-coordinate of P is zero

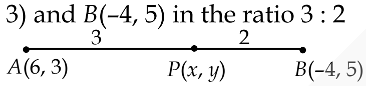
$\therefore \frac{-k+5}{k+1} = 0 \Rightarrow -k+5 = 0 \Rightarrow k = 5$

Hence, the point P divides the line segment in the ratio 5 : 1.

Also, y-coordinate of $P = \frac{-4(5)-6}{5+1} = \frac{-20-6}{5+1} = \frac{-26}{6} = \frac{-13}{3}$

$\therefore$ Coordinates of P are $\left(0, \frac{-13}{3}\right)$.

11. Let $P(x, y)$ divides the line segment joining the points $A(6, 3)$ and $B(-4, 5)$ in the ratio 3 : 2



Using section formula, we have

$$(x, y) = \left(\frac{3\times(-4)+2\times 6}{3+2}, \frac{3\times 5+2\times 3}{3+2}\right)$$

$$= \left(\frac{-12+12}{5}, \frac{15+6}{5}\right) = \left(0, \frac{21}{5}\right)$$

12. Let $P(-3, p)$ divides the line segment joining the points $A(-5, -4)$ and $B(-2, 3)$ in the ratio $k : 1$.

Using section formula,

Coordinates of P are $\left(\frac{-2k-5}{k+1}, \frac{3k-4}{k+1}\right)$

But, coordinates of P are given as $(-3, p)$.

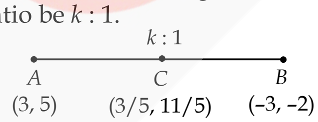
$\therefore \frac{-2k-5}{k+1} = -3$ and $\frac{3k-4}{k+1} = p$

$\Rightarrow -2k - 5 = -3k - 3 \Rightarrow k = 2$

Now, $p = \frac{3\times 2-4}{2+1} = \frac{6-4}{3} = \frac{2}{3}$

Hence, the ratio is 2 : 1 and $p = \frac{2}{3}$

13. Let the ratio be $k : 1$.



Using section formula, we have coordinates of C are $\left(\frac{-3k+3}{k+1}, \frac{-2k+5}{k+1}\right)$

But, coordinates of C are given as $\left(\frac{3}{5}, \frac{11}{5}\right)$.

$\therefore \frac{-3k+3}{k+1} = \frac{3}{5}$ and $\frac{-2k+5}{k+1} = \frac{11}{5}$

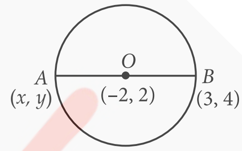
$\Rightarrow -15k + 15 = 3k + 3$ and $-10k + 25 = 11k + 11$

$\Rightarrow -18k = -12$ and $-21k = -14 \Rightarrow k = \frac{2}{3}$

Hence, the ratio is $\frac{2}{3} : 1$ *i.e.*, 2 : 3.

14. Let coordinates of the point A be (x, y) and O is the mid-point of AB.

Using mid-point formula, we have



$-2 = \frac{x+3}{2}$ and $2 = \frac{y+4}{2}$

$\Rightarrow -4 = x + 3$ and $4 = y + 4$

$\Rightarrow x = -7$ and $y = 0$

$\therefore$ Coordinates of A are (–7, 0).

15. Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram, $ABCD$ taken in order.

$\because$ The diagonals of parallelogram bisect each other.

$\therefore$ Mid-point of AC = Mid-point of BD

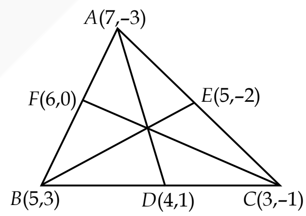
$\Rightarrow \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$

$\Rightarrow \left(\frac{1}{2}, 1\right) = \left(\frac{3+x}{2}, \frac{1+y}{2}\right) \Rightarrow \frac{3+x}{2} = \frac{1}{2}$ and $\frac{1+y}{2} = 1$

$\Rightarrow 3 + x = 1$ and $1 + y = 2 \Rightarrow x = -2$ and $y = 1$

Hence, the coordinates of fourth vertex is (–2, 1)

16. Let D, E and F be the mid-point of the sides BC, CA and AB respectively. Then, the coordinates of D, E and F are



$D\left(\frac{5+3}{2}, \frac{3-1}{2}\right) = D\ (4, 1)$,

$E = \left(\frac{3+7}{2}, \frac{-1-3}{2}\right) = E(5, -2)$

and $F\left(\frac{7+5}{2}, \frac{-3+3}{2}\right) = F(6, 0)$

$\therefore AD = \sqrt{(7-4)^2+(-3-1)^2} = \sqrt{9+16} = 5$ units;

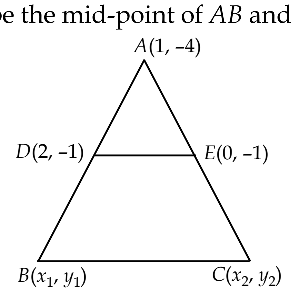
$BE = \sqrt{(5-5)^2+(-2-3)^2} = \sqrt{0+25} = 5$ units and

$CF = \sqrt{(6-3)^2+(0+1)^2} = \sqrt{9+1} = \sqrt{10}$ units

$\therefore$ Lengths of medians of ΔABC are 5 units, 5 units and $\sqrt{10}$ units.

17. Let $D(2, -1)$ and $E(0, -1)$ be the mid-point of AB and AC respectively.

Let coordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively.



$\therefore$ Mid-point of AB

$= \left(\frac{x_1+1}{2}, \frac{y_1-4}{2}\right)$

$\Rightarrow (2, -1) = \left(\frac{x_1+1}{2}, \frac{y_1-4}{2}\right)$ (Given)

$\Rightarrow 2 = \frac{x_1+1}{2}$ and $-1 = \frac{y_1-4}{2}$

$\Rightarrow$ $4 = x_1 + 1$ and $-2 = y_1 - 4$ $\Rightarrow$ $x_1 = 3$ and $y_1 = 2$

$\therefore$ Coordinates of B are (3, 2).

Now, mid-point of $AC = \left(\frac{1+x_2}{2}, \frac{-4+y_2}{2}\right)$

$\Rightarrow$ $(0,-1) = \left(\frac{1+x_2}{2}, \frac{-4+y_2}{2}\right)$ (Given)

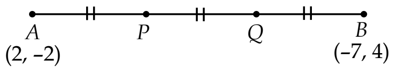
$\Rightarrow$ $0 = \frac{1+x_2}{2}$ and $-1 = \frac{-4+y_2}{2}$

$\Rightarrow$ $x_2 + 1 = 0$ and $-2 = -4 + y_2$ $\Rightarrow$ $x_2 = -1$ and $y_2 = 2$

$\therefore$ Coordinates of C are (–1, 2).

Now, mid-point of $BC = \left(\frac{3-1}{2}, \frac{2+2}{2}\right) = (1, 2)$.

18. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ divides AB into three equal parts.



$\therefore$ P divides AB in the ratio 1 : 2

Using section formula, we have

$$(x_1, y_1) = \left(\frac{-7+4}{1+2}, \frac{4-4}{1+2}\right) = \left(\frac{-3}{3}, 0\right) = (-1, 0)$$

$\therefore$ Coordinates of P are (–1, 0)

Also, Q divides AB in the ratio 2 : 1.

Using section formula, we have

$$(x_2, y_2) = \left(\frac{-14+2}{2+1}, \frac{8-2}{2+1}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

$\therefore$ Coordinates of Q are (–4, 2).

19. Let $G(x, y)$ be the centroid of triangle.

$\therefore$ $(x, y) = \left(\frac{-2+4+4}{3}, \frac{3-3+5}{3}\right) = \left(\frac{6}{3}, \frac{5}{3}\right) = \left(2, \frac{5}{3}\right)$

20. Let G be the centroid of triangle.

$\therefore$ Centroid of $\Delta ABC\ (G) = \left(\frac{-1+0-5}{3}, \frac{3+4+2}{3}\right)$

$= \left(\frac{-6}{3}, \frac{9}{3}\right) = (-2, 3)$

Since, G lies on the median $x - 2y + k = 0$

So, coordinates of G must satisfy the equation of median.

$\therefore$ $-2 - 2 \times 3 + k = 0 \Rightarrow -2 - 6 + k = 0 \Rightarrow k = 8$

21. Let $C(x, y)$ be the third vertex.

Given, centroid of $\Delta\, ABC$, $(G) = \left(\frac{5}{3}, -\frac{1}{3}\right)$

$\Rightarrow$ $\left(\frac{3-2+x}{3}, \frac{2+1+y}{3}\right) = \left(\frac{5}{3}, \frac{-1}{3}\right)$

$\Rightarrow$ $\left(\frac{1+x}{3}, \frac{3+y}{3}\right) = \left(\frac{5}{3}, \frac{-1}{3}\right)$

$\Rightarrow$ $\frac{1+x}{3} = \frac{5}{3}$ and $\frac{3+y}{3} = \frac{-1}{3}$

$\Rightarrow$ $1 + x = 5$ and $3 + y = -1$

$\Rightarrow$ $x = 4$ and $y = -4$

Hence, coordinates of C are (4, –4).

22. We have, $A(3, 0)$, $B(7,0)$ and $C(8,4)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}[3(0-4)+7(4-0)+8(0-0)]$

$= \frac{1}{2}[-12+28+0] = \frac{1}{2}(16) = 8$ sq. units

23. The points $A(9, k)$, $B(4, -2)$ and $C(3, -3)$ are collinear.

$\therefore$ Area of $\Delta ABC = 0$

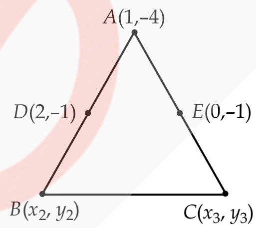
$\Rightarrow$ $\frac{1}{2}[9(-2+3)+4(-3-k)+3(k+2)] = 0$

$\Rightarrow$ $[9 - 12 - 4k + 3k + 6] = 0$

$\Rightarrow$ $[3 - k] = 0 \Rightarrow k = 3$

24. Let $A(x_1, y_1) = (1, -4)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC.

Since, $D(2, -1)$ is the mid-point of AB.



$\therefore$ $2 = \frac{1+x_2}{2}$ and $-1 = \frac{-4+y_2}{2}$

$\Rightarrow$ $x_2 = 4 - 1 = 3$

and $y_2 = -2 + 4 = 2$

$\therefore$ $B(x_2, y_2) = (3, 2)$

Also, $E(0, -1)$ is the mid-point of AC.

$\therefore$ $0 = \frac{1+x_3}{2}$ and $-1 = \frac{-4+y_3}{2}$

$\Rightarrow$ $x_3 = -1$ and $y_3 = -2 + 4 = 2$

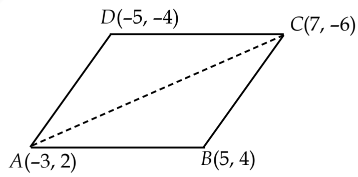
$\Rightarrow$ $C(x_3, y_3) = (-1, 2)$

Now, area of ΔABC

$= \frac{1}{2}[1(2-2)+3(2+4)+(-1)(-4-2)]$

$= \frac{1}{2}[0+18+6] = \frac{1}{2} \times 24 = 12$ sq. units

25. We have, $A(-3, 2)$, $B(5, 4)$, $C(7, -6)$ and $D(-5, -4)$



Let us join diagonal AC.

Area of $\Delta ABC = \frac{1}{2}[-3(4+6)+5(-6-2)+7(2-4)]$

$= \frac{1}{2}[-3\times10+5\times(-8)+7(-2)]$

$= \frac{1}{2}[-30-40-14] = \frac{-84}{2} = -42$

Since area of triangle cannot be negative

$\therefore$ Area of ΔABC = 42 sq.units

Area of $\Delta ACD = \frac{1}{2}[-3(-6-(-4))+7(-4-2)-5(2-(-6))]$

$= \frac{1}{2}[-3(-2)+7(-6)-5(8)] = \frac{1}{2}[6-42-40] = \frac{-76}{2} = -38$

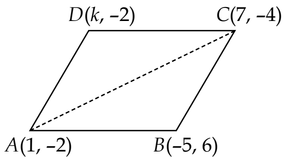
Since area of triangle cannot be negative

$\therefore$ Area of ΔACD = 38 sq.units

Now, area of quadrilateral $ABCD$

= Area of ΔABC + Area of ΔACD = 42 sq.units + 38 sq.units = 80 sq.units

26. Let $A(1, 2)$, $B(-5, 6)$, $C(7, -4)$ and $D(k, -2)$ be the vertices of quadrilateral $ABCD$.



Let us join diagonal AC.

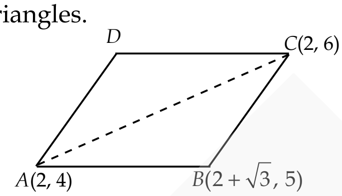
Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD

$$\Rightarrow \frac{1}{2}[1(6-(-4))+(-5)(-4-2)+7(2-6)] + \frac{1}{2}[(-4-(-2))+7(-2-2)+k(2-(-4))]=0$$

$$\Rightarrow \frac{1}{2}[10+30-28]+\frac{1}{2}[-2-28+6k]=0$$

$\Rightarrow$ $6 - 15 + 3k = 0 \Rightarrow 3k = 9 \Rightarrow k = 3$

27. Since, diagonals of parallelogram divides it into two congruent triangles.



$\therefore$ Area of ΔABC = Area of ΔACD.

$\therefore$ Area of parallelogram $ABCD$ = 2(Area of ΔABC)

$$= 2\times\frac{1}{2}[2(5-6)+(2+\sqrt{3})(6-4)+2(4-5)]$$

$$= 2(-1)+(2+\sqrt{3})(2)+2(-1)$$

$$= -2+4+2\sqrt{3}-2 = 2\sqrt{3} \text{ sq.units}$$