Pair of Linear Equations in Two Variables



SOLUTIONS

EXERCISE - 3.1

1. At present : Let Aftab's age = x years

His daughter's age = y years

Seven years ago : Aftab's age = (x - 7) years

His daughter's age = (y - 7) years

According to the condition-I, we have (x - 7) = 7(y - 7) $\Rightarrow x - 7 = 7y - 49 \Rightarrow x - 7y + 42 = 0$ (i

After three years : Aftab's age = (x + 3) years

His daughter's age = (y + 3) years

According to the condition-II, we have

(x + 3) = 3(y + 3)

$$\Rightarrow x + 3 = 3y + 9 \Rightarrow x - 3y - 6 = 0$$
 (ii

Hence, algebraic representation of given situation is

x - 7y + 42 = 0 and x - 3y - 6 = 0

Graphical representation of (i) and (ii): From equation (i), we have:

 $l_1: x - 7y + 42 = 0 \implies y = \frac{x + 42}{7}$

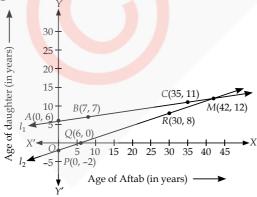
			/
x	0	7	35
у	6	7	11

From equation (ii), we have

$$l_2: x-3y-6=0 \implies y=\frac{x-6}{3}$$

x	0	6	30
y	-2	0	8

Plotting the points A(0, 6), B(7, 7) and C(35, 11) on the graph paper and joining them, we get the line l_1 . Similarly, plotting the points P(0, -2), Q(6, 0) and R(30, 8) on the graph paper and joining them, we get the line l_2 .



Clearly, the lines l_1 and l_2 intersect each other at M(42, 12).

Cost of 3 bats + Cost of 6 balls = ₹ 3900

$$\Rightarrow$$
 3x + 6y = 3900 \Rightarrow x + 2y = 1300 (i)

Also, cost of 1 bat + cost of 3 balls = ₹ 1300

$$\Rightarrow x + 3y = 1300$$
 (ii)

Thus, (i) and (ii) are the algebraic representations of the given situation.

Geometrical representation:

We have for equation (i), $l_1: x + 2y = 1300 \Rightarrow y = \frac{1300 - x}{2}$

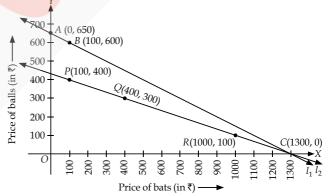
χ	0	100	1300
у	650	600	0

For equation (ii), $l_2: x + 3y = 1300 \Rightarrow y = \frac{1300 - x}{3}$

x	100	400	1000
y	400	300	100

Now, plotting the points A(0, 650), B(100, 600) and C(1300, 0) on the graph paper and joining them, we get the line l_1 .

Similarly, plotting the points P(100, 400), Q(400, 300) and R(1000, 100), on the graph paper and joining them, we get the line l_2 .



We also see from the graph that the straight lines representing the two equations intersect each other at C(1300, 0).

3. Let the cost of 1 kg of apples = $\forall x$ And the cost of 1 kg of grapes = $\forall y$

Algebraic representation:

$$2x + y = 160$$
 ...(i)
and $4x + 2y = 300 \implies 2x + y = 150$...(ii)

Geometrical representation:

We have, for equation (i), $l_1: 2x + y = 160 \Rightarrow y = 160 - 2x$

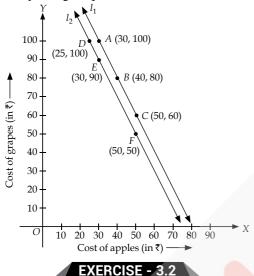
y 60 80 100	x	50	40	30
	y	60	80	100

From equation (ii), we have

$l_2: 2$	2x + y = 1	$50 \Rightarrow y =$	= 150 - 2 <i>x</i>
х	50	30	25
y	50	90	100

Plotting the points A(30, 100), B(40, 80) and C(50, 60) on the graph paper and joining them, we get the line l_1 . Similarly, plotting the points D(25, 100), E(30, 90) and F(50, 50) on the graph paper and joining them, we get the line l_2 .

The lines l_1 and l_2 are parallel.



- 1. (i) Let the number of boys be x and number of girls be y.
- $\therefore x + y = 10$
- ∴ Number of girls = [Number of boys] + 4
- y = x + 4

Now, from equation (1), we have $l_1 : y = 10 - x$

x	6	4	5
у	4	6	5

And from equation (2), we have l_2 : y = x + 4

	Y
^	9+
	8+
girls	7- (3, 7)
r of g	(2, 6)
Number of girls	5 (1,5)
Ž	4 (0, 4)
	3+
	2
l ₂ 🗸	
	l_1
	\longrightarrow

Number of boys -

Since, l_1 and l_2 intersect at the point (3, 7).

- \therefore The solution of the given pair of linear equations is x = 3, y = 7
- \therefore Required number of boys and girls are 3 and 7 respectively.
- (ii) Let the cost of a pencil is $\stackrel{?}{\underset{?}{?}} x$ and cost of a pen is $\stackrel{?}{\underset{?}{?}} y$. Since, cost of 5 pencils + Cost of 7 pens = $\stackrel{?}{\underset{?}{?}} 50$

$$\Rightarrow 5x + 7y = 50 \qquad \dots (1)$$

Also, cost of 7 pencils + cost of 5 pens = ₹ 46

$$\Rightarrow 7x + 5y = 46 \qquad \dots (2)$$

Now, from equation (1), we have

$$l_1: y = \frac{50 - 5x}{7}$$

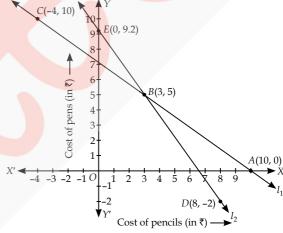
x	10	3	-4
у	0	5	10

And from equation (2), we have

$$l_2: y = \frac{46 - 7x}{5}$$

.... (1)

	_		
X	8	3	0
у	-2	5	9.2



Since, l_1 and l_2 intersect at B(3, 5).

- ∴ Cost of a pencil is ₹ 3 and cost of a pen is ₹ 5.
- 2. Comparing the given equations with $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, we have

(i) For,
$$5x - 4y + 8 = 0$$
, $7x + 6y - 9 = 0$

$$a_1 = 5$$
, $b_1 = -4$, $c_1 = 8$ and $a_2 = 7$, $b_2 = 6$, $c_2 = -9$

$$\therefore \quad \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the lines are intersecting, *i.e.*, they intersect at a unique point.

(ii) For,
$$9x + 3y + 12 = 0$$
, $18x + 6y + 24 = 0$, we have $a_1 = 9$, $b_1 = 3$, $c_1 = 12$ and $a_2 = 18$, $b_2 = 6$, $c_2 = 24$

$$\therefore \quad \frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the lines are coincident.

(iii) For,
$$6x - 3y + 10 = 0$$
, $2x - y + 9 = 0$, we have $a_1 = 6$, $b_1 = -3$, $c_1 = 10$ and $a_2 = 2$, $b_2 = -1$, $c_2 = 9$

$$\therefore \quad \frac{a_1}{a_2} = \frac{6}{2} = 3, \ \frac{b_1}{b_2} = \frac{-3}{-1} = 3, \frac{c_1}{c_2} = \frac{10}{9} \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the lines are parallel.

3. (i) For,
$$3x + 2y = 5$$
, $2x - 3y = 7$, we have $a_1 = 3$, $b_1 = 2$, $c_1 = -5$ and $a_2 = 2$, $b_2 = -3$, $c_2 = -7$

$$\therefore \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, lines are intersecting *i.e.*, they intersect at a unique point.

:. It is consistent pair of equations.

(ii) For,
$$2x - 3y = 8$$
, $4x - 6y = 9$, we have

$$a_1 = 2$$
, $b_1 = -3$, $c_1 = -8$ and $a_2 = 4$, $b_2 = -6$, $c_2 = -9$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

Here,
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, lines are parallel *i.e.*, the given pair of linear equations has no solution.

:. It is inconsistent pair of equations.

(iii) For,
$$\frac{3}{2}x + \frac{5}{3}y = 7$$
, $9x - 10y = 14$, we have

$$a_1 = \frac{3}{2}$$
, $b_1 = \frac{5}{3}$, $c_1 = -7$ and $a_2 = 9$, $b_2 = -10$, $c_2 = -14$

$$\therefore \frac{a_1}{a_2} = \frac{3/2}{9} = \frac{3}{2} \times \frac{1}{9} = \frac{1}{6},$$

$$\frac{b_1}{b_2} = \frac{5/3}{-10} = \frac{5}{3} \times \frac{1}{-10} = -\frac{1}{6}$$
 and $\frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$

Here,
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$
. So lines are intersecting.

So, the given pair of linear equations has a unique solution.

:. It is a consistent pair of equations.

(iv) For,
$$5x - 3y = 11$$
, $-10x + 6y = -22$, we have

$$a_1 = 5$$
, $b_1 = -3$, $c_1 = -11$ and $a_2 = -10$, $b_2 = 6$, $c_2 = 22$

$$\therefore \quad \frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-11}{22} = -\frac{1}{2}$$

Here,
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, lines are coincident.

.. The given pair of linear equations has infinitely many solutions.

Thus, they are consistent.

(v) For,
$$\frac{4}{2}x + 2y = 8$$
, $2x + 3y = 12$, we have

$$a_1 = \frac{4}{2}$$
, $b_1 = 2$, $c_1 = -8$ and $a_2 = 2$, $b_2 = 3$, $c_2 = -12$

$$\therefore \frac{a_1}{a_2} = \frac{4/3}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

Since,
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the lines are coincident *i.e.*, they have infinitely many solutions.

:. The given pair of linear equations are consistent.

4. (i) For,
$$x + y = 5$$
, $2x + 2y = 10$, we have

$$a_1 = 1$$
, $b_1 = 1$, $c_1 = -5$ and $a_2 = 2$, $b_2 = 2$, $c_2 = -10$

$$\therefore \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, lines are coincident.

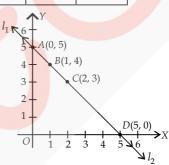
:. The given pair of linear equations are consistent.

$$l_1: x + y = 5 \implies y = 5 - x$$

x	0	5	1
у	5	0	4

$$l_2: 2x + 2y = 10 \implies x + y = 5 \implies y = 5 - x$$

	-	-	
X	2	0	5
у	3	5	0



From graph, it is clear that lines l_1 and l_2 are coincident.

- :. They have infinitely many solutions.
- (ii) For, x y = 8, 3x 3y = 16

$$a_1 = 1$$
, $b_1 = -1$, $c_1 = -8$ and $a_2 = 3$, $b_2 = -3$, $c_2 = -16$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2} \quad \because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

.. The pair of linear equations is inconsistent and lines are parallel.

- :. The given system of equations has no solution.
- (iii) For, 2x + y 6 = 0, 4x 2y 4 = 0

$$a_1 = 2$$
, $b_1 = 1$, $c_1 = -6$ and $a_2 = 4$, $b_2 = -2$, $c_2 = -4$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{-2}; \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

Here,
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

:. Lines are intersecting.

So, it is a consistent pair of linear equations.

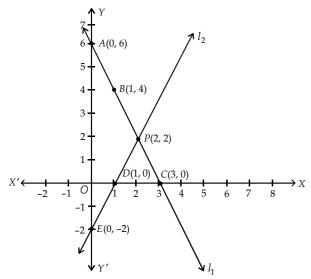
:. It has a unique solution.

$l_1: y = 6 - 2x$

x	0	3	1
у	6	0	4

and
$$l_2: y = \frac{4x - 4}{2}$$

\boldsymbol{x}	0	1	2
у	-2	0	2



- : l_1 and l_2 intersect each other at P(2, 2)
- \therefore x = 2 and y = 2
- (iv) For, 2x 2y 2 = 0, 4x 4y 5 = 0

$$a_1 = 2$$
, $b_1 = -2$, $c_1 = -2$ and $a_2 = 4$, $b_2 = -4$, $c_2 = -5$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}; \quad \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given pair of linear equations is inconsistent and lines are parallel.

Thus, the given system of equations has no solution.

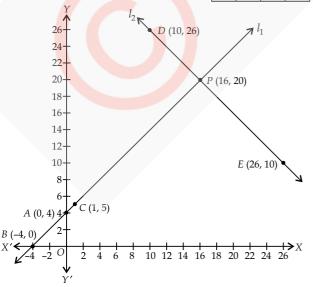
5. Let the width of the garden be x m and the length of the garden be y m.

According to question, 4 + x = y ...(i)

Also,
$$\frac{1}{2}$$
 (perimeter) = 36 $\Rightarrow y + x = 36$...(ii)

From (i),
$$l_1: y = x + 4$$
, $\begin{vmatrix} x & 0 & -4 & 1 \\ y & 4 & 0 & 5 \end{vmatrix}$

From (ii),
$$l_2: x + y = 36 \implies y = 36 - x$$
, $\begin{vmatrix} x & 10 & 26 & 16 \\ y & 26 & 10 & 20 \end{vmatrix}$



The lines l_1 and l_2 intersect each other at P(16, 20).

x = 16 and y = 20

So, Length = 20 m and width = 16 m

6. (i) Let the pair of linear equations be

2x + 3y - 8 = 0, where $a_1 = 2$, $b_1 = 3$ and $c_1 = -8$ and $a_2x + b_2y + c_2 = 0$.

For intersecting lines, we have

$$\frac{2}{a_2} \neq \frac{3}{b_2} \neq \frac{-8}{c_2}$$

- :. We can have $a_2 = 3$, $b_2 = 2$ and $c_2 = -7$
- .. The required equation will be 3x + 2y 7 = 0
- (ii) For parallel lines, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

- $\therefore \text{ Line parallel to } 2x + 3y 8 = 0, \text{ can be taken as } 2x + 3y 12 = 0$
- (iii) For coincident lines, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

 $\therefore \text{ Line coincident to } 2x + 3y - 8 = 0 \text{ can be taken as}$ 2(2x + 3y - 8 = 0)

$$\Rightarrow$$
 $4x + 6y - 16 = 0$

7. We have,
$$x - y + 1 = 0$$
 ...(i)

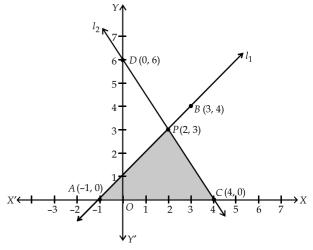
and
$$3x + 2y - 12 = 0$$
 ...(ii)

From (i), $l_1: x - y + 1 = 0 \implies y = x + 1$

x	2	-1	3
у	3	0	4

From (ii),
$$l_2: 3x + 2y - 12 = 0 \implies y = \frac{12 - 3x}{2}$$

x	2	4	0
y	3	0	6



The lines l_1 and l_2 intersect at P(2, 3). Thus, co-ordinates of the vertices of the shaded triangular region are C(4, 0), A(-1, 0) and P(2, 3).

EXERCISE - 3.3

1. (i) We have,
$$x + y = 14$$
 ...(1) and $x - y = 4$...(2)

From equation (1), we have x = (14 - y)Substituting this value of x in (2), we get

$$(14 - y) - y = 4 \implies 14 - 2y = 4$$

 \Rightarrow $-2y = -10 \Rightarrow y = 5$ Now, substituting y = 5 in (1), we get

$$x + 5 = 14 \implies x = 9$$

Hence, x = 9, y = 5 is the required solution.

(ii) We have,
$$s - t = 3$$
 ...(1)

and
$$\frac{s}{3} + \frac{t}{2} = 6$$
 ...(2)

From (1), we have
$$s = (3 + t)$$
 ...

Substituting this value of s in (2), we get

$$\frac{(3+t)}{3} + \frac{t}{2} = 6 \implies 2(3+t) + 3(t) = 6 \times 6$$

$$\Rightarrow$$
 6 + 2t + 3t = 36 \Rightarrow 5t = 30 \Rightarrow t = $\frac{30}{5}$ = 6

Substituting t = 6 in (3) we get, s = 3 + 6 = 9

Thus, s = 9, t = 6 is the required solution.

(iii) We have,
$$3x - y = 3$$
 ...(1) $9x - 3y = 9$...(2)

$$9x - 3y = 9$$
 ...(2)
From (1), $y = (3x - 3)$...(3)

Substituting this value of y in (2), we get

Substituting this value of
$$y$$
 in (2), we get $9x - 3(3x - 3) = 9$

$$\Rightarrow 9x - 9x + 9 = 9 \Rightarrow 9 = 9$$
, which is true statement.

To find these solutions, we put x = k (any real constant) in (3), we get

$$y = 3k - 3$$

x = k, y = 3k - 3 is the required solution, where k is any real number.

(iv) We have,
$$0.2x + 0.3y = 1.3$$
 ...(1)

and
$$0.4x + 0.5y = 2.3$$
 ...(2)

From (1), we have

$$y = \frac{1.3 - 0.2x}{0.3} \tag{3}$$

Substituting the value of y in (2), we get

$$0.4x + 0.5 \left[\frac{1.3 - 0.2x}{0.3} \right] = 2.3 \implies 0.4x + \left[\frac{0.65 - 0.1x}{0.3} \right] = 2.3$$

$$\Rightarrow$$
 0.3 × 0.4x + 0.65 - 0.1x = 0.3 × 2.3

$$\Rightarrow$$
 0.12x + 0.65 - 0.1x = 0.69

$$\Rightarrow$$
 0.02x = 0.69 - 0.65 = 0.04 \Rightarrow x = $\frac{0.04}{0.02}$ = 2

From (3),
$$y = \frac{1.3 - 0.2(2)}{0.3} \implies y = \frac{1.3 - 0.4}{0.3} = \frac{0.9}{0.3} = 3$$

Thus, x = 2 and y = 3 is the required solution.

(v) We have,
$$\sqrt{2}x + \sqrt{3}y = 0$$
 ...(1)

and
$$\sqrt{3}x - \sqrt{8}y = 0$$
 ...(2)

From (2), we have

$$\sqrt{3}x = \sqrt{8}y \Rightarrow x = \frac{\sqrt{8}}{\sqrt{3}}y \qquad \dots(3)$$

Substituting the value of x in (1), we get

$$\sqrt{2} \left[\frac{\sqrt{8}}{\sqrt{3}} y \right] + \sqrt{3} y = 0$$

$$\Rightarrow \frac{\sqrt{16}}{\sqrt{3}} y + \sqrt{3} y = 0 \Rightarrow \frac{4}{\sqrt{3}} y + \sqrt{3} y = 0$$

$$\Rightarrow \frac{\sqrt{16}}{\sqrt{3}}y + \sqrt{3}y = 0 \Rightarrow \frac{4}{\sqrt{3}}y + \sqrt{3}y = 0$$

$$\Rightarrow \left[\frac{4}{\sqrt{3}} + \sqrt{3}\right] y = 0 \Rightarrow y = 0$$

Substituting y = 0 in (3), we get x = 0

Thus, x = 0 and y = 0 is the required solution.

(vi) We have,
$$\frac{3x}{2} - \frac{5y}{3} = -2$$
 ...(1) and $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$...(2)

From (2), we have
$$\frac{x}{3} = \frac{13}{6} - \frac{y}{2}$$

$$\Rightarrow x = 3 \times \left(\frac{13}{6} - \frac{y}{2}\right) \Rightarrow x = \left[\frac{13}{2} - \frac{3}{2}y\right] \qquad \dots(3)$$

Substituting the value of x in (1), we get

$$\frac{3}{2} \left[\frac{13}{2} - \frac{3}{2}y \right] - \frac{5y}{3} = -2$$

$$\Rightarrow \frac{39}{4} - \frac{9y}{4} - \frac{5y}{3} = -2 \Rightarrow \frac{117 - 27y - 20y}{12} = -2$$

$$\Rightarrow$$
 117 - 47y = -24 \Rightarrow -47y = -24 - 117 = -141

$$...(1) \qquad \Rightarrow \qquad 4 \qquad 3$$

$$...(2) \qquad \Rightarrow \qquad 117 - 47y = -$$

$$...(3) \qquad \Rightarrow \qquad y = \frac{-141}{-47} = 3$$
Now, substituting

Now, substituting y = 3 in (3), we get

$$x = \frac{13}{2} - \frac{3}{2}(3) \implies x = \frac{13}{2} - \frac{9}{2} = \frac{4}{2} = 2$$

Thus, x = 2 and y = 3 is the required solution.

2. We have,
$$2x + 3y = 11$$
 ...(1)

and
$$2x - 4y = -24$$
 ...(2)

From (1), we have

$$2x = 11 - 3y$$

$$\Rightarrow x = \left\lceil \frac{11 - 3y}{2} \right\rceil \qquad \dots (3)$$

Substituting this value of x in (2), we get

$$2\left[\frac{11-3y}{2}\right] - 4y = -24 \implies 11 - 3y - 4y = -24$$

$$\Rightarrow -7y = -24 - 11 = -35 \implies y = \frac{-35}{7} = 5$$

Substituting y = 5 in (3), we get

$$x = \frac{11 - 3(5)}{2} \Rightarrow x = \frac{11 - 15}{2} \Rightarrow x = \frac{-4}{2} = -2$$

Thus, x = -2 and y = 5 is the required solution.

Also,
$$y = mx + 3 \implies 5 = m(-2) + 3$$

$$\Rightarrow$$
 $-2m = 5 - 3 \Rightarrow -2m = 2 \Rightarrow m = -1.$

(i) Let the two numbers be x and y such that x > y.

Difference between two numbers = 26

$$\Rightarrow x - y = 26 \qquad ...(1)$$

Again, one number = 3[the other number]

$$\Rightarrow x = 3y \qquad [\because x > y] \qquad \dots(2)$$

Substituting x = 3y in (1), we get

$$3y - y = 26 \implies 2y = 26 \implies y = \frac{26}{2} = 13$$

Now, substituting y = 13 in (2), we get $x = 3(13) \implies x = 39$

Thus, two numbers are 39 and 13.

(ii) Let the two angles be x and y such that x > y.

The larger angle exceeds the smaller by 18°

$$\therefore x = y + 18^{\circ}$$

Also, sum of two supplementary angles = 180°

$$\therefore \quad x + y = 180^{\circ}$$

Substituting the value of x from (1) in (2), we get $(18^{\circ} + y) + y = 180^{\circ}$

$$\Rightarrow$$
 2y = 180° - 18° = 162° \Rightarrow y = $\frac{162^{\circ}}{2}$ = 81°

Substituting $y = 81^{\circ}$ in (1), we get

$$x = 18^{\circ} + 81^{\circ} = 99^{\circ}$$

Thus, $x = 99^{\circ}$ and $y = 81^{\circ}$ is the required solution.

(iii) Let the cost of a bat = $\forall x$ and the cost of a ball = $\forall y$.

$$\Rightarrow 7x + 6y = 3800 \qquad \dots ($$

Again, [cost of 3 bats] + [cost of 5 balls] = ₹ 1750

$$\Rightarrow 3x + 5y = 1750 \qquad \dots (2$$

From (2), we have

$$y = \left[\frac{1750 - 3x}{5} \right] \tag{3}$$

Substituting this value of y in (1), we get

$$7x + 6\left[\frac{1750 - 3x}{5}\right] = 3800$$

$$\Rightarrow$$
 35x + 10500 - 18x = 19000

$$\Rightarrow$$
 17x = 19000 - 10500 \Rightarrow x = $\frac{8500}{17}$ = 500

Substituting x = 500 in (3), we get

$$y = \frac{1750 - 3(500)}{5} \Rightarrow y = \frac{1750 - 1500}{5} = \frac{250}{5} = 50$$

Thus, x = 500 and y = 50

Cost of a bat = ₹ 500 and cost of a ball = ₹ 50

(iv) Let fixed charges be ₹ x and charges per km be ₹ y.

Charges for the journey of 10 km = ₹ 105

$$\therefore x + 10y = 105$$
 ...(1)

and charges for the journey of 15 km = ₹ 155

$$\therefore x + 15y = 155$$
 ...(2)

From (1), we have

$$x = 105 - \frac{10}{y}$$
 ...(3)

Substituting the value of x in (2), we get

$$(105 - 10y) + 15y = 155$$

$$\Rightarrow$$
 5y = 155 - 105 = 50 \Rightarrow y = 10

Substituting y = 10 in (3), we get

$$x = 105 - 10(10) \implies x = 105 - 100 = 5$$

Thus, x = 5 and y = 10

So, fixed charges = ₹ 5 and charges per km = ₹ 10.

Now, charges for 25 km = x + 25y = 5 + 25(10)

The charges for 25 km journey = ₹ 255

(v) Let the numerator = x and the denominator = y

Fraction = x/y

Case I:
$$\frac{x+2}{y+2} = \frac{9}{11} \implies 11(x+2) = 9(y+2)$$

$$\Rightarrow$$
 11x + 22 = 9y + 18

$$\Rightarrow$$
 11x - 9y + 4 = 0

Case II: $\frac{x+3}{y+3} = \frac{5}{6} \implies 6(x+3) = 5(y+3)$

$$\Rightarrow 6x + 18 = 5y + 15$$

$$\Rightarrow 6x - 5y + 3 = 0 \qquad \dots(2)$$

...(1) Now, from (2),
$$x = \left[\frac{5y - 3}{6}\right]$$
 ...(3)

Substituting this value of x in (1), we get

$$11\left[\frac{5y-3}{6}\right] - 9y + 4 = 0$$

...(2)

$$\Rightarrow$$
 55y - 33 - 54y + 24 = 0 \Rightarrow y - 9 = 0 \Rightarrow y = 9

Now, substituting y = 9 in (3), we get

$$x = \frac{5(9) - 3}{6} \implies x = \frac{45 - 3}{6} = \frac{42}{6} = 7$$

x = 7 and $y = 9 \Rightarrow$ Fraction = 7/9

(vi) Let the present age of Jacob = x years and the present age of his son = y years.

 \therefore 5 years hence, age of Jacob = (x + 5) years and age of his son = (y + 5) years

Given, [Age of Jacob after 5 years] = 3[Age of his son after 5 years]

$$\begin{array}{ccc} \therefore & x + 5 = 3(y + 5) \Rightarrow x + 5 = 3y + 15 \\ \Rightarrow & x - 3y - 10 = 0 \end{array}$$
 ...(1)

5 years ago, age of Jacob = (x - 5) years and age of his son = (y - 5) years

Also, five years ago [Age of Jacob] = 7[Age of his son]

$$(x-5) = 7(y-5) \Rightarrow x-5 = 7y-35$$

$$\Rightarrow x - 7y + 30 = 0 \qquad \dots (2)$$

From (i),
$$x = 10 + 3y$$
 ...(3)

Substituting this value of x in (2), we get

$$(10 + 3y) - 7y + 30 = 0 \Rightarrow -4y = -40 \Rightarrow y = 10$$

Now, substituting y = 10 in (3), we get

$$x = 10 + 3(10) \implies x = 10 + 30 = 40$$

Thus, x = 40 and y = 10

:. Present age of Jacob = 40 years and present age of his son = 10 years

EXERCISE - 3.4

1. (i) Elimination method :
$$x + y = 5$$
 ...(1)

$$2x - 3y = 4 \qquad \dots (2)$$

Multiplying (1) by 3, we get

$$3x + 3y = 15$$
 ...(3)

Adding (2) & (3), we get $5x = 19 \implies x = \frac{19}{5}$

Now, putting $x = \frac{19}{5}$ in (1), we get

$$\frac{19}{5} + y = 5 \implies y = 5 - \frac{19}{5} = \frac{25 - 19}{5} = \frac{6}{5}$$

Thus,
$$x = \frac{19}{5}$$
 and $y = \frac{6}{5}$

Substitution Method:

...(1)

We have,
$$x + y = 5 \Rightarrow y = 5 - x$$
 ...(1)

and
$$2x - 3y = 4$$
 ...(2)

Put
$$y = 5 - x$$
 in (2), we get

$$2x - 3(5 - x) = 4 \implies 2x - 15 + 3x = 4$$

$$\Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$$

From. (1),
$$y = 5 - \frac{19}{5} = \frac{25 - 19}{5} = \frac{6}{5}$$

Hence,
$$x = \frac{19}{5}$$
 and $y = \frac{6}{5}$

(ii) Elimination method:

$$3x + 4y = 10$$

$$2x - 2y = 2$$

Multiplying (2) by 2, we get

$$4x - 4y = 4$$

Adding (1) and (3), we get

$$\therefore 7x = 14 \implies x = \frac{14}{7} = 2$$

Putting x = 2 in (1), we get 3(2) + 4y = 10

$$\Rightarrow 4y = 10 - 6 = 4 \Rightarrow y = \frac{4}{4} = 1$$

Thus, x = 2 and y = 1

Substitution Method:

$$3x + 4y = 10 \implies y = \frac{10 - 3x}{4}$$
$$2x - 2y = 2 \implies x - y = 1$$

$$2x - 2y = 2 \implies x - y = 1$$

Putting $y = \frac{10-3x}{4}$ in (2), we get

$$x - \left(\frac{10 - 3x}{4}\right) = 1 \implies 4x - 10 + 3x = 4$$

$$\Rightarrow$$
 $7x = 14 \Rightarrow x = \frac{14}{7} = 2$

Putting
$$x = 2$$
 in (1), we get
$$y = \frac{10 - 3 \times 2}{4} = \frac{10 - 6}{4} = \frac{4}{4} = 1$$

Hence, x = 2 and y = 1

(iii) Elimination method:

$$3x - 5y - 4 = 0$$

$$9x = 2y + 7$$
 or $9x - 2y - 7 = 0$

Multiplying (1) by 3, we get

$$9x - 15y - 12 = 0$$

Subtracting (2) from (3), we get

$$\therefore 13y + 5 = 0 \implies y = \frac{-5}{13}$$

Substituting the value of y in (1), we get

$$3x - 5\left(\frac{-5}{13}\right) - 4 = 0 \implies 3x + \frac{25}{13} - 4 = 0$$

$$\Rightarrow 3x = \frac{-25 + 52}{13} = \frac{27}{13} \Rightarrow x = \frac{27}{13} \times \frac{1}{3} = \frac{9}{13}$$

Thus,
$$x = \frac{9}{13}$$
 and $y = -\frac{5}{13}$

Substitution Method:

$$3x - 5y - 4 = 0 \implies y = \frac{3x - 4}{5}$$
 ...(1)
 $9x - 2y - 7 = 0$...(2)

Putting $y = \frac{3x-4}{5}$ in (2), we get

$$9x - 2\left(\frac{3x - 4}{5}\right) - 7 = 0 \implies 45x - 6x + 8 - 35 = 0$$

$$\Rightarrow$$
 39 $x = 27 \Rightarrow x = \frac{27}{39} = \frac{9}{13}$

Putting $x = \frac{9}{12}$ in (1), we get

$$y = \frac{3 \times \frac{9}{13} - 4}{5} = \frac{27 - 52}{65} = \frac{-25}{65} \implies y = \frac{-5}{13}$$

Hence, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

(iv) Elimination method:

...(1)

...(2)

...(3)

$$\frac{x}{2} + \frac{2y}{3} = -1$$
 ...(1)

$$x - \frac{y}{3} = 3$$
 ...(2)

Multiplying (2) by 2, we get

$$2x - \frac{2y}{3} = 6$$
 ...(3)

Adding (1) and (3), we have

$$\frac{x}{2} + 2x = 5 \implies \frac{5}{2}x = 5 \implies x = 5 \times \frac{2}{5} = 2$$

Putting x = 2 in (1), we get

...(1)
$$\frac{2}{2} + \frac{2y}{3} = -1 \implies 1 + \frac{2y}{3} = -1$$

..(2)
$$\Rightarrow \frac{2y}{3} = -1 - 1 = -2 \Rightarrow y = -2 \times \frac{3}{2} = -3$$

Thus, x = 2 and y = -3

Substitution Method:

$$\frac{x}{2} + \frac{2y}{3} = -1$$
 ...(1)

$$x - \frac{y}{3} = 3 \implies y = 3(x - 3)$$
 ...(2)

Putting y = 3(x - 3) in (1) from (2), we get

$$\frac{x}{2} + \frac{2}{3} \times 3(x - 3) = -1 \implies \frac{x}{2} + 2x - 6 = -1$$

$$\Rightarrow \frac{5x}{2} = 5 \Rightarrow x = 5 \times \frac{2}{5} = 2$$

Putting x = 2 in (2), we get

$$y = 3(2 - 3) = 3(-1) = -3$$

Hence, x = 2 and y = -3.

2. (i) Let the numerator = x and the denominator = y

 \therefore Fraction = x/y

Case I:
$$\frac{x+1}{y-1} = 1 \Rightarrow x+1 = y-1$$

$$\Rightarrow x - y = -2 \qquad \dots (1)$$

Case II: $\frac{x}{y+1} = \frac{1}{2} \implies x = \frac{1}{2}(y+1)$

$$\Rightarrow \quad x - \frac{y}{2} = \frac{1}{2} \qquad \qquad \dots (2)$$

Subtracting (2) from (1), we have

$$-y + \frac{y}{2} = -2 - \frac{1}{2} \implies -\frac{1}{2}y = -\frac{5}{2} \implies y = 5$$

Now, putting y = 5 in (2), we hav

$$x - \frac{5}{2} = \frac{1}{2}$$
 \Rightarrow $x = \frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$

Thus, x = 3 and y = 5.

Hence, the required fraction = 3/5

(ii) Let the present age of Nuri = x years and the present age of Sonu = y years

5 years ago:

Age of Nuri = (x - 5) years and age of Sonu = (y - 5) years According to the question,

Age of Nuri = 3[Age of Sonu]

$$\Rightarrow x - 5 = 3[y - 5] \Rightarrow x - 5 = 3y - 15$$

$$\Rightarrow x - 3y + 10 = 0 \qquad \dots (1)$$

10 years later:

Age of Nuri = (x + 10) years and age of Sonu

= (y + 10) years

According to the question:

Age of Nuri = 2[Age of Sonu]

$$\Rightarrow x + 10 = 2(y + 10) \Rightarrow x + 10 = 2y + 20$$

$$\Rightarrow x - 2y - 10 = 0 \qquad \dots (2$$

Subtracting (1) from (2), $y - 20 = 0 \implies y = 20$

Putting y = 20 in (1), we get

$$x - 3(20) + 10 = 0 \Rightarrow x - 50 = 0 \Rightarrow x = 50$$

Thus, x = 50 and y = 20

- ∴ Age of Nuri = 50 years and age of Sonu = 20 years
- (iii) Let the digit at unit's place = x and the digit at ten's place = y
- \therefore The number = 10y + x

The number obtained by reversing the digits = 10x + y

∴ 9[The number] = 2[Number obtained by reversing the digits]

$$\therefore 9[10y + x] = 2[10x + y] \Rightarrow 90y + 9x = 20x + 2y$$

$$\Rightarrow$$
 11x - 88y = 0

$$\Rightarrow x - 8y = 0 \qquad ...(1)$$

Also, sum of the digits = 9

$$\therefore \quad x + y = 9 \qquad \qquad \dots (2)$$

Subtracting (1) from (2), we have

$$9y = 9 \Rightarrow y = 1$$

Putting y = 1 in (2), we get $x + 1 = 9 \Rightarrow x = 8$

Thus, x = 8 and y = 1

- $\therefore \text{ The required number} = 10y + x = (10 \times 1) + 8$
- = 10 + 8 = 18
- (iv) Let the number of 50 rupees notes = x

and the number of 100 rupees notes = y

According to the question,

Total number of notes = 25

$$\therefore x + y = 25 \qquad \dots (1)$$

∴ The value of all the notes = ₹ 2000

$$50x + 100y = 2000$$

$$\Rightarrow x + 2y = 40$$
...(

Subtracting (1) from (2), we get

y = 15

Putting y = 15 in (1), we get

$$x + 15 = 25 \implies x = 25 - 15 = 10$$

Thus, x = 10 and y = 15

- \therefore Number of 50 rupees notes = 10 and number of 100 rupees notes = 15.
- (v) Let the fixed charge (for the three days) = $\overline{\xi}$ x and the additional charge for each extra day = $\overline{\xi}$ y
- : Charge for 7 days = ₹ 27
- ⇒ x + 4y = 27 ...(1) [: Extra days = 7 3 = 4]

Charge for 5 days = ₹21

$$\Rightarrow x + 2y = 21$$
 ...(2) [: Extra days = 5 - 3 = 2]

Subtracting (2) from (1), we get

$$2y = 6 \implies y = 3$$

Putting y = 3 in (2), we have

$$x + 2(3) = 21 \Rightarrow x = 21 - 6 = 15$$

So, x = 15 and y = 3

∴ Fixed charge = ₹ 15 and additional charge per day = ₹ 3.

EXERCISE - 3.5

1. Compare the given equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

(i) For,
$$x - 3y - 3 = 0$$
, $3x - 9y - 2 = 0$

$$a_1 = 1$$
, $b_1 = -3$, $c_1 = -3$ and $a_2 = 3$, $b_2 = -9$, $c_2 = -2$

Now,
$$\frac{a_1}{a_2} = \frac{1}{3}$$
, $\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

- The given system has no solution.
- (ii) For 2x + y 5 = 0, 3x + 2y 8 = 0

$$a_1 = 2$$
, $b_1 = 1$, $c_1 = -5$ and $a_2 = 3$, $b_2 = 2$, $c_2 = -8$

Now,
$$\frac{a_1}{a_2} = \frac{2}{3}$$
, $\frac{b_1}{b_2} = \frac{1}{2} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

:. The given system has a unique solution.

To solve the equation, we have

$$\frac{x}{1} \times \frac{-5}{-8} = \frac{y}{-5} \times \frac{2}{3} = \frac{1}{2} \times \frac{1}{3}$$

...(1)
$$\Rightarrow \frac{x}{(-8)-(-10)} = \frac{y}{(-15)-(-16)} = \frac{1}{4-3}$$

...(2)
$$\Rightarrow \frac{x}{2} = \frac{y}{1} = 1 \Rightarrow \frac{x}{2} = 1 \text{ and } \frac{y}{1} = 1 \Rightarrow x = 2 \text{ and } y = 1$$

(iii) For
$$3x - 5y - 20 = 0$$
, $6x - 10y - 40 = 0$

$$a_1 = 3$$
, $b_1 = -5$, $c_1 = -20$ and $a_2 = 6$, $b_2 = -10$, $c_2 = -40$

Since,
$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$$
, $\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

... The given system of linear equations has infinitely many solutions.

(iv) For
$$x - 3y - 7 = 0$$
, $3x - 3y - 15 = 0$
 $a_1 = 1$, $b_1 = -3$, $c_1 = -7$, $a_2 = 3$, $b_2 = -3$, $c_2 = -15$

$$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-3} = 1 \text{ and } \frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: The given system has unique solution.

Now, using cross multiplication method, we have

$$\frac{x}{-3} = \frac{y}{-7} = \frac{1}{1}$$

$$\frac{-7}{-3} = \frac{1}{1}$$

$$\frac{-3}{-15} = \frac{1}{3}$$

$$\Rightarrow \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$$

$$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6} \Rightarrow x = \frac{24}{6} = 4, y = \frac{-6}{6} = -1$$

Thus, x = 4 and y = -1.

2. (i) For,
$$2x + 3y - 7 = 0$$
, $(a - b)x + (a + b)y - (3a + b - 2) = 0$
 $a_1 = 2$, $b_1 = 3$, $c_1 = -7$ and
 $a_2 = (a - b)$, $b_2 = (a + b)$, $c_2 = -(3a + b - 2)$

For an infinite number of solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \frac{2}{(a-b)} = \frac{3}{(a+b)} = \frac{-7}{-(3a+b-2)}$$

From the first two terms, we get

$$\frac{2}{a-b} = \frac{3}{a+b}$$

$$\Rightarrow$$
 2a + 2b = 3a - 3b \Rightarrow 2a - 3a + 2b + 3b = 0

$$\Rightarrow$$
 $-a + 5b = 0 \Rightarrow a - 5b = 0$

From the last two terms, we get

$$\frac{3}{a+b} = \frac{7}{3a+b-2}$$

$$\Rightarrow$$
 9a + 3b - 6 = 7a + 7b

$$\Rightarrow$$
 2a - 4b = 6 \Rightarrow a - 2b - 3 = 0

Now, using cross-multiplication method, we have

$$\frac{a}{\overset{-5}{-2}} \times \overset{0}{\overset{-3}{-3}} = \frac{b}{\overset{0}{-3}} \times \overset{1}{\overset{1}{1}} = \frac{1}{\overset{1}{1}} \times \overset{-5}{\overset{-2}{-2}}$$

$$\Rightarrow \frac{a}{15-0} = \frac{b}{0+3} = \frac{1}{-2+5}$$

$$\Rightarrow \frac{a}{15} = \frac{b}{3} = \frac{1}{3} \Rightarrow a = \frac{1}{3} \times 15 = 5, b = \frac{1}{3} \times 3 = 1$$

Thus, a = 5 and b = 1.

(ii) For, 3x + y - 1 = 0,

$$(2k-1)x + (k-1)y - (2k+1) = 0$$

$$a_1 = 3$$
, $b_1 = 1$, $c_1 = -1$ and $a_2 = 2k - 1$, $b_2 = k - 1$, $c_2 = -(2k + 1)$

For no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \implies \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-(2k+1)}$$

Taking first two terms, we get

$$3(k-1) = 2k-1 \implies 3k-3 = 2k-1$$

$$\Rightarrow$$
 $3k - 2k = -1 + 3 \Rightarrow k = 2$.

Substitution method:

$$8x + 5y = 9$$
 ...(1) and $3x + 2y = 4$...(2)

Given equations are
$$8x + 5y = 9$$
 ...(1) and $3x + 2y = 4$...(2)
From (2), $y = \frac{4-3x}{2}$...(3)

Substituting this value of *y* in (1), we have

$$8x + 5\left[\frac{4 - 3x}{2}\right] = 9$$

$$\Rightarrow$$
 16x + 20 - 15x = 18 \Rightarrow x = 18 - 20 = -2

Substituting x = -2 in (3), we get

$$y = \frac{4 - 3(-2)}{2} = \frac{4 + 6}{2} = \frac{10}{2} = 5$$

Thus, x = -2 and y = 5

Cross-multiplication method:

For,
$$8x + 5y - 9 = 0$$
, $3x + 2y - 4 = 0$

By cross-multiplication method, we have

$$\frac{x}{5} = \frac{x}{-9} = \frac{1}{9} = \frac{1}{8} = \frac{1}{8 \times 5}$$

$$\frac{x}{2} - 4 - 4 \times 3 = \frac{y}{-27 + 32} = \frac{1}{16 - 15}$$

$$\Rightarrow x = 1 \times (-2) = -2 \text{ and } y = 1 \times 5 = 5$$

Thus, x = -2 and y = 5

4. (i) Let the fixed charges = $\forall x$ and cost of food per day = ₹ y

For student A: Number of days = 20

Cost of food for 20 days = ₹ 20y

According to the question, x + 20y = 1000

$$\Rightarrow x + 20y - 1000 = 0$$
 ...(1)

For student B: Number of days = 26

Cost of food for 26 days = ₹ 26y

According to the question : x + 26y = 1180

$$\Rightarrow x + 26y - 1180 = 0$$
 ...(2)

Solving these by cross-multiplication, we get

$$\frac{x}{20} \times \frac{-1000}{-1180} = \frac{y}{-1000} \times \frac{1}{1} = \frac{1}{1} \times \frac{20}{26}$$

$$\Rightarrow \frac{x}{-23600 + 26000} = \frac{y}{-1000 + 1180} = \frac{1}{26 - 20}$$

$$\Rightarrow \frac{x}{2400} = \frac{y}{180} = \frac{1}{6}$$

...(1)

...(2)

$$\therefore x = \frac{1}{6} \times 2400 = 400, y = \frac{1}{6} \times 180 = 30$$

Thus, x = 400 and y = 30

∴ Fixed charges = ₹ 400 and cost of food per day = ₹ 30

(ii) Let the numerator = x and the denominator = y

Fraction = x/y

Condition-I:
$$\frac{x-1}{y} = \frac{1}{3}$$

$$\Rightarrow 3x - 3 = y$$

$$\Rightarrow 3x - y - 3 = 0 \qquad \dots(1)$$

Condition-II: $\frac{x}{y+8} = \frac{1}{4}$

$$\Rightarrow 4x = y + 8$$

$$\Rightarrow 4x - y - 8 = 0 \qquad \dots(2)$$

From (1) and (2), we have

$$a_1 = 3$$
, $b_1 = -1$, $c_1 = -3$ and $a_2 = 4$, $b_2 = -1$, $c_2 = -8$

Using cross-multiplication method, we get

$$\frac{x}{\stackrel{-1}{\sim} \stackrel{-3}{\sim} = \frac{1}{\stackrel{-3}{\sim} \stackrel{-3}{\sim} \frac{3}{4}} = \frac{1}{\stackrel{3}{\sim} \stackrel{-1}{\sim} \stackrel{-1}{\sim} \frac{1}{1}}$$

$$\Rightarrow \frac{x}{8-3} = \frac{y}{-12+24} = \frac{1}{-3+4} \Rightarrow \frac{x}{5} = \frac{y}{12} = \frac{1}{1}$$

$$\therefore x = \frac{1}{1} \times 5 \text{ and } y = \frac{1}{1} \times 12 = 12$$

Thus, x = 5 and y = 12 : Fraction = 5/12.

(iii) Let the number of correct answers = x and the number of wrong answers = y

Case-I : Marks for all correct answers = $(3 \times x) = 3x$

Marks for all wrong answers = $(1 \times y) = y$

According to the condition :

$$3x - y = 40$$
 ...(1)

Case-II : Marks for all correct answers = $(4 \times x) = 4x$

Marks for all wrong answers = $(2 \times y) = 2y$

According to the condition:

$$4x - 2y = 50 \Rightarrow 2x - y = 25$$
 ...(2)

From (1) and (2), we have

$$a_1 = 3$$
, $b_1 = -1$, $c_1 = -40$ and

$$a_2 = 2$$
, $b_2 = -1$, $c_2 = -25$

Using cross-multiplication method, we get

$$\frac{x}{-1} \xrightarrow{-40} = \frac{y}{-40} \xrightarrow{3} = \frac{1}{3} \xrightarrow{-1}$$

$$\Rightarrow \frac{x}{25 - 40} = \frac{y}{-80 + 75} = \frac{1}{-3 + 2} \Rightarrow \frac{x}{-15} = \frac{y}{-5} = \frac{1}{-1}$$

$$\Rightarrow x = \frac{1}{-1} \times (-15) = 15, y = \frac{1}{-1} \times (-5) = 5$$

 \therefore x = 15 and y = 5

Now, total number of questions = [Number of correct answers] + [Number of wrong answers] = 15 + 5 = 20Thus, required number of questions = 20.

(iv) Let the speed of car-I be x km/hr and the speed of car-II be y km/hr.

Case-I:
$$A \stackrel{\text{Car-I} \rightarrow}{\bullet} Car-II \rightarrow C$$

Distance travelled by car-I in 1 hour = AC

$$\Rightarrow$$
 speed × time = 5 × x km, $AC = 5x$

Distance travelled by car-II, BC = 5y

Since
$$AB = AC - BC$$
, $100 = 5x - 5y$

$$\Rightarrow$$
 5x - 5y - 100 = 0

$$\Rightarrow x - y - 20 = 0 \qquad \dots (1)$$

Distance travelled by car-I in 1 hour = AD

$$\therefore AD = 1 \times x = x$$

Distance travelled by car-II in 1 hour = BD

$$\therefore BD = 1 \times y = y$$

Now,
$$AB = AD + DB \Rightarrow 100 = x + y$$

$$\Rightarrow x + y = 100$$

From (1) and (2), we have

$$a_1 = 1$$
, $b_1 = -1$, $c_1 = -20$ and $a_2 = 1$, $b_2 = 1$, $c_2 = -100$,

Using cross-multiplication method, we get
$$\frac{x}{-1} < -20 = \frac{y}{-20} < 1 = \frac{1}{1} < -1$$

$$\Rightarrow \frac{x}{100 + 20} = \frac{y}{-20 + 100} = \frac{1}{1 + 1} \Rightarrow \frac{x}{120} = \frac{y}{80} = \frac{1}{2}$$

$$\Rightarrow x = \frac{1}{2} \times 120 = 60, y = \frac{1}{2} \times 80 = 40$$

Thus, speed of car-I = 60 km/hr and speed of car-II = 40 km/hrkm/hr.

(v) Let the length of the rectangle = x units and the breadth of the rectangle = y units

 \therefore Area of the rectangle = $x \times y = xy$

Condition-I:

(Length - 5) × (Breadth + 3) = Area - 9
⇒
$$(x-5)(y+3) = xy - 9 \Rightarrow 3x - 5y - 15 = -9$$

⇒ $3x - 5y - 6 = 0$...(1)

Condition-II:

(Length + 3) × (Breadth + 2) = Area + 67

$$\Rightarrow$$
 (x + 3)(y + 2) = xy + 67 \Rightarrow 2x + 3y + 6 = 67

$$\Rightarrow 2x + 3y - 61 = 0 \qquad \dots (2)$$

From (1) and (2), we have
$$a_1 = 3$$
, $b_1 = -5$, $c_1 = -6$ and $a_2 = 2$, $b_2 = 3$, $c_2 = -61$

Using cross-multiplication method, we get

$$\frac{x}{-5} = \frac{y}{-61} = \frac{1}{3} = \frac$$

Thus, length of the rectangle = 17 units and breadth of the rectangle = 9 units.

EXERCISE - 3.6

1. (i) Put $\frac{1}{x} = u$ and $\frac{1}{u} = v$

$$\therefore \frac{1}{2x} + \frac{1}{3u} = 2 \Rightarrow \frac{u}{2} + \frac{v}{3} = 2 \qquad \dots (1)$$

And
$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6} \Rightarrow \frac{u}{3} + \frac{v}{2} = \frac{13}{6}$$
 ...(2)

Multiplying (1) by $\frac{1}{2}$ and (2) by $\frac{1}{2}$, we get

$$\frac{1}{3} \left[\frac{u}{2} + \frac{v}{3} = 2 \right] \Rightarrow \frac{u}{6} + \frac{v}{9} = \frac{2}{3} \qquad \dots (3)$$

$$\frac{1}{2} \left[\frac{u}{3} + \frac{v}{2} = \frac{13}{6} \right] \Rightarrow \frac{u}{6} + \frac{v}{4} = \frac{13}{12} \qquad \dots (4)$$

Subtracting (3) from (4), we ge

$$\frac{v}{4} - \frac{v}{9} = \frac{13}{12} - \frac{2}{3} \Rightarrow \frac{9v - 4v}{36} = \frac{13 - 8}{12} \Rightarrow \frac{5}{36}v = \frac{5}{12}$$
$$\Rightarrow v = \frac{5}{12} \times \frac{36}{5} = 3$$

Substituting v = 3 in (3), we get

$$\frac{u}{6} + \frac{3}{9} = \frac{2}{3} \Rightarrow \frac{u}{6} = \frac{2}{3} - \frac{3}{9} = \frac{6 - 3}{9} = \frac{3}{9} \Rightarrow u = \frac{3}{9} \times 6 = 2$$

But,
$$u = \frac{1}{x} \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$$

And,
$$v = \frac{1}{y} \Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$$

 \therefore The required solution is $x = \frac{1}{2}$, $y = \frac{1}{3}$.

(ii) We have
$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$
 ...(1)

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1 \tag{2}$$

Putting
$$\frac{1}{\sqrt{x}} = u$$
 and $\frac{1}{\sqrt{y}} = v$, we get
$$2u + 3v - 2 = 0$$

$$4u - 9v + 1 = 0$$
...(3)
...(4)

Solving (3) and (4) by cross multiplication method, we

$$\frac{u}{\overset{3}{\cancel{3}} \cancel{\cancel{3}} \overset{-2}{\cancel{1}}} = \frac{v}{\overset{-2}{\cancel{1}} \cancel{\cancel{4}}} = \frac{1}{\overset{2}{\cancel{4}} \cancel{\cancel{3}}} \xrightarrow{3}$$

$$\Rightarrow \frac{u}{3-18} = \frac{v}{-8-2} = \frac{1}{-18-12} \Rightarrow \frac{u}{-15} = \frac{v}{-10} = \frac{1}{-30}$$

$$\Rightarrow u = \frac{1}{-30} \times (-15) = \frac{1}{2}, v = \frac{1}{-30} \times (-10) = \frac{1}{3}$$

But
$$u = \frac{1}{\sqrt{x}}$$
 and $v = \frac{1}{\sqrt{y}}$

$$\therefore \quad \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \frac{1}{x} = \frac{1}{4} \quad \Rightarrow \quad x = 4$$

and
$$\frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \frac{1}{y} = \frac{1}{9} \Rightarrow y = 9$$

 \therefore The required solution is x = 4, y = 9.

(iii) We have
$$\frac{4}{x} + 3y = 14$$
 ...(1

$$\frac{3}{x} - 4y = 23$$
 ...(2)

Let $\frac{1}{x} = u$ in (1) and (2), we get

$$4u + 3y - 14 = 0$$
 ...(3)
 $3u - 4y - 23 = 0$...(4)

$$3u - 4y - 23 = 0$$
 ...(4)

Solving (3) and (4) by cross multiplication method, we

$$\frac{u}{\overset{3}{\cancel{4}} \overset{-14}{\cancel{-14}} = \frac{y}{\overset{-14}{\cancel{4}} \overset{4}{\cancel{4}}} = \frac{1}{\overset{4}{\cancel{4}} \overset{3}{\cancel{4}} \overset{3}{\cancel{4}} \overset{3}{\cancel{4}} \overset{3}{\cancel{4}}$$

$$\Rightarrow \frac{u}{-69-56} = \frac{y}{-42+92} = \frac{1}{-16-9}$$

$$\Rightarrow \frac{u}{-125} = \frac{y}{50} = \frac{1}{-25}$$

$$\Rightarrow u = \frac{1}{-25} \times (-125) = 5$$

And
$$y = \frac{1}{-25} \times 50 = -2$$

Since,
$$u = \frac{1}{x}$$
 :: $\frac{1}{x} = 5 \Rightarrow x = \frac{1}{5}$

 \therefore The required solution is $x = \frac{1}{\pi}$, y = -2.

(iv) We have,
$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$
 ...(1)

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$
 ...(2)

Let
$$\frac{1}{x-1} = u$$
 and $\frac{1}{y-2} = v$

Equations (1) and (2) can be expressed as

$$5u + v - 2 = 0 \qquad ...(3)$$

Solving (3) and (4) by cross multiplication method, we

$$\Rightarrow \frac{u}{-1-6} = \frac{v}{-12+5} = \frac{1}{-15-6} \Rightarrow \frac{u}{-7} = \frac{v}{-7} = \frac{1}{-21}$$

$$\therefore u = \frac{1}{(-21)} \times (-7) = \frac{1}{3}, v = \frac{1}{(-21)} \times (-7) = \frac{1}{3}$$

But,
$$u = \frac{1}{x - 1} \implies \frac{1}{x - 1} = \frac{1}{3}$$

$$\Rightarrow$$
 3 = $x - 1 \Rightarrow x = 4$

And,
$$v = \frac{1}{y-2} \Rightarrow \frac{1}{y-2} = \frac{1}{3}$$

$$\Rightarrow$$
 3 = $y - 2 \Rightarrow y = 5$

Thus, the required solution is x = 4, y = 5.

(v) We have,
$$\frac{7x - 2y}{xy} = 5$$
 ...(1)

$$\frac{8x + 7y}{xy} = 15$$
 ...(2)

...(1) From equation (1), $\frac{7x}{xy} - \frac{2y}{xy} = 5$

From equation (2), $\frac{8x}{xy} + \frac{7y}{xy} = 15$

$$\Rightarrow \frac{8}{y} + \frac{7}{x} = 15 \qquad \dots (4)$$

Let $\frac{1}{x} = u$ and $\frac{1}{u} = v$

$$\frac{7v}{2u} - 2u - 5 = 0$$
 ...(5)

Using cross multiplication method to solve (5) and (6),

$$\frac{v}{\stackrel{-2}{\cancel{-15}}} = \frac{u}{\stackrel{-5}{\cancel{-15}}} = \frac{1}{7} \times \frac{1}{7}$$

$$\Rightarrow \frac{v}{30+35} = \frac{u}{-40+105} = \frac{1}{49+16} \Rightarrow \frac{v}{65} = \frac{u}{65} = \frac{1}{65}$$

$$\therefore u = \frac{1}{65} \times 65 = 1, v = \frac{1}{65} \times 65 = 1$$

Since,
$$u = \frac{1}{x} \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$$

and
$$v = \frac{1}{y} \Rightarrow \frac{1}{y} = 1 \Rightarrow y = 1$$

Thus, the required solution is x = 1, y = 1.

(vi) We have,
$$6x + 3y = 6xy$$
 ...(1)

$$2x + 4y = 5xy \qquad \qquad \dots (2)$$

From (1), we get

$$\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy} \Rightarrow \frac{6}{y} + \frac{3}{x} = 6 \qquad \dots(3)$$

$$\frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy} \Rightarrow \frac{2}{y} + \frac{4}{x} = 5 \qquad \dots (4)$$

Let
$$\frac{1}{x} = u$$
 and $\frac{1}{y} = v$

$$6v + 3u = 6$$
 ...(5)

...(7)

Multiplying (6) by 3, we get 6v + 12u = 15

Subtracting (5) from (7), we get $9u = 9 \Rightarrow u = 1$ Substituting p = 1 in (5), we get 6v + 3(1) = 6

$$\Rightarrow 6v = 6 - 3 = 3 \Rightarrow v = \frac{3}{6} = \frac{1}{2}$$

Since,
$$u = \frac{1}{x} \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$$

and
$$v = \frac{1}{y} \Rightarrow \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$$

Thus, the required solution is x = 1, y = 2.

(vii) We have,
$$\frac{10}{x+y} + \frac{2}{x-y} = 4$$
 ...(1)

and
$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$
 ...(2)

Let
$$\frac{1}{x+y} = u$$
 and $\frac{1}{x-y} = v$

 \therefore Equations (1) and (2) can be expressed as 10u + 2v - 4 = 0

$$10u + 2v - 4 = 0 \qquad ...(3)$$

15u - 5v + 2 = 0 \qquad ...(4)

Solving (3) and (4) by cross-multiplication method, we get

$$\frac{u}{2} = \frac{v}{-4} = \frac{v}{-4} = \frac{1}{10} = \frac{1}{10} = \frac{1}{10} = \frac{v}{15} = \frac{1}{10} = \frac{v}{-5} = \frac{v}{-5} = \frac{v}{-60-20} = \frac{1}{-50-30} = \frac{v}{-16} = \frac{v}{-80} = \frac{1}{-80} = \frac{v}{-80} = \frac{v}{-80}$$

$$\therefore u = \frac{1}{(-80)} \times (-16) = \frac{1}{5} \text{ and } v = \frac{1}{(-80)} \times (-80) = 1$$

But,
$$u = \frac{1}{x+y} = \frac{1}{5} \implies x+y=5$$
 ...(5)

And
$$v = \frac{1}{x - y} = \frac{1}{1} \implies x - y = 1$$
 ...(6)

Adding (5) and (6), we have

$$2x = 6 \Rightarrow x = 3$$

From (5), $3 + y = 5 \Rightarrow y = 2$

Thus, the required solution is x = 3, y = 2.

(viii) We have,
$$\frac{1}{(3x+y)} + \frac{1}{(3x-y)} = \frac{3}{4}$$
 ...(1)

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8} \qquad \dots (2)$$

Let
$$\frac{1}{(3x+y)} = u$$
 and $\frac{1}{(3x-y)} = v$

 \therefore (1) and (2) can be expressed as

$$u+v=\frac{3}{4}$$
 ...(3)

$$\frac{u}{2} - \frac{v}{2} = -\frac{1}{8} \qquad ...(4)$$

Multiplying equation (3) by 1/2, we get

$$\frac{u}{2} + \frac{v}{2} = \frac{3}{8}$$
 ...(5)

Adding (4) and (5), we get

$$\left(\frac{u}{2} + \frac{v}{2}\right) = \left(\frac{3}{8} - \frac{1}{8}\right) \Rightarrow u = \frac{2}{8} = \frac{1}{4}$$

From (3),
$$\frac{1}{4} + v = \frac{3}{4} \Rightarrow v = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

But,
$$u = \frac{1}{3x + y} \Rightarrow \frac{1}{3x + y} = \frac{1}{4} \Rightarrow 3x + y = 4$$
 ...(6)

And
$$v = \frac{1}{3x - y} \Rightarrow \frac{1}{3x - y} = \frac{1}{2} \Rightarrow 3x - y = 2$$
 ...(7)

Adding (6) and (7), we get

$$6x = 6 \Rightarrow x = 1$$

Subtracting (7) from (6), we get

$$2y = 2 \implies y = 1$$

Thus, the required solution is x = 1, y = 1.

2. (i) Let the speed of rowing in still water = x km/hr and the speed of the water current = y km/hr.

: Downstream speed = (x + y) km/hr

Upstream speed = (x - y) km/hr

According to the question,

$$2 = \frac{20}{x+y}$$

$$\Rightarrow x+y=10$$

$$\therefore \text{ Time} = \frac{\text{Distance}}{\text{Speed}}$$

and
$$2 = \frac{4}{x - y}$$

$$\Rightarrow x - y = 2 \qquad \dots (2)$$

Adding (1) and (2), we get

$$2x = 12 \quad \Rightarrow \quad x = \frac{12}{2} = 6$$

From (1), 6 + y = 10

$$\Rightarrow$$
 $y = 10 - 6 = 4$

Thus, speed of rowing in still water = 6 km/hr, speed of water current = 4 km/hr

(ii) Let the time taken to finish the task by one woman alone = x days and by one man alone = y days.

 \therefore one woman's 1 day work = 1/x

one man's 1 day work = 1/y

Since, [2 women + 5 men] finish the task in 4 days

$$...(1) \therefore 4 \times \left[\frac{2}{x} + \frac{5}{y}\right] = 1$$

$$...(2) \Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4} ...(1)$$

Again, 3 women + 6 men, finish the task in 3 days

$$\therefore 3 \times \left[\frac{3}{x} + \frac{6}{y} \right] = 1 \quad \Rightarrow \quad \frac{3}{x} + \frac{6}{y} = \frac{1}{3} \qquad \dots (2)$$

Let
$$\frac{1}{x} = u$$
 and $\frac{1}{y} = v$

: Equations (1) and (2) can be expressed as

$$2u + 5v = \frac{1}{4} \Rightarrow 8u + 20v - 1 = 0$$

$$3u + 6v = \frac{1}{3} \Rightarrow 9u + 18v - 1 = 0$$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

...(1)

$$a_1 = 8$$
, $b_1 = 20$, $c_1 = -1$, $a_2 = 9$, $b_2 = 18$, $c_2 = -1$

Using cross multiplication method, we get

$$\frac{u}{20} = \frac{v}{-1} = \frac{v}{-1} \times \frac{8}{9} = \frac{1}{8 \times 20}$$

$$\Rightarrow \frac{u}{-20 + 18} = \frac{v}{-9 + 8} = \frac{1}{144 - 180}$$

$$\Rightarrow \frac{u}{-2} = \frac{v}{-1} = \frac{1}{-36}$$

$$\therefore u = \frac{1}{-36} \times (-2) = \frac{1}{18}, v = \frac{1}{-36} \times (-1) = \frac{1}{36}$$

Since,
$$u = \frac{1}{x} \Rightarrow \frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$$

and
$$v = \frac{1}{y} \Rightarrow \frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$$

- .. One man can finish the work in 36 days and one woman can finish the work in 18 days.
- (iii) Let the speed of the train = x km/hr and the speed of the bus = y km/hr

Case I: Total journey = 300 km

- :. Journey travelled by train = 60 km
- \therefore Journey travelled by bus = (300 60) km = 240 km
- ∵ Total time taken = 4 hours

$$\therefore \frac{60}{x} + \frac{240}{y} = 4$$

$$\Rightarrow \frac{1}{x} + \frac{4}{y} = \frac{1}{15}$$
...(1)

Case II : Distance travelled by train = 100 km
Distance travelled by bus = (300 - 100) km = 200 km

Total time taken = 4 hrs 10 mins = $\left(4 + \frac{10}{60}\right)$ hrs = $\frac{25}{6}$ hrs

$$\therefore \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

$$\Rightarrow \frac{4}{x} + \frac{8}{y} = \frac{1}{6}$$
...(2)

Multiplying (1) by 4, we get

$$\frac{4}{x} + \frac{16}{y} = \frac{4}{15}$$
 ...(3)

Subtracting (3) from (2), we get

$$\frac{8}{y} - \frac{16}{y} = \frac{1}{6} - \frac{4}{15} \implies \frac{-8}{y} = \frac{5 - 8}{30} = \frac{-3}{30}$$
$$\implies \frac{1}{y} = \frac{-3}{30} \times \frac{1}{(-8)} = \frac{1}{80} \implies y = 80$$

From (1),
$$\frac{1}{x} + \frac{4}{80} = \frac{1}{15}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{15} - \frac{4}{80} = \frac{1}{15} - \frac{1}{20} \Rightarrow \frac{1}{x} = \frac{4-3}{60} = \frac{1}{60}$$

$$\therefore x = 60$$

Thus, speed of the train = 60 km/hr and speed of the bus = 80 km/hr.

EXERCISE - 3.7

1. Let the age of Ani = x years and the age of Biju = y years

Case I: y > x

According to 1st condition :
$$y - x = 3$$

Now, [Age of Ani's father] = 2[Age of Ani] = 2x years

Also, [Age of Biju's sister] =
$$\frac{1}{2}$$
 [Age of Biju] = $\frac{1}{2}y$

According to 2^{nd} condition: $2x - \frac{1}{2}y = 30$

$$\Rightarrow 4x - y = 60 \qquad \dots (2)$$

Adding (1) and (2), we get y - x + 4x - y = 63

$$\Rightarrow$$
 3x = 63 \Rightarrow x = $\frac{63}{3}$ = 21

Substituting the value of x in equation (1),

we get
$$y - 21 = 3 \Rightarrow y = 3 + 21 = 24$$

Case II: x > y

$$\therefore x - y = 3 \qquad \dots (1)$$

According to the condition: $2x - \frac{1}{2}y = 30$

$$\Rightarrow 4x - y = 60 \qquad \dots (2)$$

Subtracting, (1) from (2), we get 4x - y - x + y = 60 - 3

$$\Rightarrow$$
 3x = 57 \Rightarrow x = $\frac{57}{2}$ = 19

Substituting the value of x in equation (1), we get

$$19 - y = 3 \implies y = 16$$

- ∴ Age of Ani = 19 years Age of Biju = 16 years
- 2. Let the capital of 1st friend = \mathbb{Z} x,

and the capital of 2^{nd} friend = ξy

According to the condition,

x + 100 = 2(y - 100)

$$\Rightarrow$$
 $x + 100 - 2y + 200 = 0 \Rightarrow $x - 2y + 300 = 0$...(1)$

Also,
$$6(x - 10) = y + 10 \implies 6x - y - 70 = 0$$
 ...(2)

From (1),
$$x = -300 + 2y$$
 ...(3)

Substituting the value of x in equation (2), we get 6[-300 + 2y] - y - 70 = 0

$$\Rightarrow$$
 -1870 + 11y = 0 \Rightarrow y = $\frac{1870}{11}$ = 170

Now, Substituting the value of y in equation (3), we get, x = -300 + 2y

$$= -300 + 2(170) = -300 + 340 = 40$$

Thus, 1^{st} friend has ₹ 40 and the 2^{nd} friend has ₹ 170.

3. Let the actual speed of the train = x km/hr and the actual time taken = y hours

$$\therefore$$
 Distance = speed × time

According to 1st condition: $(x + 10) \times (y - 2) = xy$

$$\Rightarrow xy - 2x + 10y - 20 = xy$$

$$\Rightarrow 2x - 10y + 20 = 0$$
 ...(1)

According to 2^{nd} condition : $(x - 10) \times (y + 3) = xy$

$$\Rightarrow xy + 3x - 10y - 30 = xy$$

$$\Rightarrow 3x - 10y - 30 = 0$$
 ...(2)

Using cross multiplication for solving (1) and (2), we get

$$\frac{x}{-10} = \frac{y}{20} = \frac{1}{20} = \frac{1}{2}$$

$$-10 - 30 - 30 - 30 - 3$$

$$\Rightarrow \frac{x}{300 + 200} = \frac{y}{60 + 60} = \frac{1}{-20 + 30} \Rightarrow \frac{x}{500} = \frac{y}{120} = \frac{1}{10}$$
$$\Rightarrow x = \frac{1}{10} \times 500 = 50, y = \frac{1}{10} \times 120 = 12$$

Thus, the distance covered by the train $= 50 \times 12 \text{ km} = 600 \text{ km}$

4. Let the number of students = x

and the number of rows = y

:. Number of students in each row

$$= \frac{\text{Number of students}}{\text{Number of rows}} = \frac{x}{y}$$

According to 1st condition:
$$\left(\frac{x}{y} + 3\right) \times (y - 1) = x$$

[: Number of students in a row × Number of rows = Number of students]

$$\Rightarrow x - \frac{x}{y} + 3y - 3 = x$$

$$\Rightarrow \frac{x}{y} - 3y + 3 = 0 \qquad \dots (1)$$

Also, according to 2nd condition:

$$\left(\frac{x}{y} - 3\right) \times (y + 2) = x$$

$$\Rightarrow x + \frac{2x}{y} - 3y - 6 = x$$

$$\Rightarrow \frac{2x}{y} - 3y - 6 = 0 \qquad \dots (2)$$

Let
$$\frac{x}{y} = p$$

∴ Equation (1) and (2) can be expressed as :

and
$$2p - 3y - 6 = 0$$
 ...(4)

Subtracting (3) from (4), we get

$$2p - 3y - 6 - p + 3y - 3 = 0$$

$$\Rightarrow p-9=0 \Rightarrow p=9$$

Substituting value of p in equation (3), we get,

$$9 - 3y + 3 = 0 \Rightarrow -3y = -12$$

$$\Rightarrow y = \frac{-12}{-3} = 4$$

We have,
$$\frac{x}{y} = 9$$
 $[\because p = 9]$

$$\therefore \quad \frac{x}{4} = 9 \Rightarrow x = 4 \times 9 = 36$$

Thus, number of students in the class, x = 36

5. : Sum of angles of a triangle =
$$180^{\circ}$$

$$\therefore \angle A + \angle B + \angle C = 180^{\circ} \qquad \dots (1)$$

$$\therefore \quad \angle C = 3 \angle B = 2(\angle A + \angle B) \qquad \dots (2)$$

From (1) and (2), we have

$$\angle A + \angle B + 2(\angle A + \angle B) = 180^{\circ}$$

$$\Rightarrow \angle A + \angle B + 2\angle A + 2\angle B = 180^{\circ}$$

$$\Rightarrow \angle A + \angle B = 60^{\circ}$$
 ...(3)

Also,
$$\angle A + \angle B + 3\angle B = 180^{\circ}$$

$$\Rightarrow \angle A + 4\angle B = 180^{\circ}$$
 ...(4)

Subtracting (3) from (4), we get

$$\angle A + 4\angle B - \angle A - \angle B = 180^{\circ} - 60^{\circ}$$

$$\Rightarrow 3 \angle B = 120^{\circ} \Rightarrow \angle B = \frac{120^{\circ}}{3} = 40^{\circ}$$

Substituting $\angle B = 40^{\circ}$ in (4) we get,

$$\angle A + 4(40^{\circ}) = 180^{\circ}$$

$$\Rightarrow$$
 $\angle A = 180^{\circ} - 160^{\circ} = 20^{\circ}$

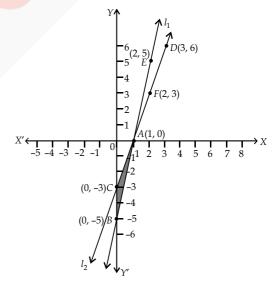
$$\therefore$$
 $\angle C = 3 \angle B = 3 \times 40^{\circ} = 120^{\circ}$

Thus, $\angle A = 20^{\circ}$, $\angle B = 40^{\circ}$ and $\angle C = 120^{\circ}$.

6. To draw the graph of 5x - y = 5, we get

and for equation 3x - y = 3, we get

Plotting the points (1, 0), (2, 5) and (0, -5), we get a straight line l_1 . Plotting the points (2, 3), (3, 6) and (0, -3), we get a straight line l_2 .



From the figure, obviously, the vertices of the triangle formed are A(1, 0), B(0, -5) and C(0, -3).

7. (i) We have,
$$px + qy = p - q$$
 ...(1)

$$qx - py = p + q \qquad \dots (2)$$

Multiplying (1) by p and (2) by q, we get

$$p^2x + qpy = p^2 - pq \qquad \dots (3)$$

$$q^2x - pqy = q^2 + pq \qquad \dots (4)$$

$$p^{2}x + q^{2}x = p^{2} + q^{2}$$

$$\Rightarrow (p^{2} + q^{2})x = p^{2} + q^{2}$$

$$\Rightarrow x = \frac{p^{2} + q^{2}}{p^{2} + q^{2}} = 1$$

Substituting x = 1 in (1) we get,

$$p(1) + qy = p - q \Rightarrow p + qy = p - q$$

 $\Rightarrow y = -1$

Thus, the required solution is x = 1, y = -1

(ii) We have,
$$ax + by = c \Rightarrow ax + by - c = 0$$

 $bx + ay = (1 + c) \Rightarrow bx + ay - (1 + c) = 0$
By cross multiplication, we have

$$\frac{x}{b} \times \frac{-c}{-(1+c)} = \frac{y}{-c} \times \frac{a}{b} = \frac{1}{a} \times \frac{b}{a}$$

$$\Rightarrow \frac{x}{-b-bc+ac} = \frac{y}{-bc+a+ac} = \frac{1}{a^2-b^2}$$

$$\therefore \quad x = \frac{-b - bc + ac}{a^2 - b^2}, y = \frac{-bc + a + ac}{a^2 - b^2}$$

$$\Rightarrow x = \frac{c(a-b)-b}{a^2-b^2}, y = \frac{c(a-b)+a}{a^2-b^2}$$

(iii) We have,
$$\frac{x}{a} - \frac{y}{b} = 0$$

$$ax + by = a^2 + b^2$$

From (1), we have

$$\frac{x}{a} = \frac{y}{b} \Longrightarrow y = \left(\frac{x}{a} \times b\right)$$

Substituting $y = \left(\frac{b}{a}x\right)$ in (2), we have

$$ax + b\left(\frac{b}{a}x\right) = a^2 + b^2$$

$$\Rightarrow x \left[\frac{a^2 + b^2}{a} \right] = a^2 + b^2 \Rightarrow x = \frac{a^2 + b^2}{a^2 + b^2} \times a \Rightarrow x = a$$

Substituting x = a in (3), we get

$$y = \frac{a}{a} \times b \Longrightarrow y = b$$

Thus, the required solution is x = a, y = b.

(iv) We have,

$$(a - b)x + (a + b)y = a^2 - 2ab - b^2$$

$$(a + b) (x + y) = a^2 + b^2$$

From (2),

$$(a + b)x + (a + b)y = a^2 + b^2$$

Subtracting (3) from (1), we get

$$x[(a-b)-(a+b)] = a^2 - 2ab - b^2 - a^2 - b^2$$

$$\Rightarrow x[a-b-a-b] = -2ab - 2b^2$$

$$\Rightarrow x(-2b) = -2b(a+b)$$

$$\Rightarrow x = \frac{-2b(a+b)}{-2b} \Rightarrow x = a+b$$

Substituting x = (a + b) in (1), we get

$$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$$

$$\Rightarrow$$
 $(a + b)y = a^2 - 2ab - b^2 - a^2 + b^2$

$$\Rightarrow$$
 $(a+b)y = -2ab \Rightarrow y = \frac{-2ab}{(a+b)}$

Thus, the required solution is

$$x = a + b, y = -\frac{2ab}{a+b}$$

...(1) (v) We have,
$$152x - 378y = -74$$
 ...(1)

$$-378x + 152y = -604$$
 ...(2)

Adding (1) and (2), we have

$$-226x - 226y = -678$$

$$\Rightarrow x + y = 3 \qquad \dots (3)$$

Subtracting (1) from (2), we get

$$-530x + 530y = -530$$

$$\Rightarrow$$
 $-x + y = -1$

$$\Rightarrow x - y = 1$$
 ...(4)

Adding (3) and (4), we get

$$2x = 4 \implies x = 2$$

Subtracting (3) from (4), we get

...(1)
$$-2y = -2 \implies y = \frac{-2}{-2} = 1$$

Thus, the required solution is x = 2 and y = 1...(2)

8. : ABCD is a cyclic quadrilateral.

$$(3)$$
 \therefore $\angle A + \angle C = 180^{\circ}$ and $\angle B + \angle D = 180^{\circ}$

$$\Rightarrow$$
 $[4y + 20^{\circ}] + [-4x] = 180^{\circ}$

$$\Rightarrow 4y - 4x + 20^{\circ} - 180^{\circ} = 0 \Rightarrow 4y - 4x - 160^{\circ} = 0$$

$$\Rightarrow y - x - 40^{\circ} = 0 \qquad \dots (1)$$

And
$$[3y - 5^{\circ}] + [-7x + 5^{\circ}] = 180^{\circ}$$

$$\Rightarrow$$
 3y - 5° - 7x + 5° - 180° = 0

$$\Rightarrow 3y - 7x - 180^\circ = 0 \qquad \dots (2)$$

Multiplying (1) by 7, we get

$$7y - 7x - 280^{\circ} = 0 \qquad ...(3)$$

Subtracting (3) from (2), we get

$$3y - 7x - 180^{\circ} - 7y + 7x + 280^{\circ} = 0$$

$$3y - 7x - 180^{\circ} - 7y + 7x + 280^{\circ} = 0$$

$$\Rightarrow$$
 $-4y + 100^{\circ} = 0 \Rightarrow y = \frac{-100^{\circ}}{-4} = 25^{\circ}$

Now, substituting
$$y = 25^{\circ}$$
 in (1), we get

...(2)
$$-x = 40^{\circ} - 25^{\circ} = 15^{\circ}$$

$$\Rightarrow x = -15^{\circ}$$

...(3)
$$\therefore \angle A = 4y + 20^{\circ} = 4(25^{\circ}) + 20^{\circ} = 100^{\circ} + 20^{\circ} = 120^{\circ}$$

$$\angle B = 3y - 5^{\circ} = 3(25^{\circ}) - 5^{\circ} = 75^{\circ} - 5^{\circ} = 70^{\circ}$$

$$\angle C = -4x = -4(-15^{\circ}) = 60^{\circ}$$

$$\angle D = -7x + 5^{\circ} = -7(-15^{\circ}) + 5^{\circ} = 105^{\circ} + 5^{\circ} = 110^{\circ}$$

Thus,
$$\angle A = 120^{\circ}$$
, $\angle B = 70^{\circ}$, $\angle C = 60^{\circ}$, $\angle D = 110^{\circ}$.

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