Pair of Linear Equations in Two Variables

CHAPTER 3

TRY YOURSELF — SOLUTIONS

1. The given system of equations is

$3x - 2y = 4$ ...(i) $\quad 2x + y = 5$ ...(ii)

Putting $x = 2$ and $y = 1$ in (i), we get

L.H.S. $= 3 \times 2 - 2 \times 1 = 4 =$ R.H.S.

Putting $x = 2$ and $y = 1$ in (ii), we get

L.H.S. $= 2 \times 2 + 1 = 5 =$ R.H.S.

Thus, $x = 2$ and $y = 1$ satisfy both the equations of the given system.

Hence, $x = 2$, $y = 1$ is a solution of the given system of equations.

2. The given system of equations is

$2x + 7y = 11$ ...(i) $\quad x - 3y = -3$ ...(ii)

Putting $x = 3$ and $y = 2$ in (ii), we get

L.H.S. $= 2 \times 3 + 7 \times 2 = 20 \neq$ R.H.S.

So, $x = 3$ and $y = 2$ does not satisfy (i)

Putting $x = 3$ and $y = 2$ in (i), we get

L.H.S. $= 3 - 3 \times 2 = -3 =$ R.H.S.

So, $x = 3$ and $y = 2$ satisfy (ii), but not (i).

Hence, $x = 3$, $y = 2$ is not a solution of the given system of equations.

3. Given system of equations is

$4x + 5y = 9 \Rightarrow y = \frac{9 - 4x}{5}$ ...(i)

$8x + 10y = 18 \Rightarrow y = \frac{18 - 8x}{10}$ ...(ii)

Table of solutions for (i) is :

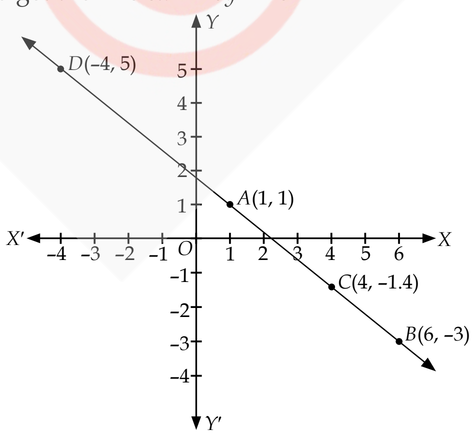
x	1	6
y	1	–3

Table of solution for (ii) is :

x	4	–4
y	–1.4	5

Now plot the points $A(1, 1)$ and $B(6, -3)$ and join them to get the line $4x + 5y = 9$.

Similarly, plot the points $C(4, -1.4)$ and $D(-4, 5)$ and join them to get the line $8x + 10y = 18$



Here, the lines represented by (i) and (ii) are coincident to each other.

4. Given system of equations is

$2x + y - 7 = 0 \Rightarrow y = 7 - 2x$ ...(i)

$-x + 2y - 4 = 0 \Rightarrow y = \frac{x + 4}{2}$ ...(ii)

Table of solutions for (i) is :

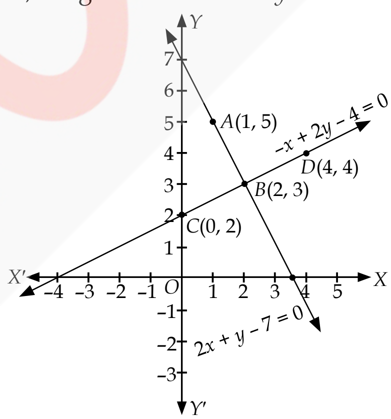
x	1	2
y	5	3

Table of solutions for (ii) is :

x	0	4
y	2	4

By plotting the points $A(1, 5)$ and $B(2, 3)$ and joining them, we get the line $2x + y - 7 = 0$.

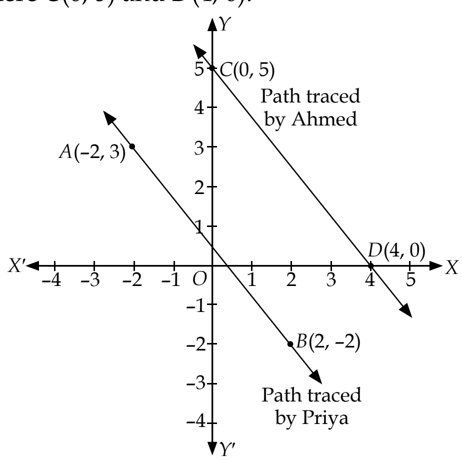
Similarly, plotting the points $C(0, 2)$ and $D(4, 4)$ and joining them, we get the line $-x + 2y - 4 = 0$.



Clearly, both lines intersect each other at $B(2, 3)$.

5. Let path of Priya is represented by the straight line AB, where $A(-2, 3)$ and $B(2, -2)$.

And path of Ahmed is represented by the straight line CD, where $C(0, 5)$ and $D(4, 0)$.



Plotting points $A(-2, 3)$, $B(2, -2)$ and joining them, we get path traced by priya.
Similarly, plotting points $C(0, 5)$, $D(4, 0)$ and joining them, we get path traced by Ahmed.
Here, the two lines do not intersect *i.e.*, two lines (or path traced) are parallel to each other.

6. Let the cost of one book be ₹ x and that of one pen be ₹ y.
According to the condition-I, we have
$$5x + 7y = 79 \quad \text{...(i)}$$
According to the condition-II, we have
$$7x + 5y = 77 \quad \text{...(ii)}$$
∴ (i) and (ii) are the required algebraic representation of the given situation.

7. Let the digit in the units place be x and digit in the tens place be y. Then, $x = 2y$
and number $= 10y + x$
Number obtained by reversing the digits $= 10x + y$
Also, Number + 27 = Number obtained by interchanging the digits.
∴ $10y + x + 27 = 10x + y$
⇒ $9x - 9y = 27 \Rightarrow x - y = 3$
Thus, the algebraic representation of given situation is $x - 2y = 0$ and $x - y = 3$.

8. Let the age of the father be x years and the sum of the ages of his 2 children be y years.
Then, $x = 2y$
After 18 years,
Age of the father $= (x + 18)$ years
Sum of the ages of his 2 children
$= (y + 18 + 18)$ years $= (y + 36)$ years
According to the question, $x + 18 = y + 36$
⇒ $x - y = 18$
Thus, the algebraic representation of the given situation is $x - 2y = 0$ and $x - y = 18$.

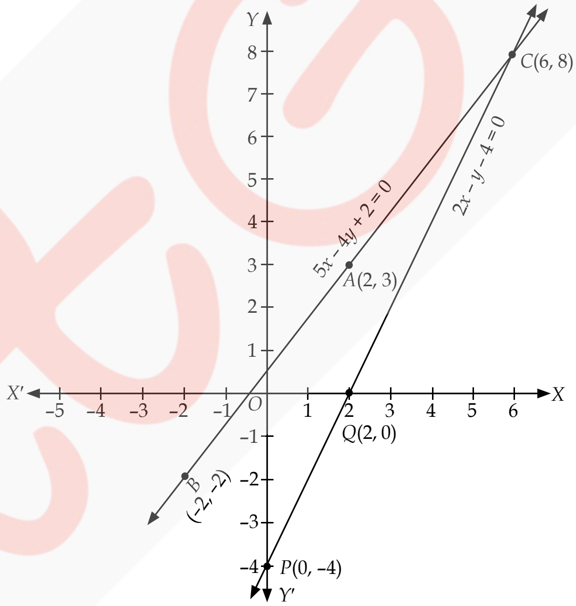
9. Let the numerator of the fraction be x and denominator be y. Then, the fraction is $\frac{x}{y}$.
Now, according to the condition-I, we have $\frac{x+2}{y+2} = \frac{4}{5}$
⇒ $5x + 10 = 4y + 8$ (Cross-multiply both side)
⇒ $5x - 4y + 2 = 0$ ...(i)
Also, according to the condition-II, we have
$\frac{x-4}{y-4} = \frac{1}{2}$ (Cross-multiply both side)
⇒ $2x - 8 = y - 4 \Rightarrow 2x - y - 4 = 0$ ...(ii)
Thus, the algebraic representation of the given problem is
$5x - 4y + 2 = 0$ and $2x - y - 4 = 0$.
To represent it graphically, let us find atleast two solutions of each of the above equations, as shown in the following tables.
Table of solutions for (i) is :

x	2	–2	6
$y = \frac{5x+2}{4}$	3	–2	8

Table of solutions for (ii) is :

x	0	2	6
$y = 2x - 4$	–4	0	8

So, we plot the points $A(2, 3)$, $B(-2, -2)$ and $C(6, 8)$ on graph paper and join them to get a straight line representing $5x - 4y + 2 = 0$.
Similarly, plot the points $P(0, -4)$, $Q(2, 0)$ and $C(6, 8)$ on same graph paper and join them to get a straight line representing $2x - y - 4 = 0$.



Clearly, the lines representing (i) and (ii) are intersecting each other at point C(6, 8).

10. Let cost of one pencil be ₹ x and cost of one eraser be ₹ y.
According to the question, we have
$$3x + 2y = 8.5 \Rightarrow y = \frac{8.5 - 3x}{2} \quad \text{...(i)}$$
and $6x + 3y = 15$
or $2x + y = 5 \Rightarrow y = 5 - 2x$ ...(ii)
Thus, equations represented by (i) and (ii) is the algebraic representation of given situation.
Table of solutions for (i) is :

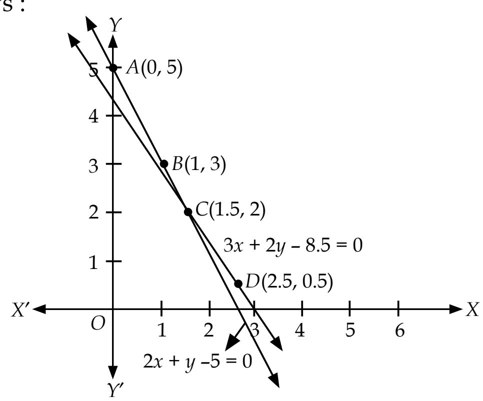
x	1.5	2.5
y	2	0.5

Table of solutions for (ii) is :

x	0	1
y	5	3

Plotting the points $C(1.5, 2)$ and $D(2.5, 0.5)$ on the graph paper and joining them, we get the line $3x + 2y = 8.5$.
Similarly, plotting the points $A(0, 5)$ and $B(1, 3)$ on graph paper and joining them, we get the line $2x + y = 5$.

∴ The graphical solution of given situations is as follows :



Graphically, we see that the lines are intersecting at point $C(1.5, 2)$.
Hence, cost of 1 pencil and 1 eraser is ₹ 1.5 and ₹ 2 respectively.

11. The given pair of linear equation is

$6x - 4y - 1 = 0 \Rightarrow y = \frac{6x-1}{4}$ ...(i)

and $2x - \frac{4}{3}y + 5 = 0 \Rightarrow y = \frac{3(2x+5)}{4}$ ...(ii)

Table of solutions for (i) is :

x	0.5	1
y	0.5	1.25

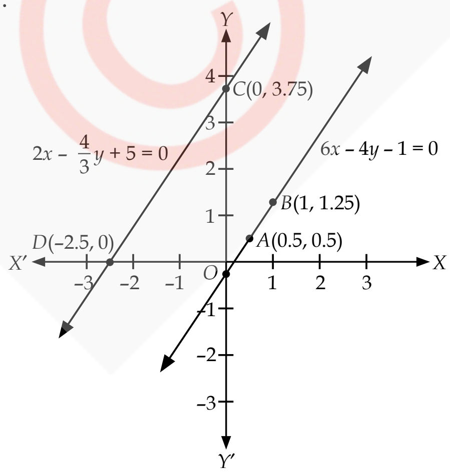
Table of solutions for (ii) is :

x	0	–2.5
y	3.75	0

Now, plotting the points $A(0.5, 0.5)$ and $B(1, 1.25)$ on the graph paper and joining them, we get the line $6x - 4y - 1 = 0$.

Similarly, plotting the points $C(0, 3.75)$ and $D(-2.5, 0)$ on the graph paper and joining them, we get the line $2x - \frac{4}{3}y + 5 = 0$.

The graphical representation of given equations is as follows :



Thus, given pair of equations has no solution as the two lines are parallel.

12. Given system of linear equations is

$2x - y - 4 = 0 \Rightarrow y = 2x - 4$ ...(i)

$x + y + 1 = 0 \Rightarrow y = -1 - x$ ...(ii)

Table of solutions for (i) is :

x	1	2
y	–2	0

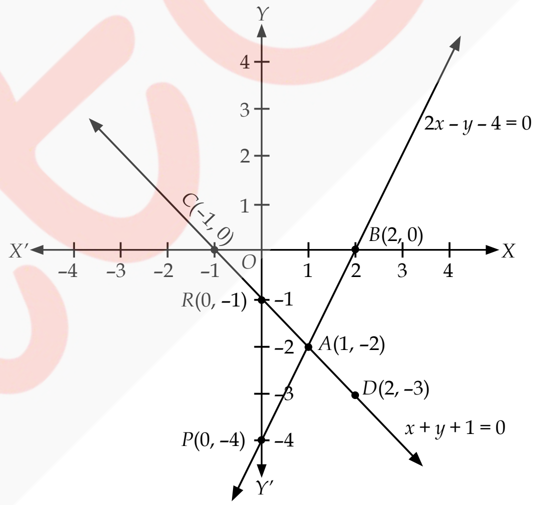
Table of solutions for (ii) is :

x	–1	2
y	0	–3

Now, plotting the points $A(1, -2)$ and $B(2, 0)$ on the graph paper and joining them, we get the line $2x - y - 4 = 0$.

Similarly, potting the points $C(-1, 0)$ and $D(2, -3)$ on the graph paper and joining them, we get the line $x + y + 1 = 0$.

The graphical representation of given equations is as follows :



We see that both lines intersect at point $A(1, -2)$ *i.e.*, given system of equations has a unique solution given by $x = 1$ and $y = -2$.

From graph, line $2x - y - 4 = 0$ meets y-axis at $P(0, -4)$ and line $x + y + 1 = 0$ meets y-axis at $R(0, -1)$.

13. Compare the given equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

(i) For, $7x - 5y + 8 = 0$ and $7x + 8y - 9 = 0$, we have

$a_1 = 7, b_1 = -5$ and $c_1 = 8$

and $a_2 = 7, b_2 = 8$ and $c_2 = -9$

Now, $\frac{a_1}{a_2} = \frac{7}{7} = 1;\ \frac{b_1}{b_2} = \frac{-5}{8};\ \frac{c_1}{c_2} = \frac{8}{-9} = \frac{-8}{9}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ Lines are intersecting at a unique point.

(ii) For, $5x + 3y - 7 = 0$ and $15x + 9y - 21 = 0$, we have

$a_1 = 5, b_1 = 3, c_1 = -7$

and $a_2 = 15, b_2 = 9, c_2 = -21$

Now, $\frac{a_1}{a_2} = \frac{5}{15} = \frac{1}{3};\ \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3};\ \frac{c_1}{c_2} = \frac{-7}{-21} = \frac{1}{3}.$

Since, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\therefore$ Lines are coincident.

14. Compare the given equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

(i) For, $6x - 3y - 18 = 0$, $2x - y - 4 = 0$

$a_1 = 6, b_1 = -3, c_1 = -18$

and $a_2 = 2, b_2 = -1, c_2 = -4$

Here, $\frac{a_1}{a_2}=\frac{6}{2}=3;\ \frac{b_1}{b_2}=\frac{-3}{-1}=3$ and $\frac{c_1}{c_2}=\frac{-18}{-4}=\frac{9}{2}$

$\because \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

So, given pair of equations is inconsistent.

(ii) For, $x - 4y + 5 = 0$ and $16y - 2x + 20 = 0$, we have

$a_1 = 1,\ b_1 = -4, c_1 = 5$

and $a_2 = -2, b_2 = 16, c_2 = 20$

Now, $\frac{a_1}{a_2}=\frac{-1}{2},\ \frac{b_1}{b_2}=\frac{-4}{16}=\frac{-1}{4},\ \frac{c_1}{c_2}=\frac{5}{20}=\frac{1}{4}$

$\because \frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So, given pair of equations is consistent.

15. We have, $7x - 15y = 2$ ...(i)

and $x + 2y = 3$ ...(ii)

From (ii), $x = 3 - 2y$ ...(iii)

Substituting the value of x from (iii) in (i), we get

$7(3 - 2y) - 15y = 2$

$\Rightarrow 21 - 14y - 15y = 2$

$\Rightarrow -29y = -19 \Rightarrow y=\frac{19}{29}$

Substituting the value of y in (iii), we get

$x=3-2\left(\frac{19}{29}\right)=\frac{49}{29}$

$\therefore x=\frac{49}{29}, y=\frac{19}{29}$ is the required solution.

16. We have, $\sqrt{2}x+\sqrt{5}y=0$ ...(i)

and $\sqrt{6}x+\sqrt{15}y=0$ ...(ii)

From (i), $x=-\frac{\sqrt{5}}{\sqrt{2}}y$ ...(iii)

Substituting the value of x from (iii) in (ii), we get

$\sqrt{6}\times\left(-\frac{\sqrt{5}}{\sqrt{2}}y\right)+\sqrt{15}y=0$

$\Rightarrow -\sqrt{15}y+\sqrt{15}y=0 \Rightarrow 0=0$, which is a true statement.

Hence, given pair of linear equations has infinitely many solutions.

Now, let us find these solutions.

Put $y = k$ (any real constant) in (iii), we get

$x=-\frac{\sqrt{5}}{\sqrt{2}}k$

Hence, $x=-\frac{\sqrt{5}}{\sqrt{2}}k,\ y=k$ is the required solution, where k is any real number.

17. Let the cost of 1 yellow candy is ₹ x and 1 orange candy is ₹ y,

According to the question,

$x + 8y = 19$ ...(i)

and $2x + 11y = 28$ ...(ii)

From (i), $x = 19 - 8y$ ...(iii)

Substituting the value of x from (iii) in (ii), we get

$2(19 - 8y) + 11y = 28 \Rightarrow 38 - 16y + 11y = 28$

$\Rightarrow 38 - 5y = 28 \Rightarrow 5y = 10 \Rightarrow y = 2$

Substituting $y = 2$ in (iii), we get

$x = 19 - 8(2) = 3$

Hence, cost of 1 yellow candy is ₹ 3 and 1 orange candy is ₹ 2.

18. We have, $5ax + 6by = 28$ ...(i)

$3ax + 4by = 18$ ...(ii)

Multiplying (i) by 3 and (ii) by 5, we get

$15ax + 18by = 84$ ...(iii)

$15ax + 20by = 90$ ...(iv)

Subtracting (iii) from (iv), we get

$2by = 6 \Rightarrow y=\frac{3}{b}$

Putting $y=\frac{3}{b}$ in (i), we get

$5ax+6b\left(\frac{3}{b}\right)=28 \Rightarrow 5ax + 18 = 28$

$\Rightarrow 5ax = 10 \Rightarrow x=\frac{2}{a}$

$\therefore x=\frac{2}{a}$ and $y=\frac{3}{b}$ is the required solution.

19. We have, $99x + 101y = 499$ ...(i)

$101x + 99y = 501$ ...(ii)

Adding (i) and (ii), we get

$200x + 200y = 1000$

$\Rightarrow x + y = 5$ ...(iii)

Subtracting (ii) from (i), we get

$-2x + 2y = -2$

$\Rightarrow x - y = 1$ ...(iv)

Adding (iii) and (iv), we get

$2x = 6 \Rightarrow x = 3$

Put $x = 3$ in *(iv)*, we get

$3 - y = 1 \Rightarrow y = 2$

Hence, $x = 3$ and $y = 2$ is the required solution.

20. Let digit at ten's place = x and digit at unit's place = y

Then, original number = $10x + y$

Number obtained by reversing the digit of given number = $10y + x$

According to the question, $x + y = 10$

and $(10x + y) + 36 = 10y + x$

$\Rightarrow 9x - 9y = -36$ or $x - y = -4$

$\therefore$ Given system of linear equations becomes

$x + y = 10$ ...(i)

and $x - y = -4$ ...(ii)

Adding (i) and (ii), we get

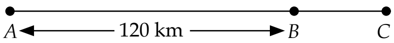
$2x = 6 \Rightarrow x = 3$

Using $x = 3$ in (i), we get

$3 + y = 10 \Rightarrow y = 7$

$\therefore$ Required number = $10x + y = 10(3) + 7 = 37$

21. Let the speed of two cars be x km/hr. and y km/hr.
Case I : When both cars travel in same direction and meet at point C (say)



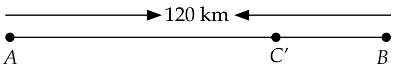
Then, $AC - BC = AB$

$\Rightarrow$ (Distance travelled by car at A) – (Distance travelled by car at B) = 120

$\Rightarrow$ $6x - 6y = 120$ ($\because$ Distance = speed × time and time = 6 hrs.)

$\Rightarrow$ $x - y = 20$ ...(i)

Case II : When both cars travel towards each other and meet at point C' (say)



Then, $AC' + BC' = AB$

$\Rightarrow$ (Distance travelled by car at A) + (Distance travelled by car at B) = 120

$\Rightarrow$ $x \times 1 + y \times 1 = 120$ ($\because$ Time = 1 hr., given)

$\Rightarrow$ $x + y = 120$ ...(ii)

Adding (i) and (ii), we get

$2x = 140 \Rightarrow x = 70$

Putting $x = 70$ in (i), we get $y = 50$.

Hence, speed of car starting from A is 70 km/hr and that starting from B is 50 km/hr.

22. Given system of equations is

$2x + 3y - 7 = 0$ ...(i)

$5x - 4y - 6 = 0$ ...(ii)

Solving (i) and (ii) by cross-multiplication method, we get

$$\frac{x}{\begin{matrix}3 & -7\\ -4 & -6\end{matrix}} = \frac{y}{\begin{matrix}-7 & 2\\ -6 & 5\end{matrix}} = \frac{1}{\begin{matrix}2 & 3\\ 5 & -4\end{matrix}}$$

$$\Rightarrow \frac{x}{-18-28} = \frac{y}{-35+12} = \frac{1}{-8-15}$$

$$\Rightarrow \frac{x}{-46} = \frac{y}{-23} = \frac{1}{-23}$$

$$\Rightarrow \frac{x}{-46} = \frac{-1}{23} \text{ and } \frac{y}{-23} = \frac{-1}{23}$$

$$\Rightarrow x = \frac{46}{23} \text{ and } y = \frac{23}{23}$$

$\therefore$ $x = 2$ and $y = 1$ is the required solution.

23. Let the fare from bus stand to Pitampura is ₹ x and that to Dilshad garden is ₹ y.

Then, according to question, we have

$2x + 3y = 46$ or $2x + 3y - 46 = 0$ ...(i)

and $3x + 5y = 74$ or $3x + 5y - 74 = 0$ ...(ii)

Solving (i) and (ii) by cross-multiplication method, we get

$$\frac{x}{\begin{matrix}3 & -46\\ 5 & -74\end{matrix}} = \frac{y}{\begin{matrix}-46 & 2\\ -74 & 3\end{matrix}} = \frac{1}{\begin{matrix}2 & 3\\ 3 & 5\end{matrix}}$$

$$\Rightarrow \frac{x}{3(-74)-(-46)5} = \frac{y}{(-46)3-2(-74)} = \frac{1}{2\times(5)-3\times(3)}$$

$$\Rightarrow \frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9} \Rightarrow \frac{x}{8} = \frac{y}{10} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{8} = 1 \text{ and } \frac{y}{10} = 1 \Rightarrow x = 8 \text{ and } y = 10$$

Hence, fare from the bus stand to Pitampura and Dilshad garden are ₹ 8 and ₹ 10 respectively.

24. Given equations are

$4x - 5y - k = 0$ and $2x - 3y - 12 = 0$

Here, $a_1 = 4$, $b_1 = -5$, $c_1 = -k$

and $a_2 = 2$, $b_2 = -3$, $c_2 = -12$

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{4}{2} \neq \frac{5}{3}, \text{ which is true.}$$

Hence, given system of equations has unique solution for any value of k *i.e.*, for all real values.

25. Given equations are

$kx + 2y - 5 = 0$ and $8x + ky - 20 = 0$

Here, $a_1 = k$, $b_1 = 2$, $c_1 = -5$

and $a_2 = 8$, $b_2 = k$, $c_2 = -20$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{k}{8} = \frac{2}{k} \neq \frac{-5}{-20}$$

From first two terms, $\frac{k}{8} = \frac{2}{k}$

$\Rightarrow$ $k^2 = 16 \Rightarrow k = 4$ and -4

Thus, values of k are 4 and –4.

26. Given equations are

$x + (k + 1)y - 5 = 0$ ...(i)

and $(k + 1)x + 9y - (8k - 1) = 0$ ...(ii)

Here, $a_1 = 1$, $b_1 = k + 1$, $c_1 = -5$

and $a_2 = k + 1$, $b_2 = 9$, $c_2 = -(8k - 1)$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{1}{k+1} = \frac{k+1}{9} = \frac{-5}{-(8k-1)}$$

Taking first two terms, we have

$$\frac{1}{k+1} = \frac{k+1}{9} \Rightarrow (k+1)^2 = 9 \Rightarrow k + 1 = \pm 3$$

$\Rightarrow$ $k = 2$ or $k = -4$

Taking last two terms, we have

$$\frac{k+1}{9} = \frac{5}{8k-1} \quad \text{...(iii)}$$

Putting $k = 2$ in (iii), we have

$$\frac{3}{9} = \frac{5}{16-1} = \frac{1}{3}, \text{ which is true.}$$

Putting $k = -4$ in (iii), we have

$$\frac{-4+1}{9} = \frac{5}{-32-1}$$

$$\Rightarrow \frac{-3}{9} = \frac{-5}{-33}, \text{ which is not true.}$$

Hence, $k = 2$.

27. Given system of equations is

$$\frac{3}{x} + \frac{2}{y} - 13 = 0 \text{ and } \frac{4}{x} - \frac{5}{y} + 2 = 0$$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in the given equations, we get

$3u + 2v - 13 = 0$ ...(i)

and $4u - 5v + 2 = 0$ ...(ii)

Form (i), $v=\frac{13-3u}{2}$ ...(iii)

Substituting the value of v in (ii), we get

$$4u-5\left(\frac{13-3u}{2}\right)+2=0$$

$$\Rightarrow \quad 4u-\frac{65}{2}+\frac{15u}{2}+2=0$$

$$\Rightarrow \quad \frac{23u}{2}=\frac{61}{2} \Rightarrow u=\frac{61}{23}$$

Putting $u=\frac{61}{23}$ in (iii), we get

$$v=\frac{13-3\left(\frac{61}{23}\right)}{2}=\frac{299-183}{23\times 2}=\frac{116}{23\times 2}=\frac{58}{23}$$

$$\therefore \quad u=\frac{61}{23} \text{ and } v=\frac{58}{23} \;\Rightarrow\; \frac{1}{x}=\frac{61}{23} \text{ and } \frac{1}{y}=\frac{58}{23}$$

$$\Rightarrow \quad x=\frac{23}{61} \text{ and } y=\frac{23}{58}$$

28. We have, $\frac{xy}{x+y}=\frac{1}{9} \Rightarrow \frac{x+y}{xy}=\frac{9}{1}$

$\Rightarrow \frac{1}{y}+\frac{1}{x}=9$ ...(i)

and $\frac{xy}{x-y}=\frac{1}{11} \Rightarrow \frac{x-y}{xy}=11 \Rightarrow \frac{1}{y}-\frac{1}{x}=11$ ...(ii)

Now, putting $\frac{1}{y}=u$ and $\frac{1}{x}=v$ in (i) and (ii), we get

$u + v = 9$ ...(iii)

$u - v = 11$ ...(iv)

Adding (iii) and (iv), we get

$2u = 20 \Rightarrow u = 10$

Putting the value of $u = 10$ in (iii), we get

$10 + v = 9 \Rightarrow v = -1$

Now, $\frac{1}{y}=10 \Rightarrow y=\frac{1}{10}$ and $\frac{1}{x}=-1 \Rightarrow x=-1$

29. Let speed of sailor in still water be x km/hr and speed of current be y km/hr.

$\therefore$ Speed of sailor in going downstream = $(x + y)$ km/hr and speed of sailor in going upstream = $(x - y)$ km/hr.

According to the question,

Time taken to go 8 km downstream = 40 minutes

$$\Rightarrow \frac{8}{x+y}=\frac{40}{60}$$

$\Rightarrow \frac{1}{x+y}=\frac{1}{12} \Rightarrow x+y=12$ ...(i)

Also, time taken to go 8 km upstream = 1 hour

$\Rightarrow \frac{8}{x-y}=1 \Rightarrow x-y=8$ ...(ii)

On adding (i) and (ii) we get

$2x = 20 \Rightarrow x = 10$

Putting $x = 10$ in (i) we get

$10 + y = 12 \Rightarrow y = 2$

Hence, speed of sailor in still water is 10 km/hr and speed of current is 2 km/hr.

30. Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

Then, speed of boat in upstream = $(x - y)$ km/hr

and speed of boat in downstream = $(x + y)$ km/hr

According to the question,

$\frac{40}{x-y}+\frac{36}{x+y}=8$ ...(i)

and $\frac{48}{x-y}+\frac{72}{x+y}=12$ ...(ii)

Putting $\frac{1}{x-y}=u$ and $\frac{1}{x+y}=v$, in (i) and (ii), we get

$40u + 36v = 8$ ...(iii)

$48u + 72v = 12$ ...(iv)

Multiplying (iii) by 48 and (iv) by 40, we get

$1920u + 1728v = 384$ ...(v)

$1920u + 2880v = 480$ ...(vi)

Subtracting (v) from (vi), we get

$1152v = 96$

$$\Rightarrow v=\frac{1}{12}$$

Substituting the value of v in (iii), we get

$$40u+36\left(\frac{1}{12}\right)=8$$

$$\Rightarrow 40u+3=8 \Rightarrow 40u=5 \Rightarrow u=\frac{1}{8}$$

$$\therefore \frac{1}{x-y}=\frac{1}{8}$$

$\Rightarrow x - y = 8$ ...(vii)

and $\frac{1}{x+y}=\frac{1}{12}$

$\Rightarrow x + y = 12$ ...(viii)

Solving (vii) and (viii), we get

$x = 10, y = 2$

$\therefore$ The speed of the boat = 10 km/hr and speed of the stream = 2 km/hr.

31. Suppose that one man and one boy alone can do the piece of work in x and y days respectively.

So, one day's work of a man = $\frac{1}{x}$ and

one day's work of a boy = $\frac{1}{y}$.

According to the question,

$$\left(\frac{4}{x}+\frac{6}{y}\right)=\frac{1}{5} \Rightarrow \frac{20}{x}+\frac{30}{y}=1$$

and $$\left(\frac{3}{x}+\frac{4}{y}\right)=\frac{1}{7} \Rightarrow \frac{21}{x}+\frac{28}{y}=1$$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in both the equations, we have

$20u + 30v - 1 = 0$ and $21u + 28v - 1 = 0$

Solving by cross-multiplication method, we have

$$\frac{u}{\begin{matrix}30 & -1\\ 28 & -1\end{matrix}} = \frac{v}{\begin{matrix}-1 & 20\\ -1 & 21\end{matrix}} = \frac{1}{\begin{matrix}20 & 30\\ 21 & 28\end{matrix}}$$

$$\Rightarrow \frac{u}{-30+28} = \frac{v}{-21+20} = \frac{1}{28\times 20 - 30\times 21}$$

$$\Rightarrow \frac{u}{-2} = \frac{v}{-1} = \frac{1}{560-630} \quad \Rightarrow \frac{u}{-2} = \frac{v}{-1} = \frac{1}{-70}$$

$$\Rightarrow u = \frac{-2}{-70} = \frac{1}{35} \text{ and } v = \frac{-1}{-70} = \frac{1}{70}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{35} \text{ and } \frac{1}{y} = \frac{1}{70}$$

$\Rightarrow$ $x = 35$ and $y = 70$

Hence, one man and one boy alone can finish the piece of work in 35 days and 70 days respectively.

32. Let 1 woman can finish the work in x days and 1 man can finish the work in y days.

$\therefore$ 1 woman's one day work $= \frac{1}{x}$

and 1 man's one day work $= \frac{1}{y}$

According to the question,

$$\frac{3}{x} + \frac{6}{y} = \frac{1}{5} \qquad \ldots\text{(i)}$$

and $$\frac{4}{x} + \frac{7}{y} = \frac{1}{4} \qquad \ldots\text{(ii)}$$

Multiplying (i) by 4 and (ii) by 3, we get

$$\frac{12}{x} + \frac{24}{y} = \frac{4}{5} \qquad \ldots\text{(iii)}$$

$$\frac{12}{x} + \frac{21}{y} = \frac{3}{4} \qquad \ldots\text{(iv)}$$

Subtracting (iv) from (iii), we get

$$\frac{3}{y} = \frac{4}{5} - \frac{3}{4} = \frac{16-15}{20} = \frac{1}{20} \Rightarrow y = 60$$

Substituting the value of y in (i), we get

$$\frac{3}{x} + \frac{6}{60} = \frac{1}{5} \Rightarrow \frac{3}{x} = \frac{1}{5} - \frac{1}{10} \Rightarrow \frac{3}{x} = \frac{1}{10}$$

$\Rightarrow$ $x = 30$

Hence, 1 woman can do the work in 30 days and 1 man can do it in 60 days.