Quadratic Equations

CHAPTER 4

TRY YOURSELF — SOLUTIONS

1. We have, $(x-3)^2 + 2 = 5x - 9$

$\Rightarrow x^2 + 9 - 6x + 2 = 5x - 9 \Rightarrow x^2 - 11x + 20 = 0$

It is of the form $ax^2 + bx + c = 0, a \neq 0$.

So, the given equation is a quadratic equation.

2. We have, $x(7x - 12) = 7(x^2 - 7x + 9)$

$\Rightarrow 7x^2 - 12x = 7x^2 - 49x + 63 \Rightarrow 37x - 63 = 0$

It is not of the form $ax^2 + bx + c = 0, a \neq 0$.

So, the given equation is not a quadratic equation.

3. We have, $x(6x + 5) = 8x^2 + 6x$

$\Rightarrow 6x^2 + 5x = 8x^2 + 6x \Rightarrow 2x^2 + x = 0$

It is of the form $ax^2 + bx + c = 0, a \neq 0$.

So, the given equation is a quadratic equation.

4. We have, $2x - \frac{3}{x} - 15 = 5\left(2x - \frac{3}{x}\right), x \neq 0$

$\Rightarrow 2x - \frac{3}{x} - 15 = 10x - \frac{15}{x}$

$\Rightarrow 2x^2 - 3 - 15x = 10x^2 - 15$ $[\because x \neq 0]$

$\Rightarrow 8x^2 + 15x - 12 = 0$

It is of the form $ax^2 + bx + c = 0, a \neq 0$.

So, the given equation is a quadratic equation.

5. Let two consecutive odd integers be $2x + 1$ and $2x + 3$.

According to question, $(2x + 1)(2x + 3) = 783$

$\Rightarrow 4x^2 + 6x + 2x + 3 = 783$

$\Rightarrow 4x^2 + 8x - 780 = 0 \Rightarrow x^2 + 2x - 195 = 0$

This is the required quadratic equation.

6. Let the present age of Raju be x years.

$\therefore$ His father's present age = $(x + 30)$ years

After 5 years, Raju's age = $(x + 5)$ years

After 5 years, father's age = $(x + 30 + 5) = (x + 35)$ years

According to question, $(x + 5)(x + 35) = 450$

$\Rightarrow x^2 + 35x + 5x + 175 = 450 \Rightarrow x^2 + 40x - 275 = 0$

This is the required quadratic equation.

7. Given equation is in the form $p(x) = 0$,

where $p(x) = 5x^2 - 126x + 25$ ...(i)

On putting $x = 25$ in (i), we get

$p(25) = 5(25)^2 - 126(25) + 25 = 3125 - 3150 + 25 = 0$

On putting $x = \frac{1}{10}$ in (i), we get

$p\left(\frac{1}{10}\right) = 5\left(\frac{1}{10}\right)^2 - 126\left(\frac{1}{10}\right) + 25$

$= \frac{5 - 1260 + 2500}{100} = \frac{1245}{100} \neq 0.$

Hence, $x = 25$ is a solution but $x = \frac{1}{10}$ is not a solution of the given quadratic equation.

8. Given equation is in the form $p(x) = 0$,

where $p(x) = x^2 + 8x + 4$ ...(i)

On putting $x = -2$ in (i), we get

$p(-2) = (-2)^2 + 8(-2) + 4 = 4 - 16 + 4 = -8 \neq 0$

On putting $x = -4$ in (i), we get

$p(-4) = (-4)^2 + 8(-4) + 4 = 16 - 32 + 4 = -12 \neq 0$

So, $x = -2$ and -4 both are not the solutions of the given equation.

9. Given equation is in the form $p(x) = 0$,

where $p(x) = x^2 - 4\sqrt{2}x + 2\sqrt{2}$ ...(i)

On putting $x = \sqrt{2}$ in (i), we get

$p(\sqrt{2}) = (\sqrt{2})^2 - 4\sqrt{2}(\sqrt{2}) + 2\sqrt{2}$

$= 2 - 8 + 2\sqrt{2} = 2\sqrt{2} - 6 \neq 0$

On putting $x = \sqrt{2} + 1$ in (i), we get

$p(\sqrt{2}+1) = (\sqrt{2}+1)^2 - 4\sqrt{2}(\sqrt{2}+1) + 2\sqrt{2}$

$= 2 + 1 + 2\sqrt{2} - 8 - 4\sqrt{2} + 2\sqrt{2} = -5 \neq 0$

So, $x = \sqrt{2}$ and $\sqrt{2} + 1$ both are not the solutions of the given equation.

10. Given, $x^2 + kx - 192 = 0$

Since, $x = 12$ is a root of the given equation, so it will satisfy the given equation.

$\therefore (12)^2 + k(12) - 192 = 0 \Rightarrow 144 + 12k - 192 = 0$

$\Rightarrow 12k - 48 = 0 \Rightarrow k = \frac{48}{12} = 4$

11. Given, $5x^2 - 8x + k = 0$

Since, $x = -2/5$ is a root of the given equation, so it will satisfy the given equation.

$\therefore 5\left(\frac{-2}{5}\right)^2 - 8\left(\frac{-2}{5}\right) + k = 0$

$\Rightarrow \frac{4}{5} + \frac{16}{5} + k = 0 \Rightarrow 4 + k = 0 \Rightarrow k = -4$

12. Given, $3x^2 - 2ax + 2b = 0$ ...(i)

Since, $x = 2$ and $x = 3$ are the roots of (i), so these will satisfy the given equation.

On putting $x = 2$ in (i), we get

$3(2)^2 - 2a(2) + 2b = 0$

$\Rightarrow \quad 12 - 4a + 2b = 0 \Rightarrow 4a - 2b = 12$ ...(ii)

On putting $x = 3$ in (i), we get

$3(3)^2 - 2a(3) + 2b = 0$

$\Rightarrow \quad 27 - 6a + 2b = 0 \Rightarrow 6a - 2b = 27$ ...(iii)

Subtracting (ii) from (iii), we get

$2a = 15 \Rightarrow a = \frac{15}{2}$

Substituting the value of a in (ii), we get

$4\left(\frac{15}{2}\right) - 2b = 12 \Rightarrow 30 - 2b = 12 \Rightarrow 2b = 18 \Rightarrow b = 9$

13. We have, $11x^2 - 26x - 21 = 0$

$\Rightarrow \quad 11x^2 - 33x + 7x - 21 = 0 \Rightarrow 11x(x - 3) + 7(x - 3) = 0$

$\Rightarrow \quad (x - 3)(11x + 7) = 0 \Rightarrow x - 3 = 0$ or $11x + 7 = 0$

$\Rightarrow \quad x = 3$ or $x = \frac{-7}{11}$

Hence, 3 and $\frac{-7}{11}$ are the two roots of the given equation.

14. We have, $2x^2 - 17x + 21 = 0$

$\Rightarrow \quad 2x^2 - 14x - 3x + 21 = 0 \Rightarrow 2x(x - 7) - 3(x - 7) = 0$

$\Rightarrow \quad (x - 7)(2x - 3) = 0 \Rightarrow x - 7 = 0$ or $2x - 3 = 0$

$\Rightarrow \quad x = 7$ or $x = \frac{3}{2}$

Hence, 7 and $\frac{3}{2}$ are the two roots of the given equation.

15. We have, $2ax^2 - (2a - b^2)x - b^2 = 0$

$\Rightarrow \quad 2ax^2 - 2ax + b^2x - b^2 = 0 \Rightarrow 2ax(x - 1) + b^2(x - 1) = 0$

$\Rightarrow \quad (x - 1)(2ax + b^2) = 0 \Rightarrow x - 1 = 0$ or $2ax + b^2 = 0$

$\Rightarrow \quad x = 1$ or $x = -\frac{b^2}{2a}$

Hence, 1 and $-\frac{b^2}{2a}$ are the two roots of the given equation.

16. We have, $x^2 + (1 + \sqrt{5})x + \sqrt{5} = 0$

$\Rightarrow \quad x^2 + x + \sqrt{5}x + \sqrt{5} = 0$

$\Rightarrow \quad x(x + 1) + \sqrt{5}(x + 1) = 0 \Rightarrow (x + 1)(x + \sqrt{5}) = 0$

$\Rightarrow \quad x + 1 = 0$ or $x + \sqrt{5} = 0 \Rightarrow x = -1$ or $x = -\sqrt{5}$

Hence, -1 and $-\sqrt{5}$ are the two roots of the given equation.

17. Let original average speed of the train be x km/hr.

According to question,

$$\frac{63}{x} + \frac{72}{x + 6} = 3 \Rightarrow \frac{7}{x} + \frac{8}{x + 6} = \frac{1}{3} \Rightarrow \frac{7(x + 6) + 8x}{x(x + 6)} = \frac{1}{3}$$

$\Rightarrow \quad 3(7x + 42 + 8x) = x^2 + 6x \Rightarrow 45x + 126 = x^2 + 6x$

$\Rightarrow \quad x^2 - 39x - 126 = 0 \Rightarrow x^2 - 42x + 3x - 126 = 0$

$\Rightarrow \quad (x - 42)(x + 3) = 0 \Rightarrow x - 42 = 0$ or $x + 3 = 0$

$\Rightarrow \quad x = 42$ $\quad (\because x > 0 \text{ so } x \neq -3)$

Hence, the original speed of the train is 42 km/hr.

18. Let the marked price of the book be ₹ x.

Total cost = ₹ 300

$\therefore$ Number of books $= \frac{300}{x}$

If price of the book is ₹ $(x - 5)$, then

Number of books $= \frac{300}{x - 5}$

According to question,

$$\frac{300}{x - 5} - \frac{300}{x} = 5 \Rightarrow \frac{300x - 300(x - 5)}{(x - 5)x} = 5$$

$\Rightarrow \quad 1500 = 5(x^2 - 5x) \Rightarrow x^2 - 5x - 300 = 0$

$\Rightarrow \quad x^2 - 20x + 15x - 300 = 0 \Rightarrow x(x - 20) + 15(x - 20) = 0$

$\Rightarrow \quad (x - 20)(x + 15) = 0 \Rightarrow x = 20$ or $x = -15$

Since, x has to be a positive integer, so $x = -15$ is rejected.

$\therefore \quad x = 20$

Hence, original marked price of the book is ₹ 20.

19. We have, $4x^2 + 4bx - (a^2 - b^2) = 0$

$\Rightarrow \quad x^2 + bx - \left(\frac{a^2 - b^2}{4}\right) = 0 \Rightarrow x^2 + bx = \frac{a^2 - b^2}{4}$

Adding $\left(\frac{b}{2}\right)^2$ on both sides, we get

$$x^2 + 2\left(\frac{b}{2}\right)x + \left(\frac{b}{2}\right)^2 = \frac{a^2 - b^2}{4} + \left(\frac{b}{2}\right)^2$$

$\Rightarrow \quad \left(x + \frac{b}{2}\right)^2 = \frac{a^2}{4} \Rightarrow x + \frac{b}{2} = \pm\frac{a}{2}$

$\Rightarrow \quad x = \frac{-b}{2} \pm \frac{a}{2} \Rightarrow x = \frac{-b - a}{2}$ or $x = \frac{-b + a}{2}$

Hence, the roots are $-\left(\frac{a + b}{2}\right)$ and $\left(\frac{a - b}{2}\right)$.

20. We have, $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

$\Rightarrow \quad x^2 - (\sqrt{3} + 1)x = -\sqrt{3}$

Adding $\left(\frac{\sqrt{3} + 1}{2}\right)^2$ on both sides, we get

$$x^2 - 2\left(\frac{\sqrt{3} + 1}{2}\right)x + \left(\frac{\sqrt{3} + 1}{2}\right)^2 = -\sqrt{3} + \left(\frac{\sqrt{3} + 1}{2}\right)^2$$

$\Rightarrow \quad \left(x - \frac{\sqrt{3} + 1}{2}\right)^2 = \frac{-4\sqrt{3} + (\sqrt{3} + 1)^2}{4}$

$\Rightarrow \quad \left(x - \frac{\sqrt{3} + 1}{2}\right)^2 = \left(\frac{\sqrt{3} - 1}{2}\right)^2$

$$\Rightarrow \quad x - \frac{\sqrt{3}+1}{2} = \pm\frac{\sqrt{3}-1}{2} \Rightarrow x = \frac{\sqrt{3}+1}{2} \pm \frac{\sqrt{3}-1}{2}$$

$$\Rightarrow \quad x = \sqrt{3} \text{ or } x = 1$$

Hence, the roots are $\sqrt{3}$ and 1.

21. We have, $3x^2 + 11x + 10 = 0$

$$\Rightarrow \quad x^2 + \frac{11}{3}x + \frac{10}{3} = 0 \Rightarrow x^2 + \frac{11}{3}x = \frac{-10}{3}$$

Adding $\left(\frac{11}{6}\right)^2$ on both sides, we get

$$x^2 + 2\left(\frac{11}{6}\right)x + \left(\frac{11}{6}\right)^2 = \frac{-10}{3} + \left(\frac{11}{6}\right)^2$$

$$\Rightarrow \quad \left(x + \frac{11}{6}\right)^2 = \frac{-10}{3} + \frac{121}{36} \Rightarrow \left(x + \frac{11}{6}\right)^2 = \frac{1}{36}$$

$$\Rightarrow \quad x + \frac{11}{6} = \pm\sqrt{\frac{1}{36}} = \pm\frac{1}{6}$$

$$\Rightarrow \quad x = -\frac{11}{6} \pm \frac{1}{6} \Rightarrow x = \frac{-11}{6} + \frac{1}{6} \text{ or } x = \frac{-11}{6} - \frac{1}{6}$$

$$\Rightarrow \quad x = \frac{-10}{6} = \frac{-5}{3} \text{ or } x = \frac{-12}{6} = -2$$

Hence, the roots are $\frac{-5}{3}$ and -2.

22. We have, $\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$

$$\Rightarrow \quad x^2 + \frac{10}{\sqrt{3}}x + 7 = 0 \Rightarrow x^2 + \frac{10}{\sqrt{3}}x = -7$$

Adding $\left(\frac{10}{2\sqrt{3}}\right)^2$ on both sides, we get

$$x^2 + 2\left(\frac{10}{2\sqrt{3}}\right)x + \left(\frac{10}{2\sqrt{3}}\right)^2 = -7 + \left(\frac{10}{2\sqrt{3}}\right)^2$$

$$\Rightarrow \quad \left(x + \frac{10}{2\sqrt{3}}\right)^2 = -7 + \frac{100}{4\times 3}$$

$$\Rightarrow \quad \left(x + \frac{10}{2\sqrt{3}}\right)^2 = \frac{16}{12} = \frac{4}{3} \Rightarrow x + \frac{10}{2\sqrt{3}} = \pm\sqrt{\frac{4}{3}}$$

$$\Rightarrow \quad x = \frac{-10}{2\sqrt{3}} \pm \frac{2}{\sqrt{3}} \Rightarrow x = \frac{-10+4}{2\sqrt{3}} \text{ or } x = \frac{-10-4}{2\sqrt{3}}$$

$$\Rightarrow \quad x = -\sqrt{3} \text{ or } x = \frac{-7}{\sqrt{3}}$$

Hence, the roots are $-\sqrt{3}$ and $\frac{-7}{\sqrt{3}}$.

23. We have, $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = p^2$, $b = p^2 - q^2$ and $c = -q^2$

$$\therefore \quad b^2 - 4ac = (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2)$$
$$= (p^2 - q^2)^2 + 4p^2q^2 = (p^2 + q^2)^2 > 0$$

So, the given equation has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(p^2 - q^2) \pm (p^2 + q^2)}{2p^2}$$

$$\Rightarrow \quad x = \frac{-p^2 + q^2 + p^2 + q^2}{2p^2} = \frac{q^2}{p^2}$$

$$\text{or} \quad x = \frac{-p^2 + q^2 - p^2 - q^2}{2p^2} = -1$$

Hence, the roots are $\frac{q^2}{p^2}$ and -1.

24. We have, $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Comparing this equation with $Ax^2 + Bx + C = 0$, we have

$A = 9$, $B = -9(a + b)$ and $C = 2a^2 + 5ab + 2b^2$

$\therefore \quad B^2 - 4AC = 81(a + b)^2 - 36\,(2a^2 + 5ab + 2b^2)$

$= 9a^2 + 9b^2 - 18ab = 9(a - b)^2 \geq 0$

So, the roots of the given equation are real and are given by

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{9(a+b) \pm 3(a-b)}{18}$$

$$\Rightarrow \quad x = \frac{9(a+b) + 3(a-b)}{18} = \frac{12a + 6b}{18} = \frac{2a + b}{3}$$

$$\text{or} \quad x = \frac{9(a+b) - 3(a-b)}{18} = \frac{6a + 12b}{18} = \frac{a + 2b}{3}$$

Hence, the roots are $\frac{2a+b}{3}$ and $\frac{a+2b}{3}$.

25. We have, $abx^2 + (b^2 - ac)x - bc = 0$

Comparing this equation with $Ax^2 + Bx + C = 0$, we have

$A = ab$, $B = b^2 - ac$ and $C = -bc$

$\therefore \quad B^2 - 4AC = (b^2 - ac)^2 - 4(ab)(-bc)$

$= (b^2 - ac)^2 + 4ab^2c = b^4 - 2ab^2c + a^2c^2 + 4ab^2c = (b^2 + ac)^2 \geq 0$

So, the given equation has real roots, which are given by

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(b^2 - ac) \pm (b^2 + ac)}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab} \text{ or } x = \frac{-(b^2 - ac) - (b^2 + ac)}{2ab}$$

$$\Rightarrow \quad x = \frac{2ac}{2ab} \text{ or } x = \frac{-2b^2}{2ab} \Rightarrow x = \frac{c}{b} \text{ or } x = \frac{-b}{a}$$

Hence, the roots are $\frac{c}{b}$ and $\frac{-b}{a}$.

26. We have, $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$, $x \neq 3, -5$

$$\Rightarrow \quad \frac{(x+5) - (x-3)}{(x-3)(x+5)} = \frac{1}{6} \Rightarrow (8)6 = x^2 + 2x - 15$$

$$\Rightarrow \quad x^2 + 2x - 63 = 0$$

Comparing the equation with $ax^2 + bx + c = 0$, we get $a = 1$, $b = 2$ and $c = -63$

$\therefore \quad b^2 - 4ac = (2)^2 - 4(1)(-63) = 4 + 252 = 256 > 0$

So, the given equation has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{256}}{2(1)} = \frac{-2 \pm 16}{2}$$

$$\Rightarrow \quad x = \frac{-2+16}{2} \text{ or } x = \frac{-2-16}{2} \Rightarrow x = 7 \text{ or } x = -9$$

Hence, the roots are 7 and –9.

27. We have, $x^2 + x + 7 = 0$

Here, $a = 1, b = 1$ and $c = 7$.

$\therefore \quad D = b^2 - 4ac = (1)^2 - 4(1)(7) = 1 - 28 = -27$

28. We have, $(4x - 3)^2 + 20x = 11$

$\Rightarrow \quad 16x^2 + 9 - 24x + 20x = 11$

$\Rightarrow \quad 16x^2 - 4x - 2 = 0 \Rightarrow 8x^2 - 2x - 1 = 0$

Here, $a = 8, b = -2$ and $c = -1$.

$\therefore \quad D = b^2 - 4ac = (-2)^2 - 4(8)(-1) = 4 + 32 = 36$

29. We have, $x^2 - 8x + 16 = 0$

Here, $a = 1, b = -8$ and $c = 16$.

$\therefore \quad D = b^2 - 4ac = (-8)^2 - 4(1)(16) = 64 - 64 = 0$

Thus, the given equation has real and equal roots.

30. We have, $4x^2 - 2\sqrt{3}\,x + 9 = 0$

Here, $a = 4, b = -2\sqrt{3}$ and $c = 9$

$\therefore \quad D = b^2 - 4ac = \left(-2\sqrt{3}\right)^2 - 4(4)(9) = 12 - 144 = -132 < 0$

Thus, the given equation has no real roots.

31. Let length of park = x m and

breadth of park = y m.

Perimeter of park = $2(x + y) = 80$ (Given)

$\Rightarrow \quad x + y = 40 \Rightarrow y = 40 - x$

Area of park = $xy = 300$ (Given)

$\Rightarrow \quad x(40 - x) = 300 \Rightarrow 40x - x^2 = 300$

$\Rightarrow \quad x^2 - 40x + 300 = 0$ ...(i)

Here, $a = 1, b = -40$ and $c = 300$.

$\therefore \quad D = b^2 - 4ac = (-40)^2 - 4(1)(300) = 1600 - 1200 = 400 > 0$

$\therefore$ Roots of (i) are real and distinct.

Hence, it is possible to design the given rectangular park.

32. We have, $kx^2 + 2x - 3 = 0$

Here, $a = k, b = 2$ and $c = -3$.

$\therefore \quad D = b^2 - 4ac = (2)^2 - 4(k)(-3) = 4 + 12k$

Now, the given equation has real and equal roots, so $D = 0$

$$\Rightarrow \quad 4 + 12k = 0 \Rightarrow k = \frac{-4}{12} = \frac{-1}{3}$$

33. We have, $2x^2 - 10x + k = 0$

Here, $a = 2, b = -10$ and $c = k$.

$\therefore \quad D = b^2 - 4ac = (-10)^2 - 4(2)(k) = 100 - 8k$

Now, the given equation has real and equal roots, so $D = 0$

$$\Rightarrow \quad 100 - 8k = 0 \Rightarrow k = \frac{100}{8} = \frac{25}{2}$$

34. We have, $5x^2 + kx - 4 = 0$

Here, $a = 5, b = k$ and $c = -4$.

$\therefore \quad D = b^2 - 4ac = k^2 - 4(5)(-4) = k^2 + 80$

Now, the given equation has real and equal roots, if $D = 0$

$\Rightarrow \quad k^2 + 80 = 0$

But k^2 is always positive.

So, for no value of k, $D = 0$.

Hence, equation has no real and equal roots.

35. We have, $(n + 3)x^2 - (5 - n)x + 1 = 0$

Here, $a = n + 3, b = -(5 - n)$ and $c = 1$.

$$\begin{aligned}\therefore \quad D &= b^2 - 4ac = (-(5 - n))^2 - 4(n + 3)(1)\\ &= 25 + n^2 - 10n - 4n - 12\\ &= n^2 - 14n + 13 = n^2 - 13n - n + 13\\ &= n(n - 13) - 1(n - 13) = (n - 13)(n - 1)\end{aligned}$$

Now, the given equation has coincident roots, *i.e.*, equal roots, so $D = 0$

$\Rightarrow \quad (n - 13)(n - 1) = 0 \Rightarrow n = 1$ or $n = 13$