Triangles

TRY YOURSELF

SOLUTIONS

1. (i) True :
Since, a regular quadrilateral is always a square.
Hence, in any two squares, all the angles are equal (each 90°) and sides would be proportional.
$\therefore$ Two regular quadrilaterals are always similar.
(ii) True :
Since, two right angled triangles can also be similar.
$\therefore$ If two triangles are similar, then they are not necessarily equilateral.
(iii) False :
Since, two congruent figures are always similar but two similar figures need not be congruent always.
(iv) True :
Since, all the sides are equal in equilateral triangle.
$\therefore$ The corresponding sides of equilateral triangles are always proportional.

2. In $ABCDE$, $\angle E = 540° - (\angle A + \angle B + \angle C + \angle D)$
$= 540° - (80° + 130° + 70° + 140°) = 540° - 420° = 120°$
Similarly, $\angle T = 120°$
Now, in pentagons, $ABCDE$ and $PQRST$, we have
(i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, $\angle D = \angle S$ and $\angle E = \angle T$
(ii) $\frac{AB}{PQ} = \frac{2}{1}, \frac{BC}{QR} = \frac{5}{2.5} = \frac{2}{1}, \frac{CD}{RS} = \frac{3}{1.5} = \frac{2}{1},$
$\frac{DE}{ST} = \frac{2}{1}, \frac{EA}{TP} = \frac{3.6}{1.8} = \frac{2}{1}$
Hence, the two figures are similar as their corresponding angles are equal and their corresponding sides are in the same ratio *i.e.* 2/1.

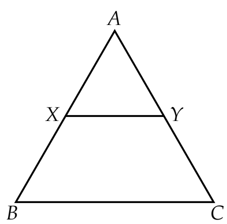
3. Given, $\Delta ABC \sim \Delta DFE$
$\Rightarrow \angle A = \angle D, \angle B = \angle F$ and $\angle C = \angle E$ ...(i)
Now, in ΔABC, $\angle A = 30°$, $\angle C = 50°$
$\therefore \angle B = 180° - (\angle A + \angle C)$
$= 180° - (30° + 50°) = 180° - 80° = 100°$
Hence, using (i), $\angle F = 100°$
Also, $\frac{AB}{DF} = \frac{AC}{DE}$ $(\because \Delta ABC \sim \Delta DFE)$
$\Rightarrow \frac{5}{7.5} = \frac{8}{DE} \Rightarrow DE = \frac{8 \times 7.5}{5} = 12$ cm

4. Given, $\Delta ABC \sim \Delta PQR$, $AC = 4\sqrt{3}$ cm, $BC = 8$ cm, $PQ = 3$ cm, $QR = 6$ cm
Now, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \Rightarrow \frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$
From last two terms, we get
$\frac{8}{6} = \frac{4\sqrt{3}}{y} \Rightarrow y = \frac{6 \times 4\sqrt{3}}{8} = 3\sqrt{3}$ cm ...(i)
From first two terms, we get
$\frac{z}{3} = \frac{8}{6} \Rightarrow z = \frac{24}{6} \Rightarrow z = 4$ cm ...(ii)
From (i) and (ii), we get $y + z = (3\sqrt{3} + 4)$ cm

5. Given, $\Delta PQR \sim \Delta TSM$
$\therefore \angle P = \angle T, \angle Q = \angle S$ and $\angle R = \angle M$ ...(i)
and $\frac{PQ}{TS} = \frac{QR}{SM} = \frac{RP}{MT}$ ...(ii)
Given, $\angle P = 55°$, $\angle S = 25°$ ...(iii)
and $PQ = 7$ cm, $QR = 9$ cm, $TS = 21$ cm, $MT = 24$ cm ...(iv)
Now, using (iv) in (ii), we have
$\frac{7}{21} = \frac{9}{SM} = \frac{RP}{24}$
Using first two terms, $\frac{7}{21} = \frac{9}{SM}$
$\Rightarrow SM = \frac{9 \times 21}{7} \Rightarrow SM = 27$ cm
Using first and last term, $\frac{7}{21} = \frac{RP}{24}$
$\Rightarrow RP = \frac{7 \times 24}{21} = 8$ cm
$\therefore$ Difference of remaining two sides
$= SM - RP = 27 - 8 = 19$ cm
Using (iii) in (i), we have
$\angle P = 55° = \angle T$
$\angle Q = \angle S = 25°$
$\angle R = \angle M = 180° - (55° + 25°) = 180° - 80° = 100°$

6. Given, $PX = 3$ cm, $PY = 7.5$ cm and $XQ = 2$ cm
In ΔXYZ, $PQ \parallel YZ$
$\therefore$ By basic proportionality theorem, we have
$\frac{XP}{PY} = \frac{XQ}{QZ}$
$\Rightarrow \frac{3}{7.5} = \frac{2}{QZ} \Rightarrow QZ = \frac{2 \times 7.5}{3} = 5$ cm
Now, $XZ = XQ + QZ = (2 + 5)$ cm $= 7$ cm

7. **Given :** A ΔABC, X and Y are points on AB and AC respectively such that $XY \parallel BC$ and $BX = CY$
To prove : ABC is an isosceles triangle.



Proof : Since $XY \parallel BC$
$\therefore \frac{AX}{BX} = \frac{AY}{CY}$
[By basic proportionality theorem]

$\Rightarrow \quad \frac{AX}{BX} = \frac{AY}{BX}$ $\quad [\because BX = CY]$

$\Rightarrow \quad AX = AY$ ...(i)

Also, $BX = CY$ ...(ii)

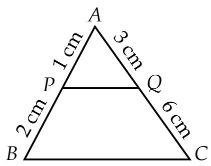
Adding (i) and (ii), we get

$AX + BX = AY + CY$

$\Rightarrow \quad AB = AC$

$\Rightarrow \quad ABC$ is an isosceles triangle.

8. In ΔABC, we have $\frac{AP}{PB} = \frac{1}{2}$



and $\frac{AQ}{QC} = \frac{3}{6} = \frac{1}{2}$

Hence, $\frac{AP}{PB} = \frac{AQ}{QC}$

$\therefore \quad PQ \parallel BC$

[By converse of basic proportionality theorem]

In ΔAPQ and ΔABC

$\angle APQ = \angle ABC$ and $\angle AQP = \angle ACB$

[$\because$ Corresponding angles as $PQ \parallel BC$]

$\angle PAQ = \angle BAQ$ [Common]

$\therefore \quad \Delta APQ \sim \Delta ABC$ [By AAA similarity criterion]

$\Rightarrow \quad \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

Now, $\frac{AP}{AB} = \frac{PQ}{BC} \Rightarrow \frac{AP}{AP+BP} = \frac{PQ}{BC}$

$\Rightarrow \quad \frac{1}{1+2} = \frac{PQ}{BC} \quad \Rightarrow BC = 3PQ$

9. Let AB be the lamp post of height 3.9 m.

Height of Rama, CD = 120 cm = 1.2 m

Distance covered by Rama in 3 seconds = 1.5 × 3 = 4.5 m

Let DE be the shadow of Rama after 3 seconds.

In ΔABE and ΔCDE

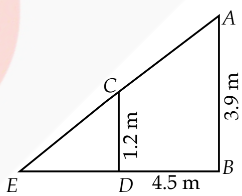
$\angle ABE = \angle CDE = 90°$

$\angle AEB = \angle CED$ [Common]

$\therefore \quad \Delta ABE \sim \Delta CDE$ [By AA similarity criterion]

$\therefore \quad \frac{AB}{CD} = \frac{BE}{DE}$

$\Rightarrow \quad \frac{3.9}{1.2} = \frac{4.5+DE}{DE}$



$\Rightarrow \quad 3.9DE = 5.4 + 1.2\,DE$

$\Rightarrow \quad (3.9 - 1.2)DE = 5.4$

$\Rightarrow \quad DE = \frac{5.4}{2.7} = 2$ m

Hence, the shadow of Rama is 2 m.

10. In ΔPRQ and ΔSTQ,

$\angle PRQ = \angle STQ = 90°$

$\angle PQR = \angle SQT$ [Common]

So, $\Delta PRQ \sim \Delta STQ$ [By AA similarity criterion]

$\Rightarrow \quad \frac{QR}{QT} = \frac{QP}{QS}$

$\Rightarrow \quad QR \times QS = QP \times QT$

11. In ΔABC and ΔBDC,

$\angle ABC = \angle BDC = 90°$

$\angle ACB = \angle BCD$ [Common]

$\therefore \quad \Delta ABC \sim \Delta BDC$ [By AA similarity criterion]

$\Rightarrow \quad \angle BAC = \angle DBC$ ...(i)

Now, in ΔADB and ΔBDC,

$\angle BDA = \angle CDB = 90°$

$\angle BAD = \angle CBD$ [From (i)]

$\therefore \quad \Delta ADB \sim \Delta BDC$ [By AA similarity criterion]

$\Rightarrow \quad \frac{BD}{CD} = \frac{AD}{BD} \Rightarrow CD = \frac{BD^2}{AD}$

$\Rightarrow \quad CD = \frac{8^2}{4} = \frac{64}{4} = 16$ cm

12. (i) In ΔABO and ΔDCO, $\frac{AO}{DO} = \frac{16}{9}$ But $\frac{BO}{CO} = \frac{9}{5}$.

Hence, $\frac{AO}{DO} \neq \frac{BO}{CO}$

Thus, ΔABO and ΔDCO are not similar.

(ii) In ΔPQR, $\angle P + \angle Q + \angle R = 180°$

$\Rightarrow \quad \angle R = 180° - 45° - 78° = 57°$

In ΔLMN, $\angle L + \angle M + \angle N = 180°$

$\Rightarrow \quad \angle N = 180° - 57° - 45° = 78°$

So, $\angle P = \angle M$, $\angle Q = \angle N$, $\angle R = \angle L$

$\therefore \quad \Delta PQR \sim \Delta MNL$ (By AAA similarity criterion)

13. In ΔABC and ΔDEF,

$\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}, \frac{BC}{EF} = \frac{8}{16} = \frac{1}{2}, \frac{AC}{DF} = \frac{9}{18} = \frac{1}{2}$

$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\therefore \quad \Delta ABC \sim \Delta DEF$ [By SSS similarity criterion]

$\Rightarrow \quad \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

$\Rightarrow \quad x = 87°, y = 58°, z = 35°$

14. In ΔABC and ΔDFE,

$\frac{AB}{DF} = \frac{3.8}{11.4} = \frac{1}{3}, \frac{BC}{FE} = \frac{6}{18} = \frac{1}{3}, \frac{AC}{DE} = \frac{3\sqrt{3}}{9\sqrt{3}} = \frac{1}{3}$

$\Rightarrow \quad \frac{AB}{DF} = \frac{BC}{FE} = \frac{AC}{DE}$

$\therefore \quad \Delta ABC \sim \Delta DFE$ (By SSS similarity criterion)

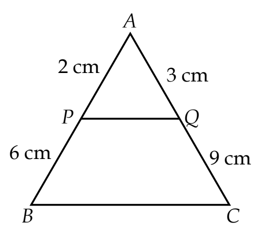
$\Rightarrow \quad \angle A = \angle D, \angle B = \angle F, \angle C = \angle E$ ...(i)

Now, in ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 75° - 65° = 40°$

$\therefore \quad \angle E = 40°$ [From (i)]

15. In ΔAPQ and ΔABC,



$\frac{AP}{AB} = \frac{2}{2+6} = \frac{2}{8} = \frac{1}{4}$

$\frac{AQ}{AC} = \frac{3}{3+9} = \frac{3}{12} = \frac{1}{4}$

$\Rightarrow \quad \frac{AP}{AB} = \frac{AQ}{AC}$

and $\angle PAQ = \angle BAC$ [Common]

$\therefore \quad \Delta APQ \sim \Delta ABC$ [By SAS similarity criterion]

$\Rightarrow \quad \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC} = \frac{1}{4}$

$\Rightarrow \quad BC = 4PQ.$

16. In ΔABC and ΔFED

$\frac{AB}{FE} = \frac{6}{4.5} = \frac{4}{3}$ and $\frac{BC}{ED} = \frac{4}{3}$

Also, $\angle ABC = \angle FED = 85°$

$\therefore \quad \Delta ABC \sim \Delta FED$ [By SAS similarity criterion]

In ΔABC and ΔQPR

$\frac{AB}{PQ} = \frac{6}{12} = \frac{1}{2}$ and $\frac{BC}{PR} = \frac{4}{9} \Rightarrow \frac{AB}{PQ} \neq \frac{BC}{PR}$

$\Rightarrow \quad \Delta ABC$ and ΔQPR are not similar.

Similarly, ΔDEF and ΔRPQ are not similar.

17. We have, $\Delta ABC \sim \Delta DEF$

$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2}$

[$\because$ Areas of two similar triangles are in the ratio of the squares of their corresponding sides]

$\Rightarrow \quad \frac{169}{121} = \frac{(26)^2}{(DE)^2} \Rightarrow (DE)^2 = \frac{676 \times 121}{169} = 484$

$\Rightarrow \quad DE = 22$ cm.

18. Let the length of median of ΔCAD be x cm.

We have, $\Delta BAT \sim \Delta CAD$

$\therefore \quad \frac{ar(\Delta BAT)}{ar(\Delta CAD)} = \frac{(12.1)^2}{x^2}$

[$\because$ Areas of two similar triangles are in the ratio of the squares of their corresponding medians]

$\Rightarrow \quad \frac{121}{64} = \frac{(12.1)^2}{x^2}$

$\Rightarrow \quad x^2 = \frac{12.1 \times 12.1 \times 64}{121} = \frac{121 \times 64}{100} = \frac{7744}{100}$

$\Rightarrow \quad x = \frac{88}{10} = 8.8$

Thus, corresponding median of ΔCAD is 8.8 cm.

19. In ΔPST and ΔPQR,

$\therefore \quad \angle PST = \angle PQR$ and $\angle PTS = \angle PRQ$

[Corresponding angles, as $ST \parallel QR$]

$\angle SPT = \angle QPR$ [Common]

$\therefore \quad \Delta PST \sim \Delta PQR$ [By AAA similarity criterion]

$\Rightarrow \frac{PS}{PQ} = \frac{PT}{PR} = \frac{ST}{QR}$

Now, $\frac{PT}{PR} = \frac{PT}{PT + TR} = \frac{2}{2+4} = \frac{2}{6} = \frac{1}{3}$

Since, areas of two similar triangles are in the ratio of the squares of their corresponding sides.

$\therefore \quad \frac{ar(\Delta PST)}{ar(\Delta PQR)} = \left(\frac{PT}{PR}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9} = 1:9$

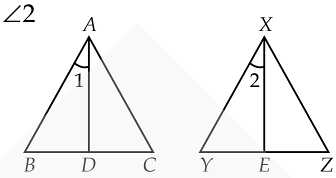
20. Given, AD and XE are angle bisectors of $\angle A$ and $\angle X$ respectively.

$\therefore \quad \angle 1 = \frac{1}{2}\angle A$ and $\angle 2 = \frac{1}{2}\angle X$

Now, $\Delta ABC \sim \Delta XYZ$

$\therefore \quad \angle A = \angle X$

$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle X \Rightarrow \angle 1 = \angle 2$



and $\angle B = \angle Y$

In ΔABD and ΔXYE,

$\angle 1 = \angle 2$ and $\angle B = \angle Y$

$\therefore \quad \Delta ABD \sim \Delta XYE$

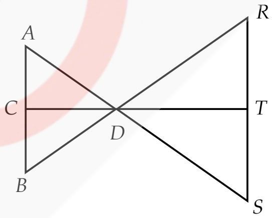
[By AA similarity criterion]

$\Rightarrow \quad \frac{ar(\Delta ABD)}{ar(\Delta XYE)} = \frac{(AD)^2}{(XE)^2}$

[$\because$ Areas of similar triangles are in the ratio of squares of their corresponding sides]

$\Rightarrow \frac{ar(\Delta ABD)}{ar(\Delta XYE)} = \frac{(4)^2}{(3)^2} = \frac{16}{9} = 16:9$

21. We have $\Delta DAB \sim \Delta DRS$



and $\frac{CD}{TD} = \frac{4}{5}$

Now,

$\frac{ar(\Delta DAB)}{ar(\Delta DRS)} = \frac{(CD)^2}{(TD)^2} = \left(\frac{CD}{TD}\right)^2$

[$\because$ Areas of similar triangles are in the ratio of squares of their corresponding medians]

$\Rightarrow \quad \frac{ar(\Delta DAB)}{ar(\Delta DRS)} = \left(\frac{4}{5}\right)^2 = \frac{16}{25} \neq \frac{12}{15}$

22. We have, $AB = 15$ cm, $BC = 17$ cm, $AC = 8$ cm

Now, $(AB)^2 + (AC)^2 = (15)^2 + (8)^2$

$= 225 + 64 = 289 = (17)^2 = (BC)^2$

Thus, by converse of Pythagoras theorem, ΔABC is a right triangle.

23. In right ΔADC, we have,

$AC^2 = AD^2 + DC^2$ [By Pythagoras theorem]

$\Rightarrow \quad b^2 = h^2 + (a - x)^2 \quad \Rightarrow \quad b^2 = h^2 + a^2 + x^2 - 2ax$

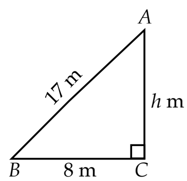
24. Let AB be the ladder and AC be the wall.

Let $AC = h$ m

In right ΔABC,

$AB^2 = AC^2 + BC^2$

[By Pythagoras theorem]



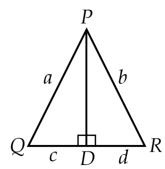
$\Rightarrow \quad 17^2 = h^2 + 8^2$

$\Rightarrow \quad h^2 = 289 - 64 = 225 \quad \Rightarrow \quad h = 15$

25. In right ΔPDQ,

$PQ^2 = PD^2 + QD^2$

[By Pythagoras theorem]



$\Rightarrow \quad a^2 = PD^2 + c^2 \quad \Rightarrow \quad PD^2 = a^2 - c^2$ ...(i)

In right ΔPDR,

$PR^2 = PD^2 + DR^2$ [By Pythagoras theorem]

$\Rightarrow \quad b^2 = PD^2 + d^2 \quad \Rightarrow \quad PD^2 = b^2 - d^2$ ...(ii)

From (i) and (ii), we get

$a^2 - c^2 = b^2 - d^2$

$\Rightarrow \quad a^2 - b^2 = c^2 - d^2$

$\Rightarrow \quad (a + b)(a - b) = (c + d)(c - d)$