Real Numbers

CHAPTER 1

TRY YOURSELF SOLUTIONS

1. Let $n = 5q + 1$, where q is a positive integer
Squaring both sides, we get
$n^2 = (5q + 1)^2 = 25q^2 + 10q + 1$
$= 5(5q^2 + 2q) + 1 = 5m + 1$,
where $m = 5q^2 + 2q$ is an integer
Hence, the square of any positive integer of the form $5q + 1$ is of the same form.

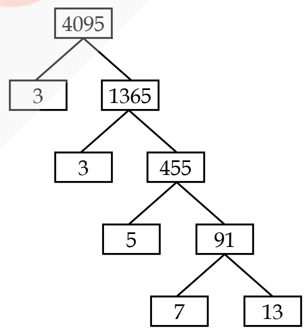
2. On dividing n by 3, let q be the quotient and r be the remainder.
Then, by Euclid's division lemma,
$n = 3q + r$, where $0 \le r < 3$
$\Rightarrow$ $n = 3q$ or $n = 3q + 1$ or $n = 3q + 2$
Case I : If $n = 3q$, which is divisible by 3
but $(n + 1)$ and $(n + 2)$ are not divisible by 3.
So, in this case, only n is divisible by 3.
Case II : If $n = 3q + 1$, then $n + 2 = 3q + 3$
$= 3(q + 1)$ which is divisible by 3
but n and $(n + 1)$ are not divisible by 3.
So, in this case, only $(n + 2)$ is divisible by 3.
Case III : If $n = 3q + 2$, then $n + 1 = 3q + 3$
$= 3(q + 1)$ which is divisible by 3
but n and $(n + 2)$ are not divisible by 3.
So, in this case, only $(n + 1)$ is divisible by 3.
Thus, one and only one out of n, $(n + 1)$ and $(n + 2)$ is divisible by 3.

3. Here, $250 > 30$
$\therefore$ Applying Euclid's division lemma, we get
$250 = 30 \times 8 + 10$
Since, remainder, $10 \neq 0$
$\therefore$ Applying Euclid's division lemma to 30 and 10, we get
$30 = 10 \times 3 + 0$
Since, remainder is 0, when divisor is 10
$\therefore$ By Euclid's division algorithm,
HCF (250, 30) = 10

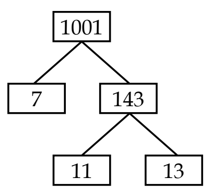
4. The required number of soaps in each box is HCF of 612 and 342.
By Euclid's division algorithm, we have
$612 = 342 \times 1 + 270$,
$342 = 270 \times 1 + 72$,
$270 = 72 \times 3 + 54$,
$72 = 54 \times 1 + 18$,
$54 = 18 \times 3 + 0$
Here, remainder is 0, when divisor is 18
$\therefore$ HCF (612, 342) is 18
So, the trader can pack 18 soaps in each box.

5. Given numbers are 1305, 1365 and 1530
$\because$ $1530 > 1365 > 1305$
$\therefore$ By applying Euclid's division lemma to 1530 and 1365, we get
$1530 = 1365 \times 1 + 165$,
$1365 = 165 \times 8 + 45$,
$165 = 45 \times 3 + 30$,
$45 = 30 \times 1 + 15$, $30 = 15 \times 2 + 0$
Since, remainder is 0 when divisor is 15
$\therefore$ HCF (1530, 1365) is 15
Now, applying Euclid's division lemma to 1305 and 15, we get
$1305 = 15 \times 87 + 0$
Since, remainder is 0, when divisor is 15
$\therefore$ HCF (1305, 15) is 15
Hence, HCF of 1305, 1365 and 1530 is 15.

6. (i) Using factor tree method, we have



$\therefore$ $4095 = 3 \times 3 \times 5 \times 7 \times 13 = 3^2 \times 5 \times 7 \times 13$
(ii) Using factor tree method, we have



$\therefore$ $1001 = 7 \times 11 \times 13$

7. We have, $9 \times 13 \times 17 + 17$
$= 17(9 \times 13 + 1) = 17(117 + 1) = 17 \times 118$, which is not a prime number because it has 17 as a factor other than 1 and the number itself.
$\therefore$ $9 \times 13 \times 17 + 17$ is a composite number.
Also, we have $5 \times 6 \times 7 \times 8 \times 9 + 7 = 7(5 \times 6 \times 8 \times 9 + 1)$, which is again not a prime number because it has 7 as a factor other than 1 and the number itself.
$\therefore$ $5 \times 6 \times 7 \times 8 \times 9 + 7$ is a composite number.

8. If any number ends with the digit 0 or 5, it is always divisible by 5.
If 12^n ends with the digit zero or five, it must be divisible by 5.
This is possible only if prime factorisation of 12^n contains the prime number 5.
Now, $12 = 2 \times 2 \times 3 = 2^2 \times 3$
$\Rightarrow \quad 12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$
Since, there is no term containing 5.
Hence, there is no value of n for which 12^n ends with the digit zero or five.

9. The prime factorisation of 144, 180 and 192 is,
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$
$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$
$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^6 \times 3$
$\therefore \quad$ HCF (144, 180, 192) $= 2^2 \times 3 = 12$
and LCM (144, 180, 192) $= 2^6 \times 3^2 \times 5 = 2880$

10. Since, the books are to be distributed equally among the students of section A or section B.
So, number of books must be a multiple of 32 as well as 36
$\therefore \quad$ Required number of books is the LCM of 32 and 36
Prime factorisation of 32 and 36 is
$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
$\therefore \quad$ LCM of 32 and 36 is $2^5 \times 3^2 = 288$
Hence, required number of books is 288.

11. Given, HCF $(a, b) = 11$
Product of a and $b = 7623$
$$\therefore \quad \text{LCM}(a, b) = \frac{\text{Product of } a \text{ and } b}{\text{HCF}(a, b)}$$
$$\Rightarrow \quad \text{LCM}(a, b) = \frac{7623}{11} = 693$$

12. Given HCF (2520, 6600) = 120
LCM (2520, 6600) $= 252 \times k$
Now, HCF (2520, 6600) × LCM (2520, 6600) = 2520 × 6600
$\Rightarrow \quad (120) \times (252 \times k) = 2520 \times 6600$
$$\Rightarrow \quad k = \frac{2520 \times 6600}{252 \times 120} = 550$$

13. We know that HCF (a, b) × LCM (a, b) = product of a and b
$\Rightarrow \quad 12 \times \text{LCM}(a, b) = 1152$
$$\Rightarrow \quad \text{LCM}(a, b) = \frac{1152}{12} = 96$$

14. Given, $x = p^2q^3$, $y = p^3q$, where p and q are primes.
LCM $(x, y) = p^3q^3$
HCF $(x, y) = p^2q$
Now, LCM $(x, y) = p^3q^3 = pq^2p^2q = pq^2 \times$ HCF (x, y)
$\therefore \quad$ LCM is a multiple of HCF.

15. Let us assume that $\sqrt{3}$ is rational
So, we can find integers a and b ($b \neq 0$ and a, b are co-prime) such that
$$\sqrt{3} = \frac{a}{b} \Rightarrow \sqrt{3}b = a$$
$\Rightarrow \quad 3b^2 = a^2$ [Squaring both sides]
$\therefore \quad$ 3 divides $a^2 \Rightarrow$ 3 divides a ...(ii)
So, we can write $a = 3m$, where m is an integer
Putting $a = 3m$ in (i), we get
$3b^2 = 9m^2 \Rightarrow b^2 = 3m^2$
$\therefore \quad$ 3 divides $b^2 \Rightarrow$ 3 divides b ...(iii)
From (ii) and (iii), 3 is a common factor of a and b, which contradicts the fact that a and b are co-prime.
$\therefore \quad$ Our assumption that $\sqrt{3}$ is rational is wrong.
Hence, $\sqrt{3}$ is irrational.

16. Let us assume that $3+\sqrt{2}$ is rational.
So, we can find two integers a and b ($b \neq 0$ and a, b are co-prime)
such that $3+\sqrt{2} = \frac{a}{b}$, $\Rightarrow \sqrt{2} = \frac{a}{b} - 3 = \frac{a-3b}{b}$
Here, $\frac{a-3b}{b}$ is rational [$\because$ a and b are integers]
$\Rightarrow \quad \sqrt{2}$ is rational, which contradicts the fact that $\sqrt{2}$ is irrational
$\therefore \quad$ Our supposition is wrong.
Hence, $3+\sqrt{2}$ is irrational.

17. We have a rational number 23.3408, whose decimal expansion terminates.
$$\text{Now, } 23.3408 = \frac{233408}{10000} = \frac{233408}{10^4} = \frac{2^6 \times 7 \times 521}{2^4 \times 5^4}$$
$$= \frac{2^2 \times 7 \times 521}{5^4} = \frac{14588}{2^0 \times 5^4}$$
Thus, 23.3408 can be expressed as $\frac{14588}{2^0 \times 5^4}$, where numerator and denominator are co-prime and denominator is of the form $2^m \times 5^n$, where m, n are non-negative integers.

18. Here, denominator,
$2^2 \times 15^3 = 2^2 \times (5 \times 3)^3 = 2^2 \times 5^3 \times 3^3$
$\therefore \quad \frac{129}{2^2 \times 5^3 \times 3^3}$ will be a non-terminating repeating decimal expansion because it contain 3 as a factor in its denominator.

19. As, we need only 10 as denominator, so we multiply $\frac{1}{3}$ with $\frac{3}{10}$
$$\therefore \quad \frac{1}{3} \times \frac{3}{10} = \frac{1}{10} = 0.1$$