Areas Related to Circles

CHAPTER 12

TRY YOURSELF — SOLUTIONS

1. Let d be the diameter of the circle whose area is equal to sum of areas of two circles of diameters $d_1 = 20$ cm and $d_2 = 48$ cm.

$\therefore \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{d_1}{2}\right)^2 + \pi\left(\frac{d_2}{2}\right)^2$

$\Rightarrow \left(\frac{d}{2}\right)^2 = \left(\frac{20}{2}\right)^2 + \left(\frac{48}{2}\right)^2$

$\Rightarrow \frac{d^2}{4} = 100 + 576 \Rightarrow \frac{d^2}{4} = 676 \Rightarrow d^2 = 676 \times 4$

$\Rightarrow d = 26 \times 2 = 52$ cm

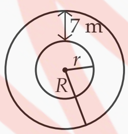
2. Let r cm be the radius of the protractor.
Given, perimeter of semi-circular protractor = 180 cm

$\Rightarrow (\pi + 2)r = 180$

$\Rightarrow \left(\frac{22}{7} + 2\right)r = 180 \Rightarrow \frac{36}{7}r = 180 \Rightarrow r = \frac{180 \times 7}{36} = 35$

Hence, diameter of protractor = $2r = 2 \times 35 = 70$ cm

3. Let r and R be the inner and outer radius of circular track.
Given, circumference of inner track = 308 m



$\Rightarrow 2\pi r = 308$

$\Rightarrow r = \frac{308}{2\pi} = \frac{308}{2 \times 22} \times 7 = 7 \times 7 = 49$ m

$\therefore$ Outer radius, $R = r +$ width of track $= 49 + 7 = 56$ m

$\therefore$ Circumference of outer track = $2\pi R$

$= 2 \times \frac{22}{7} \times 56 = 352$ m

Now, cost of putting fence of 1 m = ₹ 30

$\therefore$ Cost of putting fence of 352 m = ₹(30 × 352) = ₹ 10560

4. Let R and r be the radius of outer and inner track respectively.
Given, outer circumference = 352 m

$\Rightarrow 2\pi R = 352$

$\Rightarrow R = \frac{352}{2\pi} = \frac{352}{2 \times 22} \times 7 = 8 \times 7 = 56$ m

Also, inner circumference = 264 m [Given]

$\Rightarrow 2\pi r = 264$

$\Rightarrow r = \frac{264}{2\pi} = \frac{264}{2 \times 22} \times 7 = 6 \times 7 = 42$ m

$\therefore$ Width of track = $R - r = 56 - 42 = 14$ m

Hence, area of track = $\pi(R^2 - r^2) = \pi(R + r)(R - r)$

$= \frac{22}{7}(56 + 42) \times 14 = 22 \times 98 \times 2 = 4312$ m^2

5. Let r cm be the radius of circle.
Given, circumference of circle – diameter of circle = 75 cm

$\Rightarrow 2\pi r - 2r = 75$

$\Rightarrow 2r\left(\frac{22}{7} - 1\right) = 75 \Rightarrow 2r \times \frac{15}{7} = 75 \Rightarrow r = \frac{75 \times 7}{15 \times 2} = \frac{35}{2}$

$\therefore$ Area of circle = πr^2

$= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{11 \times 5 \times 35}{2} = 962.5$ cm^2

6. Radius of wheel = 50 cm

$\therefore$ Circumference of wheel = $2\pi r = \left(2 \times \frac{22}{7} \times 50\right)$ cm

Distance travelled = 44 km = (44 × 1000 × 100) cm

$\therefore$ Number of revolutions = $\frac{\text{Distance travelled}}{\text{Circumference of wheel}}$

$= \frac{44 \times 1000 \times 100 \times 7}{2 \times 22 \times 50} = 14000$

7. Given that, radius of a circle (r) = 28 cm and measure of central angle $(\theta) = 45°$

$\therefore$ Area of a sector of a circle $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{45°}{360°} \times \frac{22}{7} \times (28)^2 = 22 \times 4 \times 28 \times \frac{1}{8}$

$= 22 \times 14 = 308$ cm^2

Area of major sector = πr^2 – Area of minor sector

$= \frac{22}{7} \times 28 \times 28 - 308 = 2464 - 308 = 2156$ cm^2

8. Angle described by hour hand in 1 hour = 30°
Time covered from 8 a.m. to 11 a.m. = 3 hours
$\therefore$ Angle described by hour hand in 3 hours = 3 × 30° = 90°
Area swept by hour hand in 3 hours = Area of sector of angle 90° in a circle of radius 5.6 cm

$= \frac{90°}{360°} \times \frac{22}{7} \times (5.6)^2 = \frac{1}{4} \times \frac{22}{7} \times 5.6 \times 5.6 = 24.64$ cm^2

9. Given, length of arc = 26.4 cm

$\Rightarrow \frac{\theta°}{360°} \times 2\pi r = 26.4 \Rightarrow \frac{80°}{360°} \times 2\pi r = 26.4$ $[\because \theta = 80°]$

$\Rightarrow r = \frac{26.4 \times 360°}{2\pi \times 80°}$ cm

$\therefore$ Area of minor sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{80°}{360°} \times \pi \times \left(\frac{26.4 \times 360°}{2\pi \times 80°}\right)^2$

$= \frac{360°}{80°} \times \frac{7}{4 \times 22} \times 26.4 \times 26.4 = 249.48$ cm^2

10. Length of arc = 22 m [Given]

$\Rightarrow \frac{\theta}{360°} \times 2\pi r = 22 \Rightarrow \frac{\theta}{360°} \times 2 \times \frac{22}{7} \times 8.4 = 22$

$\Rightarrow \quad \theta = \frac{22 \times 7 \times 360°}{2 \times 22 \times 8.4} = 150°$

$\therefore$ Area of sector $= \frac{\theta}{360°} \times \pi r^2 = \frac{150°}{360°} \times \frac{22}{7} \times (8.4)^2$

$= \frac{5}{12} \times 22 \times 1.2 \times 8.4 = 92.4 \text{ cm}^2$

11. Let r = 7.5 cm and R = 11 cm.

$\therefore$ Area of shaded region = Area of sector of radius 11 cm – Area of sector of radius 7.5 cm

$= \frac{\theta}{360°} \times \pi R^2 - \frac{\theta}{360°} \times \pi r^2$

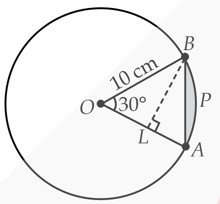
$= \frac{\theta}{360°} \pi (11^2 - 7.5^2) = \frac{120°}{360°} \times \frac{22}{7} (11 + 7.5)(11 - 7.5)$

$= \frac{1}{3} \times \frac{22}{7} \times 18.5 \times 3.5 = 67.83 \text{ cm}^2$

12. Let O be the centre and AB is the chord of the circle having radius 10 cm.

Draw $BL \perp OA$

In right angled ΔOLB,



$\frac{BL}{OB} = \sin 30° = \frac{1}{2}$

$\Rightarrow \frac{BL}{10} = \frac{1}{2} \quad (\because OB = 10 \text{ cm})$

$\Rightarrow BL = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$

Area of the segment APB = Area of the sector $OAPB$ – Area of the ΔAOB

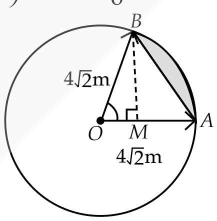
$= \frac{30°}{360°} \times \pi \times (r)^2 - \frac{1}{2} \times OA \times BL$

$= \left\{\frac{1}{12} \pi \times (10)^2 - \frac{1}{2} \times 10 \times 5\right\} \text{cm}^2$

$= \left(\frac{25\pi}{3} - 25\right) \text{cm}^2 = 25 \times \left(\frac{\pi}{3} - 1\right) \text{cm}^2$

$= 25 \times \left(\frac{3.14}{3} - 1\right) \text{cm}^2 = \left(\frac{25 \times 0.14}{3}\right) \text{cm}^2 = 1\frac{1}{6} \text{ cm}^2$

13. Draw $BM \perp OA$



In right ΔOMB, $\frac{BM}{OB} = \sin 45°$

$\Rightarrow \frac{BM}{4\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow BM = 4 \text{ m}$

Area of shaded region = Area of sector OAB – Area of ΔOAB

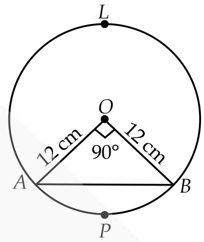
$= \frac{\theta}{360°} \pi r^2 - \frac{1}{2} \times OA \times BM$

$= \frac{45°}{360°} \times \frac{22}{7} \times 4\sqrt{2} \times 4\sqrt{2} - \frac{1}{2} \times 4\sqrt{2} \times 4$

$= \frac{1}{8} \times \frac{22}{7} \times 16 \times 2 - 2\sqrt{2} \times 4 = 12.57 - 11.28 = 1.29 \text{ m}^2$

14. Let O be the centre of circle.

We have, radius (r) = 12 cm and θ = 90°



So, area of sector $OAPB = \frac{\theta}{360°} \pi r^2$

$= \frac{90°}{360°} \times 3.14 \times 12^2 = 113.04 \text{ cm}^2$

Now, area of ΔAOB

$= \frac{1}{2} \times 12 \times 12 = 72 \text{ cm}^2$

$\therefore$ Area of the minor segment APB = Area of sector $OAPB$ – Area of ΔAOB = (113.04 – 72) cm^2 = 41.04 cm^2

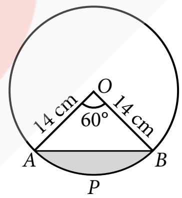
Area of circle = πr^2 = 3.14 × 12^2 = 452.16 cm^2

$\therefore$ Area of major segment ALB = Area of circle – Area of minor segment APB

= (452.16 – 41.04) cm^2 = 411.12 cm^2

15. We have, radius (r) = 14 cm and θ = 60°

Area of minor segment = Area of sector $OAPB$ – Area of ΔAOB



$= \frac{\theta \pi r^2}{360°} - \frac{1}{2} r^2 \sin\theta$

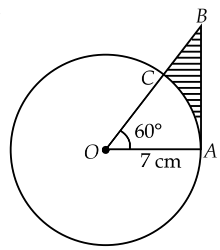
$= \frac{60° \times 22 \times 14 \times 14}{7 \times 360°} - \frac{1}{2} \times 14 \times 14 \times \sin 60°$

$= \frac{22 \times 14}{3} - 7 \times 14 \times \frac{\sqrt{3}}{2}$ = 102.67 – 84.87 = 17.8 cm^2

16. Here, AB is tangent to the circle.

$\therefore \quad \angle OAB$ = 90°

In right ΔOAB,



$\frac{AB}{OA} = \tan 60° \Rightarrow \frac{AB}{7} = \sqrt{3}$

$\Rightarrow \quad AB = 7\sqrt{3} \text{ cm}$

By Pythagoras theorem,

$OB^2 = OA^2 + AB^2 = 7^2 + (7\sqrt{3})^2 = 49 + 147$

$\Rightarrow \quad OB^2 = 196 \Rightarrow OB = 14 \text{ cm}$

$\therefore \quad BC = OB - OC = 14 - 7 = 7 \text{ cm}$

Length of arc, $\overset{\frown}{AC} = \frac{\theta}{360°} \times 2\pi r = \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 7$

$= \frac{1}{6} \times 2 \times 22 = 7.33 \text{ cm}$

$\therefore$ Perimeter of shaded region = Length of arc $\overset{\frown}{AC} + AB + BC$

$= 7.33 + 7\sqrt{3} + 7 = (14.33 + 7\sqrt{3}) \text{ cm}$

Now, area of shaded region = Area of ΔAOB – Area of sector OAC

$= \frac{1}{2} \times OA \times AB - \frac{\theta}{360°} \times \pi r^2$

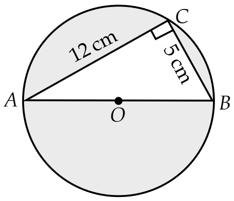
$= \frac{1}{2} \times 7 \times 7\sqrt{3} - \frac{60°}{360°} \times \frac{22}{7} \times 7 \times 7$

$= \frac{49\sqrt{3}}{2} - \frac{1}{6} \times 22 \times 7 = \frac{49\sqrt{3}}{2} - \frac{154}{6} = \frac{7}{2}\left(7\sqrt{3} - \frac{22}{3}\right) \text{cm}^2$

17. We know that angle in a semi-circle is 90°.

$\therefore \quad \angle ACB = 90°$

Since, ACB is right angle triangle.



$\therefore$ By Pythagoras theorem,

$AB^2 = AC^2 + BC^2 = 12^2 + 5^2$

$\Rightarrow \quad AB^2 = 144 + 25 = 169$

$\Rightarrow \quad AB = 13$ cm

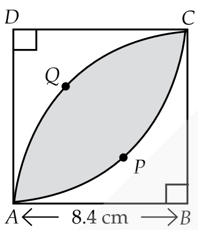
$\therefore$ Radius of circle, $r = \frac{13}{2}$ cm

Area of shaded region = Area of circle – Area of ΔABC

$$= \pi r^2 - \frac{1}{2} \times BC \times AC \; = \frac{22}{7} \times \frac{13}{2} \times \frac{13}{2} - \frac{1}{2} \times 5 \times 12$$

$= 132.78 - 30 = 102.78$ cm^2

18. Radius of quadrant, $r = 8.4$ cm.



$\therefore$ Area of a quadrant $= \frac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 8.4 \times 8.4$$

$= 22 \times 0.3 \times 8.4 = 55.44$ cm^2

$\therefore$ Area of 2 quadrants $= 2 \times 55.44$

$= 110.88$ cm^2

Side of square $ABCD$ = Radius of quadrant = 8.4 cm

$\therefore$ Area of square $ABCD = (8.4)^2 = 70.56$ cm^2

$\therefore$ Area of shaded region $APCQA$ = Area of 2 quadrants – Area of square

$= 110.88 - 70.56 = 40.32$ cm^2

19. Area of equilateral triangle $= 196\sqrt{3}$ cm^2 (Given)

$$\Rightarrow \quad \frac{\sqrt{3}}{4}(\text{side})^2 = 196\sqrt{3}$$

$\Rightarrow \quad (\text{side})^2 = 196 \times 4 \Rightarrow \text{side} = 14 \times 2 = 28$ cm

$\therefore$ Side of equilateral triangle is 28 cm.

Now, circle and equilateral triangle formed by the same wire

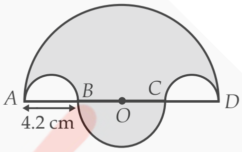
$\therefore$ Perimeter of equilateral triangle = Perimeter of circle

$$\Rightarrow \quad 3 \times 28 = 2\pi r \Rightarrow r = \left(\frac{3 \times 28}{2\pi}\right) \text{cm}$$

Area of circle $= \pi r^2 = \pi \times \left(\frac{3 \times 28}{2\pi}\right)^2$

$$= \pi \times \frac{3 \times 28}{2\pi} \times \frac{3 \times 28}{2\pi} = \frac{3 \times 28 \times 3 \times 28 \times 7}{2 \times 2 \times 22} = 561.27 \text{ cm}^2$$

20. Let the radii of largest semi-circle, smallest semi-circle and semi-circle with diameter BC be r_1, r_2 and r_3 respectively.



Given, $AD = 16.8$ cm

$\therefore \quad r_1 = \frac{16.8}{2} = 8.4$ cm

Also, $AB = 4.2$ cm

$\therefore \quad r_2 = \frac{4.2}{2} = 2.1$ cm

So, $r_3 = OA - AB = 8.4 - 4.2 = 4.2$ cm

(i) Length of boundary = Circumference of largest semi-circle $ABCD$ + Circumference of semi-circle with diameter BC.

$= \pi r_1 + 4.2 + 4.2 + \pi r_3 = \pi(8.4 + 4.2) + 8.4$

$$= \frac{22}{7} \times 12.6 + 8.4 = 39.6 + 8.4 = 48 \text{ cm}$$

(ii) Area of shaded region $= \frac{\pi r_1^2}{2} + \frac{\pi r_3^2}{2} - 2\left(\frac{1}{2}\pi r_2^2\right)$

$$= \pi\left(\frac{r_1^2}{2} + \frac{r_3^2}{2} - r_2^2\right) = \frac{22}{7}\left[\frac{(8.4)^2}{2} + \frac{(4.2)^2}{2} - (2.1)^2\right]$$

$$= \frac{22}{7}(35.28 + 8.82 - 4.41) = 124.74 \text{ cm}^2$$

21. We have, side of square $ABCD = 16.1$ cm

$\therefore$ Area of square $ABCD = (16.1)^2 = 259.21$ cm^2

Radius of quadrant, r = side of square = 16.1 cm

$\therefore$ Area of quadrant $OBCD = \frac{1}{4} \times \frac{22}{7} \times (16.1)^2$

$= 203.665$ cm^2

Area of shaded region = Area of square $ABCD$ – Area of quadrant $OBCD$

$= 259.21 - 203.665 = 55.545$ cm^2

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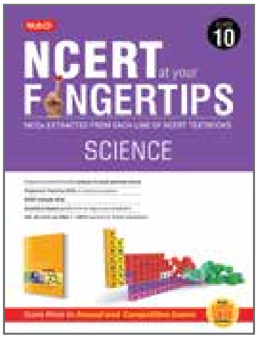

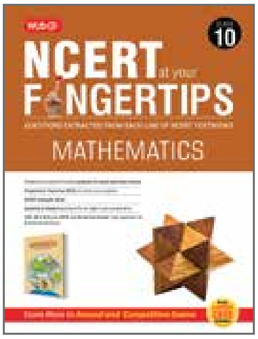

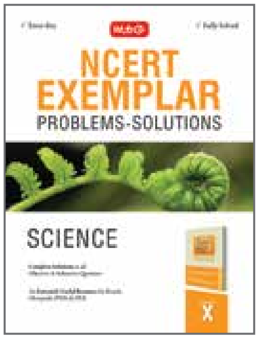

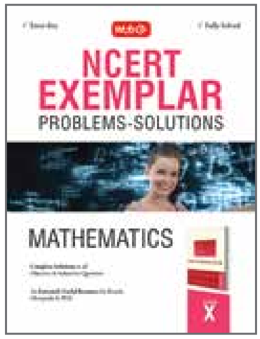

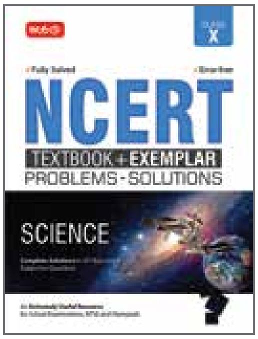

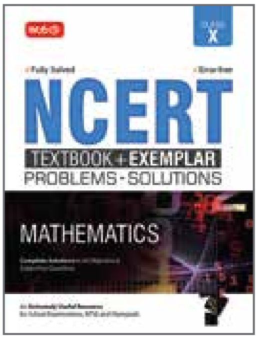

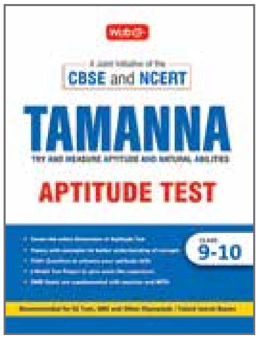

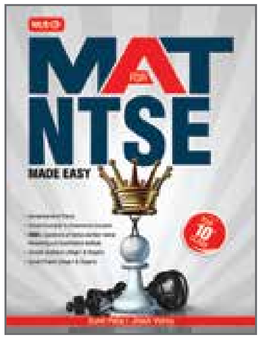

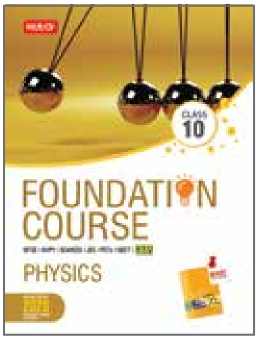

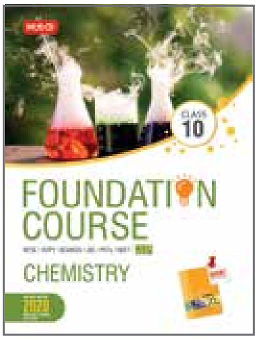

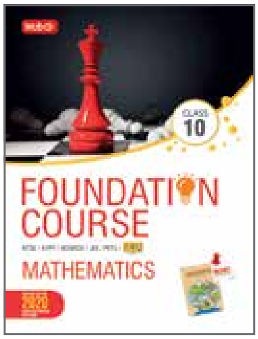

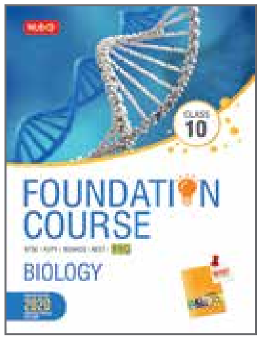

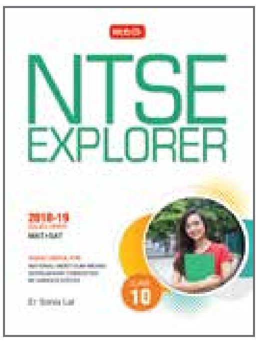

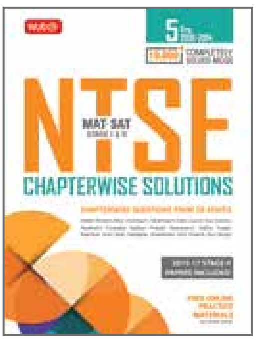

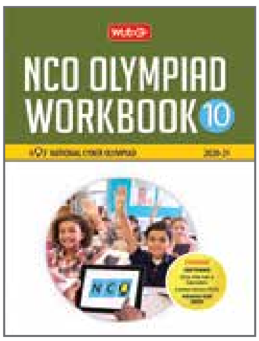

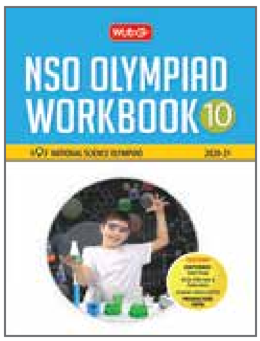

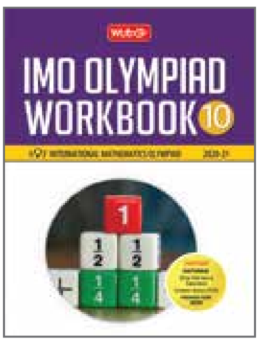

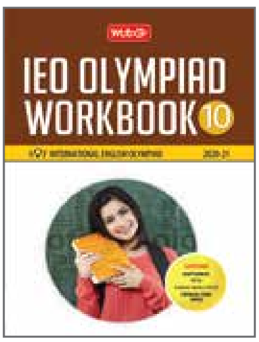

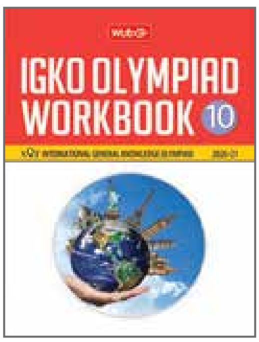

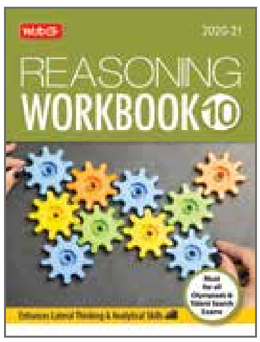

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