Surface Areas and Volumes

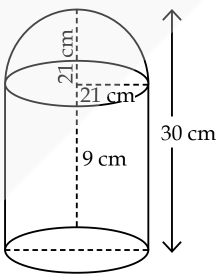
CHAPTER 13

TRY YOURSELF SOLUTIONS

1. We have, radius (r) of base of cylindrical part = radius (r) of base of conical part = 14 m
Height (h) of cylindrical part = 5 m
Height (h_1) of conical part = 15.5 – 5 = 10.5 m
Slant height (l) of conical part = $\sqrt{r^2+h_1^2}$
$=\sqrt{(14)^2+(10.5)^2}=\sqrt{196+110.25}=\sqrt{306.25}=17.5\text{m}$
Total area to be painted = curved surface area of cylindrical part + curved surface area of conical part
$= 2\pi rh + \pi rl = \pi r[2h + l] = \frac{22}{7}\times 14[2\times 5+17.5]$
$= 44[10 + 17.5] = 44 \times 27.5 = 1210 \text{ m}^2$
Hence, cost of painting the tent = ₹(1210 × 75) = ₹ 90750

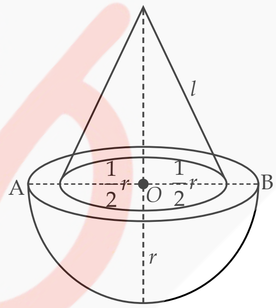
2. Total surface area of cube = $6(\text{side})^2$
$= 6 \times 6 \times 6 = 216 \text{ cm}^2$
Radius of hemisphere, r = 3.5/2 cm
Base area of hemisphere = $\pi r^2 = \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}$
$= 11 \times 0.5 \times \frac{3.5}{2} = 9.625 \text{ cm}^2$
Curved surface area of hemisphere = $2\pi r^2$
$=2\times\frac{22}{7}\times\left(\frac{3.5}{2}\right)^2=11\times 0.5\times 3.5=19.25\text{cm}^2$
∴ Total surface area of block = Total surface area of cube – Base area of hemisphere + Curved surface area of hemisphere
$= 216 - 9.625 + 19.25 = 225.625 \text{ cm}^2$

3. Radius (r) of cylinder = Radius (r) of hemisphere
$=\frac{42}{2}=21$ cm



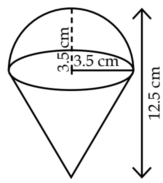
Height of cylinder (h)
= 30 – 21 = 9 cm
Inner surface area of the vessel = Curved surface area of cylinder + curved surface area of hemisphere
$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
$=2\times\frac{22}{7}\times 21\times(9+21)=44\times 3\times 30=3960\text{cm}^2$

4. Curved surface area of the hemispherical part of radius $r = 2\pi r^2$ sq. units.
Radius of conical part = $\frac{1}{2}r$ and its slant height = l
Curved surface area of the conical part
$=\pi\times\frac{1}{2}r\times l=\frac{\pi}{2}rl$ sq.units
Area of the exposed upper base of the hemisphere
$=\pi\left[r^2-\left(\frac{1}{2}r\right)^2\right]=\pi\left[r^2-\frac{1}{4}r^2\right]$
$=\frac{3}{4}\pi r^2$ sq.units.



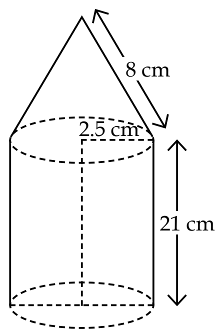
Total surface area of the solid = Curved surface area of hemispherical part + Area of exposed upper base of hemisphere + Curved surface area of conical part.
$=2\pi r^2+\frac{3}{4}\pi r^2+\frac{\pi}{2}rl=\frac{8\pi r^2+3\pi r^2}{4}+\frac{\pi}{2}rl$
$=\frac{11\pi}{4}r^2+\frac{\pi}{2}rl=\frac{\pi}{4}(11r+2l)r$ sq.units

5. Radius of hemispherical part (r) = Radius of conical part (r) = 3.5 cm
Height of conical part (h) = 12.5 – 3.5 = 9 cm
Volume of ice-cream in the cone = Volume of conical part + Volume of hemispherical part



$=\frac{1}{3}\pi r^2h+\frac{2}{3}\pi r^3=\frac{1}{3}\pi r^2[h+2r]$
$=\frac{1}{3}\times\frac{22}{7}\times(3.5)^2[9+2(3.5)]$
$=\frac{1}{3}\times\frac{22}{7}\times 12.25\times 16=\frac{4312}{21}=205.33 \text{ cm}^3$

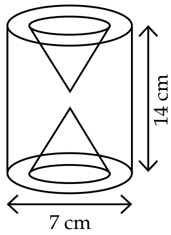
6. For cylindrical part;
Radius (r) = 2.5 cm
and height (h) = 21 cm
∴ Volume of cylinder = πr^2h
$=\frac{22}{7}\times 2.5\times 2.5\times 21$
$= 412.5 \text{ cm}^3$



For conical part;
Radius (r) = 2.5 cm
Slant height (l) = 8 cm
∴ Height (h_1) = $\sqrt{l^2-r^2}=\sqrt{(8)^2-(2.5)^2}$
$=\sqrt{64-6.25}=\sqrt{57.75}=7.6$ cm (Approx.)
∴ Volume of cone $=\frac{1}{3}\pi r^2h_1=\frac{1}{3}\times\frac{22}{7}\times 2.5\times 2.5\times 7.6$
$= 1045/21 = 49.76 \text{ cm}^3$

$\therefore$ Volume of rocket = Volume of cylindrical part + Volume of conical part = 412.5 + 49.76 = 462.26 cm^3

7. We have, radius of cylinder $(r) = 7/2$ cm



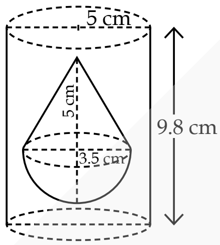
Height of cylinder (h) = 14 cm
Radius of both cones, (r_1) = 2.1 cm
Height of both cones, (h_1) = 4 cm
Volume of the remaining solid
= Volume of cylinder – 2 × Volume of cone

$= \pi r^2 h - 2 \times \frac{1}{3}\pi r_1^2 h_1$

$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 14 - \frac{2}{3}\times\frac{22}{7}\times 2.1\times 2.1\times 4$

= 539 – 36.96 = 502.04 cm^3

8. Radius of cylinder (r) = 5 cm



Height of cylinder (h) = 9.8 cm
Volume of cylinder when it is full of water = $\pi r^2 h$
$= \pi(5)^2 \times 9.8 = 245\pi$ cm^3
Now, radius of cone (r_1)
= radius of hemisphere (r_1) = 3.5 cm
Height of cone (h_1) = 5 cm
Volume of water displaced = Volume of the solid = Volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3 = \frac{1}{3}\pi r_1^2[h_1 + 2r_1]$

$= \frac{1}{3}\pi(3.5)^2[5+2(3.5)] = \frac{1}{3}\pi(12.25)[12] = 49\pi$ cm^3

Volume of water left in the tub = Volume of cylinder when it is full of water – Volume of water displaced

$= 245\pi - 49\pi = 196\pi = 196\times\frac{22}{7} = 616\,cm^3$

9. Given, diameter of metallic sphere = 4.2 cm
$\therefore$ Radius of metallic sphere (r) = 2.1 cm
Diameter of cylindrical wire = 0.2 cm
$\therefore$ Radius of cylindrical wire (r_1) = 0.1 cm
Let the length of wire be x.
Now, volume of cylindrical wire = volume of sphere

$\Rightarrow \pi r_1^2 x = \frac{4}{3}\pi r^3 \Rightarrow \pi(0.1)^2 \times x = \frac{4}{3}\pi(2.1)^3$

$\Rightarrow x = \frac{4\times 2.1\times 2.1\times 2.1}{3\times 0.1\times 0.1} = 1234.8\,cm = 12.348\,m.$

10. Given, internal and external radii of hollow sphere are r = 3 cm and R = 5 cm respectively.

Volume of hollow sphere = $\frac{4}{3}\pi(R^3 - r^3)$

$= \frac{4}{3}\pi(5^3 - 3^3) = \frac{4}{3}\pi(98)$ cm^3

Given, height of solid cylinder $(h) = \frac{8}{3}$ cm

Let r_1 be the radius of solid cylinder.
Volume of solid cylinder = $\pi r_1^2 h = \pi r_1^2 \times 8/3$
Now, volume of hollow sphere = Volume of solid cylinder

$\Rightarrow \frac{4}{3}\pi(98) = \pi r_1^2 \times \frac{8}{3} \Rightarrow r_1^2 = 49 \Rightarrow r_1 = 7$ cm

$\therefore$ Diameter of solid cylinder = 2 × 7 = 14 cm

11. Let x be the required number of marbles.
Diameter of marble = 1.4 cm

$\therefore$ Radius of marble $(r) = \frac{1.4}{2} = 0.7$ cm

Volume of a marble $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times(0.7)^3$ cm^3

Diameter of cylindrical beaker = 7 cm

$\therefore$ Radius of cylindrical beaker $(r_1) = \frac{7}{2}$ cm = 3.5 cm

Water level rise in beaker (h) = 5.6 cm
Now, volume increased = Volume of x marbles

$\Rightarrow \pi r_1^2 h = x\times\frac{4}{3}\times\pi\times(0.7)^3$

$\Rightarrow \pi\times(3.5)^2\times 5.6 = x\times\frac{4}{3}\times\pi\times(0.7)^3$

$\Rightarrow x = \frac{(3.5)^2\times 5.6\times 3}{4\times(0.7)^3} = 150$

So, 150 marbles should be dropped into the beaker so that water level rises by 5.6 cm.

12. Let the number of bottles required be x.
Diameter of hemispherical bowl = 36 cm
$\therefore$ Radius of hemispherical bowl (r) = 18 cm
Radius of cylindrical bottles (r_1) = 3 cm
Height of cylindrical bottles (h) = 6 cm
Now, volume of hemispherical bowl = x × volume of one cylindrical bottle

$\Rightarrow \frac{2}{3}\pi r^3 = x\times\pi r_1^2 h \Rightarrow \frac{2}{3}\times(18)^3 = x\times(3)^2\times 6$

$\Rightarrow x = \frac{2\times 18\times 18\times 18}{3\times 3\times 3\times 6} = 72$

$\therefore$ 72 bottles are required to empty the bowl.

13. Given, diameter of cylindrical pipe = 4 cm
$\therefore$ Radius of cylindrical pipe (r) = 2 cm
Rate of water flow = 20 m/minute = 2000 cm/minute
Volume of water flowing through the pipe in 1 minute
= (π × 2 × 2 × 2000) = π × 8000 cm^3
Diameter of conical tank = 80 cm
$\therefore$ Radius of conical tank (r_1) = 40 cm
Depth (height) of conical tank (h_1) = 72 cm

Volume of conical tank $= \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\times\pi\times 40\times 40\times 72$

= (π × 40 × 40 × 24) cm^3

Let t (in minutes) be the time taken to fill the tank.
So, the water that flows through the pipe in t minutes will be equal to volume of conical tank.

$\therefore t = \frac{\text{Volume of the conical tank}}{\text{Volume of water that flows through the pipe in 1 minute}}$

$= \dfrac{\pi \times 40 \times 40 \times 24}{\pi \times 8000} = \dfrac{24}{5}$ minutes $= 4.8$ minutes

14. Given, radii of circular ends of frustum are $r_1 = 14$ cm and $r_2 = 6$ cm

Height of frustum $(h) = 6$ cm

Slant height, $(l) = \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(14-6)^2 + 6^2}$

$\sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10\text{cm}$

$\therefore$ Lateral surface area of frustum $= \pi(r_1 + r_2)l$

$= \dfrac{22}{7}(14+6) \times 10 = 628.57 \text{ cm}^2$

Total surface area of frustum $= \pi[r_1^2 + r_2^2 + l(r_1 + r_2)]$

$= \dfrac{22}{7}[(14)^2 + (6)^2 + 10(14+6)]$

$= \dfrac{22}{7}[196 + 36 + 10(20)] = \dfrac{9504}{7} = 1357.71 \text{cm}^2$

15. Given diameters of two circular faces of frustum of cone are 35 cm and 30 cm.

$\therefore$ Radii of two circular faces are

$r_1 = \dfrac{35}{2}$ cm and $r_2 = \dfrac{30}{2} = 15$ cm

Height of frustum $(h) = 14$ cm

Volume of oil in the container $= \dfrac{1}{3}\pi[r_1^2 + r_2^2 + r_1 r_2]h$

$= \dfrac{1}{3} \times \dfrac{22}{7}\left[\left(\dfrac{35}{2}\right)^2 + (15)^2 + \left(\dfrac{35}{2}\right)(15)\right]14$

$= \dfrac{22}{3}\left[\dfrac{1225}{4} + 225 + \dfrac{525}{2}\right] \times 2$

$= \dfrac{22}{3}\left[\dfrac{1225 + 900 + 1050}{4}\right] \times 2 = \dfrac{11}{3} \times 3175 \text{cm}^3$

Mass of 1 cm^3 of oil $= 1.2$ g

$\therefore \left(\dfrac{11}{3} \times 3175\right)\text{cm}^3$ of oil $= \left[1.2 \times \dfrac{11}{3} \times 3175\right]$g

$= \left[1.2 \times \dfrac{11}{3} \times \dfrac{3175}{1000}\right]$kg $= 13.97$ kg

Cost of oil at the rate of ₹40 per kg = ₹(40 × 13.97)

= ₹558.80

16. Since, diameter of the upper and lower circular ends of the frustum of cone are 14 m and 26 m.

$\therefore$ Radius of upper and lower circular ends of the frustum of cone are $r_1 = 7$ m and $r_2 = 13$ m

Height of frustum $(h) = 8$ m

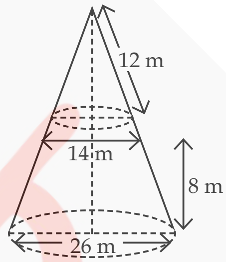
Slant height of frustum $(l) = \sqrt{h^2 + (r_2 - r_1)^2}$

$= \sqrt{8^2 + (13-7)^2} = \sqrt{64+36} = \sqrt{100} = 10\text{m}$

Curved surface area of frustum $= \pi(r_1 + r_2)l$

$= \dfrac{22}{7}(7+13) \times 10$

$= \dfrac{22}{7} \times 20 \times 10 = \dfrac{4400}{7} = 628.57 \text{m}^2$



Radius of cone $(r_1) = 7$ m

Slant height of cone $(l_1) = 12$ m

$\therefore$ Curved surface area of the cone $= \pi r_1 l_1$

$= \dfrac{22}{7} \times 7 \times 12 = 264 \text{ m}^2$

Area of canvas required to make the tent = Curved surface area of frustum + Curved surface area of cone $= 628.57 + 264 = 892.57 \text{ m}^2$

17. In ΔABC and ΔADE

$\angle BAC = \angle DAE$ (Common)

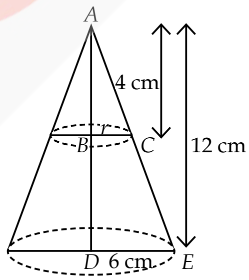
$\angle ABC = \angle ADE$ (Each 90°)

$\therefore$ $\Delta ABC \sim \Delta ADE$

(By AA Similarity Criterion)

$\therefore$ $\dfrac{AB}{AD} = \dfrac{BC}{DE}$

$\Rightarrow$ $\dfrac{4}{12} = \dfrac{BC}{6} \Rightarrow BC = \dfrac{4 \times 6}{12} = 2\text{cm}$



Now, lower and upper base radii of frustum are $r_1 = 2$ cm and $r_2 = 6$ cm respectively

Height of frustum $(h) = 12 - 4 = 8$ cm

Slant height of frustum (l)

$= \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{8^2 + (6-2)^2}$

$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5} = 4 \times 2.236 = 8.944$ cm

Total surface area of remaining solid (frustum)

$= \pi[r_2^2 + r_1^2 + l(r_2 + r_1)]$

$= \dfrac{22}{7}[(6)^2 + (2)^2 + 8.944(6+2)]$

$= \dfrac{22}{7}[36 + 4 + 71.552] = \dfrac{22}{7} \times 111.552 = 350.592 \text{ cm}^2$