Polynomials

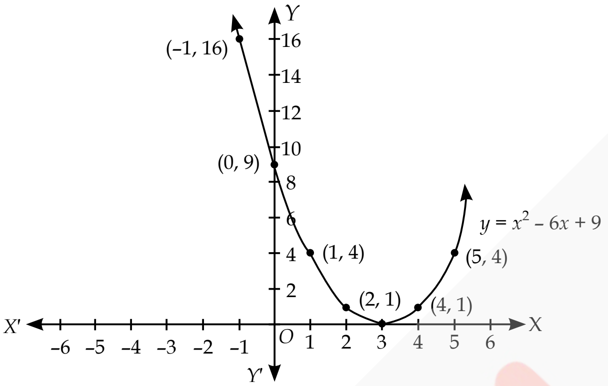
CHAPTER 2

TRY YOURSELF SOLUTIONS

1. Let $y = f(x) = x^2 - 6x + 9 = (x - 3)^2$

Now, we find values of y for different values of x.

x	–1	0	1	2	3	4	5
$y = f(x)$	16	9	4	1	0	1	4



The given curve represents a parabola opening upwards. Also, it intersect x axis at one point only. Hence, $y = f(x)$ has coincident zeroes.

∴ Number of zeroes will be 1.

2. (i) Since, curve represents a parabola opening upwards.

∴ $a > 0$.

(ii) Since, curve represents a parabola opening downwards.

∴ $a < 0$.

3. (i) Let $p(y) = y^3 - 2y^2 - \sqrt{3}y + 1/2$

Yes, it is a polynomial is $p(y)$ is of the form $a_0 + a_1y + a_2y^2 + a_3y^3$, where, $a_0 = \frac{1}{2}$, $a_1 = -\sqrt{3}$, $a_2 = -2$, $a_3 = 1$ (all being real numbers).

Clearly, $p(y)$ is of degree 3.

(ii) Let $p(x) = \sqrt{7}x^4 - \sqrt{x} + 2x - 1/3$

$p(x)$ is not of the form $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$

Hence, $p(x)$ is not a polynomial in x.

4. We know, $x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

∴ Zeroes of $x^2 - 3$ are $\sqrt{3}$ and $-\sqrt{3}$.

Now, sum of zeroes $= (\sqrt{3}) + (-\sqrt{3}) = 0$

$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of zeroes $= (\sqrt{3})(-\sqrt{3}) = -3 = \frac{(-3)}{1}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

5. Let $f(x) = x^3 - 27x + 54$

Now, $f(-6) = (-6)^3 - 27(-6) + 54$

$= -216 + 162 + 54 = -216 + 216 = 0$

$f(3) = (3)^3 - 27(3) + 54 = 27 - 81 + 54 = 81 - 81 = 0$

Hence, –6, 3, 3 are the zeroes of $f(x)$, where 3 is a repeated zero of $f(x)$.

Let $\alpha = -6$, $\beta = 3$, $\gamma = 3$ be the roots of $f(x)$.

Now, $\alpha + \beta + \gamma = -6 + 3 + 3 = 0 = \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = (-6)(3) + (3)(3) + 3(-6)$

$= -18 + 9 - 18 = -27 = \frac{(-27)}{1} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\alpha\beta\gamma = (-6)(3)(3) = -54 = \frac{-(54)}{1} = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3}$

6. Let $f(x) = x^2 + 3mx + 8m$

∵ 2 is a root of $f(x)$ ∴ $f(2) = 0$

Now, $f(2) = (2)^2 + 3m(2) + 8m = 0$

$\Rightarrow 4 + 14m = 0 \Rightarrow 14m = -4 \Rightarrow m = \frac{-2}{7}$

Let the other zero of $f(x)$ be α

Sum of the roots $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\Rightarrow 2 + \alpha = \frac{-3m}{1} = -3 \times \frac{-2}{7} = \frac{6}{7} \Rightarrow \alpha = \frac{6}{7} - 2 = \frac{6 - 14}{7} = -\frac{8}{7}$

7. Since, α and β are the zeroes of $f(x) = x^2 - x - 4$

∴ $\alpha\beta = \frac{(-4)}{1} = -4$ ...(i) and $\alpha + \beta = -\frac{(-1)}{1} = 1$ ...(ii)

Now, $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{(\beta + \alpha) - (\alpha\beta)^2}{\alpha\beta} = \frac{(\alpha + \beta) - (\alpha\beta)^2}{\alpha\beta}$

$= \frac{1 - (-4)^2}{-4}$ [Using (i) and (ii)]

$= \frac{1 - 16}{-4} = \frac{-15}{-4} = \frac{15}{4}$

8. Since, α and β are the zeroes of $f(t) = t^2 - 4t + 3$

∴ $\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } t^2} = \frac{3}{1} = 3$ ...(i)

and $\alpha + \beta = \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2} = \frac{-(-4)}{1} = 4$ ...(ii)

Now, $\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = (\alpha\beta)^3(\alpha + \beta)$

$= (3)^3(4)$ [Using (i) and (ii)]

$= 27 \times 4 = 108$

9. Since, α and β are the zeroes of $p(z) = 5z^2 - 9z + 4$

$\therefore\ \alpha + \beta = \dfrac{-(\text{Coefficient of } z)}{\text{Coefficient of } z^2} = -\dfrac{(-9)}{5} = \dfrac{9}{5}$ ...(i)

and $\alpha\beta = \dfrac{\text{Constant term}}{\text{Coefficient of } z^2} = \dfrac{4}{5}$ ...(ii)

Now, $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \dfrac{4}{5}\left(\dfrac{9}{5}\right) = \dfrac{36}{25}$

10. The quadratic polynomial $f(x)$ whose sum of zeroes, (S) and product of zeroes, (P) is given by $k(x^2 - Sx + P)$

Here, $S = \sqrt{2}$ and $P = \dfrac{1}{3}$

$\therefore\ f(x) = k\left(x^2 - \sqrt{2}x + \dfrac{1}{3}\right)$, k being any non zero real number

$= k\left(\dfrac{3x^2 - 3\sqrt{2}x + 1}{3}\right) = 3x^2 - 3\sqrt{2}x + 1$ [Taking $k = 3$]

Hence, required quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$

11. Here, sum of zeroes $(S) = \sqrt{2}$

Sum of the product of zeroes taken two at a time $(S') = \sqrt{3}$.

and product of zeroes, $P = \dfrac{1}{\sqrt{6}}$

Now, required cubic polynomial is given by

$k(x^3 - Sx^2 + S'x - P)$, k being any non zero real number.

$k\left(x^3 - \sqrt{2}x^2 + \sqrt{3}x - \dfrac{1}{\sqrt{6}}\right)$

$k\left(\dfrac{\sqrt{6}x^3 - 2\sqrt{3}x^2 + 3\sqrt{2}x - 1}{\sqrt{6}}\right) = \sqrt{6}x^3 - 2\sqrt{3}x^2 + 3\sqrt{2}x - 1$

[Taking $k = \sqrt{6}$]

12. Given, α and β are zeroes of the polynomial, $f(x) = x^2 - 2x + 3$

$\therefore\ \alpha + \beta = \dfrac{-(-2)}{1} = 2$ and $\alpha\beta = 3$

Let S and P denotes the sum and product of polynomial, whose roots are $\dfrac{\alpha - 1}{\alpha + 1}$ and $\dfrac{\beta - 1}{\beta + 1}$

Now, $S = \dfrac{\alpha - 1}{\alpha + 1} + \dfrac{\beta - 1}{\beta + 1} = \dfrac{(\alpha - 1)(\beta + 1) + (\alpha + 1)(\beta - 1)}{(\alpha + 1)(\beta + 1)}$

$= \dfrac{(\alpha\beta + \alpha - \beta - 1) + (\alpha\beta - \alpha + \beta - 1)}{\alpha\beta + (\alpha + \beta) + 1}$

$= \dfrac{2\alpha\beta - 2}{\alpha\beta + \alpha + \beta + 1} = \dfrac{2(\alpha\beta - 1)}{\alpha\beta + (\alpha + \beta) + 1} = \dfrac{2(3 - 1)}{3 + 2 + 1} = \dfrac{4}{6} = \dfrac{2}{3}$

$P = \dfrac{(\alpha - 1)(\beta - 1)}{(\alpha + 1)(\beta + 1)} = \dfrac{\alpha\beta - (\alpha + \beta) + 1}{\alpha\beta + (\alpha + \beta) + 1} = \dfrac{3 - (2) + 1}{3 + 2 + 1} = \dfrac{2}{6} = \dfrac{1}{3}$

$\therefore$ Cubic polynomial is $k(x^2 - Sx + P)$, k being any real number.

$= k\left(x^2 - \dfrac{2}{3}x + \dfrac{1}{3}\right) = \dfrac{k(3x^2 - 2x + 1)}{3}$

$= 3x^2 - 2x + 1$ [Taking $k = 3$]

Thus, one of the polynomial is given by $3x^2 - 2x + 1$.

13. We have, $p(x) = 3x^3 + x^2 + 2x + 5$

$g(x) = 1 + 2x + x^2 = x^2 + 2x + 1$

Now, on dividing $p(x)$ by $g(x)$, we get the following :

$$\begin{array}{r|l} & 3x - 5 \\ x^2 + 2x + 1 & 3x^3 + x^2 + 2x + 5 \\ & 3x^3 + 6x^2 + 3x \\ & (-)\quad (-)\quad (-) \\ \hline & -5x^2 - x + 5 \\ & -5x^2 - 10x - 5 \\ & (+)\quad (+)\quad (+) \\ \hline & 9x + 10 \end{array}$$

$\therefore$ Quotient $= 3x - 5$, Remainder $= 9x + 10$

14. Here, dividend $= f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$

and divisor $= g(x) = x^2 + 2x - 3$

Now, on dividing $f(x)$ by $g(x)$, we get the following :

$$\begin{array}{r|l} & 4x^2 - 6x + 22 \\ x^2 + 2x - 3 & 4x^4 + 2x^3 - 2x^2 + x - 1 \\ & 4x^4 + 8x^3 - 12x^2 \\ & (-)\quad (-)\quad (+) \\ \hline & -6x^3 + 10x^2 + x - 1 \\ & -6x^3 - 12x^2 + 18x \\ & (+)\quad (+)\quad (-) \\ \hline & 22x^2 - 17x - 1 \\ & 22x^2 + 44x - 66 \\ & (-)\quad (-)\quad (+) \\ \hline & -61x + 65 \end{array}$$

Thus, remainder obtained is $-61x + 65$

$\therefore$ We should add $61x - 65$ to $f(x)$ so that resulting polynomial is divisible by $x^2 + 2x - 3$.

15. Let $f(x) = 2x^3 + 3x^2 - 5x - 17$

$q(x) = x - 2$, $r(x) = -x + 3$

Then, by division algorithm, we get

$f(x) = g(x)\,q(x) + r(x) \Rightarrow \dfrac{f(x) - r(x)}{q(x)} = g(x)$

$\Rightarrow\ g(x) = \dfrac{(2x^3 + 3x^2 - 5x - 17) - (-x + 3)}{x - 2}$

$= \dfrac{2x^3 + 3x^2 - 4x - 20}{x - 2}$

On dividing $2x^3 + 3x^2 - 4x - 20$ by $x - 2$, we get the following :

$$\begin{array}{r|l} & 2x^2 + 7x + 10 \\ x - 2 & 2x^3 + 3x^2 - 4x - 20 \\ & 2x^3 - 4x^2 \\ & (-)\quad (+) \\ \hline & 7x^2 - 4x - 20 \\ & 7x^2 - 14x \\ & (-)\quad (+) \\ \hline & 10x - 20 \\ & 10x - 20 \\ & (-)\quad (+) \\ \hline & 0 \end{array}$$

Thus, $g(x) = 2x^2 + 7x + 10$

16. Let $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$

Since, two zeroes of $f(x)$ are $\sqrt{2}$ and $-\sqrt{2}$

$\therefore \quad (x-\sqrt{2})(x+\sqrt{2}) = x^2 - 2$ is a factor of $f(x)$.

On dividing $f(x)$ by $x^2 - 2$, we get the following :

$$
\begin{array}{r|l|l}
x^2-2 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 & 2x^2 - 3x + 1 \\
 & 2x^4 \qquad\quad - 4x^2 & \\
 & (-) \qquad\quad (+) & \\
\hline
 & -3x^3 + x^2 + 6x - 2 & \\
 & -3x^3 \qquad\; + 6x & \\
 & (+) \qquad\quad (-) & \\
\hline
 & x^2 - 2 & \\
 & x^2 - 2 & \\
 & (-) \quad (+) & \\
\hline
 & 0 &
\end{array}
$$

By division algorithm, we have

$f(x) = (x^2 - 2)(2x^2 - 3x + 1)$

$= (x+\sqrt{2})(x-\sqrt{2})\ (2x^2 - 2x - x + 1)$

$= (x+\sqrt{2})(x-\sqrt{2})[2x(x-1) - 1(x-1)]$

$= (x+\sqrt{2})(x-\sqrt{2})(2x-1)(x-1)$

Hence, other zeroes are $\frac{1}{2}$ and 1.

17. Let $f(x) = x^4 + 4x^3 - x^2 - 10x + 6$

Since two zeroes of $f(x)$ are $-(1+\sqrt{3})$ and $-(1-\sqrt{3})$

$\therefore \quad (x+(1+\sqrt{3}))(x+(1-\sqrt{3})) = (x+1)^2 - (\sqrt{3})^2$

$= x^2 + 1 + 2x - 3 = x^2 + 2x - 2$

Now, we divide $f(x)$ by $x^2 + 2x - 2$

$$
\begin{array}{r|l|l}
x^2+2x-2 & x^4 + 4x^3 - x^2 - 10x + 6 & x^2 + 2x - 3 \\
 & x^4 + 2x^3 - 2x^2 & \\
 & (-) \quad (-) \quad (+) & \\
\hline
 & 2x^3 + x^2 - 10x + 6 & \\
 & 2x^3 + 4x^2 - 4x + 6 & \\
 & (-) \quad (-) \quad (+) & \\
\hline
 & -3x^2 - 6x + 6 & \\
 & -3x^2 - 6x + 6 & \\
 & (+) \quad (+) \quad (-) & \\
\hline
 & 0 &
\end{array}
$$

By division algorithm, we have

$f(x) = (x^2 + 2x - 2)(x^2 + 2x - 3)$

$= (x^2 + 2x - 2)(x^2 + 3x - x - 3)$

$= (x^2 + 2x - 2)[x(x + 3) - 1(x + 3)]$

$= (x^2 + 2x - 2)(x - 1)(x + 3)$

$= [x + (1 + \sqrt{3})][x + (1 - \sqrt{3})](x - 1)(x + 3)$

Thus, other zeroes are 1, –3.

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