

Statistics

EXERCISE - 14.1

1. We have the following table:

| Number of plants | Number of Houses (f_i) | Class mark (x_i) | $f_i x_i$ |
|------------------|----------------------------|----------------------|------------------------|
| 0 - 2 | 1 | 1 | 1 |
| 2 - 4 | 2 | 3 | 6 |
| 4 - 6 | 1 | 5 | 5 |
| 6 - 8 | 5 | 7 | 35 |
| 8 - 10 | 6 | 9 | 54 |
| 10 - 12 | 2 | 11 | 22 |
| 12 - 14 | 3 | 13 | 39 |
| Total | $\Sigma f_i = 20$ | | $\Sigma f_i x_i = 162$ |

$$\therefore \text{Mean, } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{162}{20} = 8.1$$

Thus, mean number of plants per house is 8.1. We have used the direct method because values of x_i and f_i are small.

2. Let the assumed mean, $a = 550$

\therefore Class size, $h = 20$

$$\therefore u_i = \frac{x_i - a}{h} = \frac{x_i - 550}{20}$$

\therefore We have the following table :

| Class-interval | Frequency (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 550}{20}$ | $f_i u_i$ |
|----------------|---------------------|----------------------|------------------------------|------------------------|
| 500 - 520 | 12 | 510 | -2 | -24 |
| 520 - 540 | 14 | 530 | -1 | -14 |
| 540 - 560 | 8 | 550 | 0 | 0 |
| 560 - 580 | 6 | 570 | 1 | 6 |
| 580 - 600 | 10 | 590 | 2 | 20 |
| Total | $\Sigma f_i = 50$ | | | $\Sigma f_i u_i = -12$ |

$$\therefore \text{Mean, } \bar{x} = a + h \times \left\{ \frac{\Sigma f_i u_i}{\Sigma f_i} \right\}$$

$$= 550 + 20 \times \left(\frac{-12}{50} \right) = 550 - \frac{24}{5} = 550 - 4.8 = 545.2$$

Hence, the mean daily wages of workers is ₹ 545.2.

3. Let the assumed mean, $a = 18$

\therefore Class size, $h = 2$

$$\therefore u_i = \frac{x_i - a}{h} = \frac{x_i - 18}{2}$$

Now, we have the following table:

| Class-interval | Frequency (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 18}{2}$ | $f_i u_i$ |
|----------------|-------------------------|----------------------|----------------------------|---------------------------|
| 11 - 13 | 7 | 12 | -3 | -21 |
| 13 - 15 | 6 | 14 | -2 | -12 |
| 15 - 17 | 9 | 16 | -1 | -9 |
| 17 - 19 | 13 | 18 | 0 | 0 |
| 19 - 21 | f | 20 | 1 | f |
| 21 - 23 | 5 | 22 | 2 | 10 |
| 23 - 25 | 4 | 24 | 3 | 12 |
| Total | $\Sigma f_i = (f + 44)$ | | | $\Sigma f_i u_i = f - 20$ |

$$\therefore \text{Mean, } \bar{x} = a + h \times \frac{\Sigma f_i u_i}{\Sigma f_i}$$

$$\Rightarrow 18 = 18 + 2 \left(\frac{f - 20}{f + 44} \right) \Rightarrow 0 = 2 \left(\frac{f - 20}{f + 44} \right)$$

$$\Rightarrow 2(f - 20) = 0 \Rightarrow f = 20$$

Thus, missing frequency is 20.

4. Let the assumed mean, $a = 75.5$

\therefore Class size, $h = 3$

$$\therefore u_i = \frac{x_i - a}{h} = \frac{x_i - 75.5}{3}$$

Now, we have the following table :

| Class-interval | Frequency (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 75.5}{3}$ | $f_i u_i$ |
|----------------|---------------------|----------------------|------------------------------|----------------------|
| 65 - 68 | 2 | 66.5 | -3 | -6 |
| 68 - 71 | 4 | 69.5 | -2 | -8 |
| 71 - 74 | 3 | 72.5 | -1 | -3 |
| 74 - 77 | 8 | 75.5 | 0 | 0 |
| 77 - 80 | 7 | 78.5 | 1 | 7 |
| 80 - 83 | 4 | 81.5 | 2 | 8 |
| 83 - 86 | 2 | 84.5 | 3 | 6 |
| Total | $\Sigma f_i = 30$ | | | $\Sigma f_i u_i = 4$ |

$$\therefore \text{Mean, } \bar{x} = a + h \times \left\{ \frac{\Sigma f_i u_i}{\Sigma f_i} \right\} = 75.5 + 3 \times \frac{4}{30} = 75.5 + \frac{4}{10}$$

$$= 75.5 + 0.4 = 75.9$$

Thus, the mean heartbeats per minute is 75.9.

5. Let the assumed mean, $a = 57$

$$\therefore d_i = x_i - 57$$

Now, we have the following table:

| Number of Mangoes | Frequency (f_i) | Class mark (x_i) | $d_i = x_i - 57$ | $f_i d_i$ |
|-------------------|---------------------|----------------------|------------------|-----------------------|
| 50 - 52 | 15 | 51 | -6 | -90 |
| 53 - 55 | 110 | 54 | -3 | -330 |
| 56 - 58 | 135 | 57 | 0 | 0 |
| 59 - 61 | 115 | 60 | 3 | 345 |
| 62 - 64 | 25 | 63 | 6 | 150 |
| Total | $\Sigma f_i = 400$ | | | $\Sigma f_i d_i = 75$ |

$$\therefore \text{Mean, } \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 57 + \frac{75}{400} = 57 + 0.1875$$

$$= 57.1875 \approx 57.19$$

Thus, the average number of mangoes per box = 57.19.

We choose assumed mean method.

6. Let the assumed mean, $a = 225$

And class size, $h = 50$

$$\therefore u_i = \frac{x_i - a}{h} = \frac{x_i - 225}{50}$$

Now, we have the following table:

| Daily expenditure (in ₹) | Frequency (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 225}{50}$ | $f_i u_i$ |
|--------------------------|---------------------|----------------------|------------------------------|-----------------------|
| 100 - 150 | 4 | 125 | -2 | -8 |
| 150 - 200 | 5 | 175 | -1 | -5 |
| 200 - 250 | 12 | 225 | 0 | 0 |
| 250 - 300 | 2 | 275 | 1 | 2 |
| 300 - 350 | 2 | 325 | 2 | 4 |
| Total | $\Sigma f_i = 25$ | | | $\Sigma f_i u_i = -7$ |

$$\therefore \text{Mean, } \bar{x} = a + h \times \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) = 225 + 50 \left(\frac{-7}{25} \right)$$

$$= 225 + 2(-7) = 225 - 14 = 211$$

Thus, the mean daily expenditure on food is ₹ 211.

7. Let the assumed mean, $a = 0.14$

Here, class size, $h = 0.04$

$$\therefore u_i = \frac{x_i - a}{h} = \frac{x_i - 0.14}{0.04}$$

\therefore We have the following table:

| Class-intervals | Frequency (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 0.14}{0.04}$ | $f_i u_i$ |
|-----------------|---------------------|----------------------|---------------------------------|------------------------|
| 0.00-0.04 | 4 | 0.02 | -3 | -12 |
| 0.04-0.08 | 9 | 0.06 | -2 | -18 |
| 0.08-0.12 | 9 | 0.10 | -1 | -9 |
| 0.12-0.16 | 2 | 0.14 | 0 | 0 |
| 0.16-0.20 | 4 | 0.18 | 1 | 4 |
| 0.20-0.24 | 2 | 0.22 | 2 | 4 |
| Total | $\Sigma f_i = 30$ | | | $\Sigma f_i u_i = -31$ |

$$\therefore \text{Mean, } \bar{x} = a + h \times \left[\frac{\Sigma f_i u_i}{\Sigma f_i} \right] = 0.14 + 0.04 \left[\frac{-31}{30} \right]$$

$$= 0.14 - 0.041 = 0.099$$

\therefore Mean concentration of SO_2 in air is 0.099 ppm.

8. Using the direct method, we have the following table:

| Number of days | Frequency (f_i) | Class mark (x_i) | $f_i x_i$ |
|----------------|---------------------|----------------------|------------------------|
| 0 - 6 | 11 | 3 | 33 |
| 6 - 10 | 10 | 8 | 80 |
| 10 - 14 | 7 | 12 | 84 |
| 14 - 20 | 4 | 17 | 68 |
| 20 - 28 | 4 | 24 | 96 |
| 28 - 38 | 3 | 33 | 99 |
| 38 - 40 | 1 | 39 | 39 |
| Total | $\Sigma f_i = 40$ | | $\Sigma f_i x_i = 499$ |

$$\therefore \text{Mean, } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{499}{40} = 12.475$$

Thus, mean number of days a student remained absent = 12.48.

9. Let assumed mean, $a = 70$

Here, class size, $h = 10$

$$\therefore u_i = \frac{x_i - a}{h} = \frac{x_i - 70}{10}$$

Now, we have the following table:

| Literacy rate (in %) | Frequency (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 70}{10}$ | $f_i u_i$ |
|----------------------|---------------------|----------------------|-----------------------------|-----------------------|
| 45 - 55 | 3 | 50 | -2 | -6 |
| 55 - 65 | 10 | 60 | -1 | -10 |
| 65 - 75 | 11 | 70 | 0 | 0 |
| 75 - 85 | 8 | 80 | 1 | 8 |
| 85 - 95 | 3 | 90 | 2 | 6 |
| Total | $\Sigma f_i = 35$ | | | $\Sigma f_i u_i = -2$ |

$$\therefore \text{Mean, } \bar{x} = a + h \times \left[\frac{\Sigma f_i u_i}{\Sigma f_i} \right] = 70 + 10 \left[\frac{-2}{35} \right]$$

$$= 70 + \left[\frac{-4}{7} \right] = \frac{486}{7} = 69.4285 = 69.43 \text{ (approx.)}$$

Thus, the mean literacy rate is 69.43%.

EXERCISE - 14.2

1. Mode : Here, the highest frequency is 23.

The frequency 23 corresponds to the class interval 35-45.

\therefore The modal class is 35-45.

Now, class size, $h = 10$, lower limit, $l = 35$

Frequency of the modal class (f_1) = 23

Frequency of the class preceding the modal class (f_0) = 21

Frequency of the class succeeding the modal class (f_2) = 14

$$\begin{aligned}\therefore \text{Mode} &= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\ &= 35 + \left[\frac{23 - 21}{2 \times 23 - 21 - 14} \right] \times 10 = 35 + \left[\frac{2}{46 - 35} \right] \times 10 \\ &= 35 + \frac{20}{11} = 35 + 1.8 \text{ (Approx.)} = 36.8 \text{ years (Approx.)}\end{aligned}$$

Mean : Let assumed mean, $a = 40$, $h = 10$

| Age (in years) | Frequency (f_i) | Class Mark (x_i) | $u_i = \frac{x_i - 40}{10}$ | $f_i u_i$ |
|----------------|---------------------|----------------------|-----------------------------|------------------------|
| 5 - 15 | 6 | 10 | -3 | -18 |
| 15 - 25 | 11 | 20 | -2 | -22 |
| 25 - 35 | 21 | 30 | -1 | -21 |
| 35 - 45 | 23 | 40 | 0 | 0 |
| 45 - 55 | 14 | 50 | 1 | 14 |
| 55 - 65 | 5 | 60 | 2 | 10 |
| Total | $\Sigma f_i = 80$ | | | $\Sigma f_i u_i = -37$ |

$$\therefore \text{Mean, } \bar{x} = a + h \times \left[\frac{\Sigma f_i u_i}{\Sigma f_i} \right] = 40 + 10 \left[\frac{-37}{80} \right]$$

$$= 40 - \frac{37}{8} = \frac{283}{8} = 35.375$$

\therefore Required mean = 35.37 years.

Interpretation : The maximum number of patients admitted in the hospital are of age 36.8 years while the average age of patients is 35.37 years.

2. Here, the highest frequency = 61

\therefore The frequency 61 corresponds to class 60 - 80.

\therefore The modal class is 60 - 80.

\therefore We have, $l = 60$, $h = 20$, $f_1 = 61$, $f_0 = 52$, $f_2 = 38$.

$$\begin{aligned}\therefore \text{Mode} &= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\ &= 60 + \left[\frac{61 - 52}{2 \times 61 - 52 - 38} \right] \times 20 = 60 + \left[\frac{9}{122 - 90} \right] \times 20 \\ &= 60 + \frac{180}{32} = 60 + \frac{45}{8} = 60 + 5.625 = 65.625 \text{ hours.}\end{aligned}$$

Thus, the required modal lifetimes of the components is 65.625 hours.

3. Mode :

\therefore The maximum number of families is 40 having their total monthly expenditure in the interval 1500-2000.

\therefore Modal class is 1500-2000

So, $l = 1500$, $h = 500$, $f_1 = 40$, $f_0 = 24$, $f_2 = 33$

$$\begin{aligned}\therefore \text{Mode} &= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\ &= 1500 + \left[\frac{40 - 24}{2 \times 40 - 24 - 33} \right] \times 500 = 1500 + \left[\frac{16}{80 - 57} \right] \times 500 \\ &= 1500 + \frac{8000}{23} = 1500 + 347.83 = 1847.83\end{aligned}$$

Thus, the required modal monthly expenditure of the families is ₹ 1847.83.

Mean: Let assumed mean (a) = 3250 and class size, $h = 500$

\therefore We have the following table:

| Expenditure (in ₹) | Number of families (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 3250}{500}$ | $f_i u_i$ |
|--------------------|------------------------------|----------------------|--------------------------------|-------------------------|
| 1000 - 1500 | 24 | 1250 | -4 | -96 |
| 1500 - 2000 | 40 | 1750 | -3 | -120 |
| 2000 - 2500 | 33 | 2250 | -2 | -66 |
| 2500 - 3000 | 28 | 2750 | -1 | -28 |
| 3000 - 3500 | 30 | 3250 | 0 | 0 |
| 3500 - 4000 | 22 | 3750 | 1 | 22 |
| 4000 - 4500 | 16 | 4250 | 2 | 32 |
| 4500 - 5000 | 7 | 4750 | 3 | 21 |
| Total | $\Sigma f_i = 200$ | | | $\Sigma f_i u_i = -235$ |

$$\begin{aligned}\therefore \bar{x} &= a + h \times \left[\frac{\Sigma f_i u_i}{\Sigma f_i} \right] = 3250 + 500 \times \left[\frac{-235}{200} \right] \\ &= 3250 - \frac{1175}{2} = 3250 - 587.50 = 2662.50\end{aligned}$$

Thus, the mean monthly expenditure is ₹ 2662.50.

4. Mode : Since greatest frequency 10 corresponds to class 30-35.

\therefore Modal Class = 30-35 and $h = 5$, $l = 30$, $f_1 = 10$, $f_0 = 9$, $f_2 = 3$

$$\begin{aligned}\therefore \text{Mode} &= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h = 30 + \left[\frac{10 - 9}{2 \times 10 - 9 - 3} \right] \times 5 \\ &= 30 + \frac{1}{8} \times 5 = 30 + 0.625 = 30.6 \text{ (Approx.)}\end{aligned}$$

Mean : Let the assumed mean, $a = 37.5$ and class size, $h = 5$

\therefore We have the following table:

| Number of students per teacher | Frequency (f_i) | Class mark (x_i) | $u_i = \frac{x_i - 37.5}{5}$ | $f_i u_i$ |
|--------------------------------|---------------------|----------------------|------------------------------|------------------------|
| 15 - 20 | 3 | 17.5 | -4 | -12 |
| 20 - 25 | 8 | 22.5 | -3 | -24 |
| 25 - 30 | 9 | 27.5 | -2 | -18 |
| 30 - 35 | 10 | 32.5 | -1 | -10 |
| 35 - 40 | 3 | 37.5 | 0 | 0 |
| 40 - 45 | 0 | 42.5 | 1 | 0 |
| 45 - 50 | 0 | 47.5 | 2 | 0 |
| 50 - 55 | 2 | 52.5 | 3 | 6 |
| Total | $\Sigma f_i = 35$ | | | $\Sigma f_i u_i = -58$ |

$$\begin{aligned}\therefore \text{Mean, } \bar{x} &= a + h \times \left[\frac{\Sigma f_i u_i}{\Sigma f_i} \right] \\ &= 37.5 + 5 \times \left[\frac{-58}{35} \right] = 37.5 - 8.3 = 29.2.\end{aligned}$$

Interpretation : The maximum teacher-student ratio is 30.6 while average teacher-student ratio is 29.2.

5. The class 4000-5000 has the highest frequency i.e., 18

\therefore Modal class = 4000-5000

Also, $h = 1000$, $l = 4000$, $f_1 = 18$, $f_0 = 4$, $f_2 = 9$

$$\begin{aligned}\therefore \text{Mode} &= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\ &= 4000 + \left[\frac{18 - 4}{2 \times 18 - 4 - 9} \right] \times 1000 = 4000 + 1000 \left[\frac{14}{23} \right] \\ &= 4000 + 608.695 = 4608.7 \text{ (Approx.)}\end{aligned}$$

Thus, the required mode is 4608.7.

6. \therefore The class 40-50 has the maximum frequency i.e., 20

\therefore Modal class = 40-50

$\therefore l = 40, f_1 = 20, f_0 = 12, f_2 = 11$ and $h = 10$.

$$\begin{aligned}\therefore \text{Mode} &= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\ &= 40 + \left[\frac{20 - 12}{2 \times 20 - 12 - 11} \right] \times 10 = 40 + 10 \left[\frac{8}{40 - 23} \right] \\ &= 40 + \frac{80}{17} = 40 + 4.7 = 44.7\end{aligned}$$

Thus, the required mode is 44.7

EXERCISE - 14.3

1. We have the following table :

| Monthly Consumption (in units) | Number of Consumers (f_i) | Cumulative Frequency (c.f.) |
|--------------------------------|-------------------------------|-----------------------------|
| 65 - 85 | 4 | 4 |
| 85 - 105 | 5 | 4 + 5 = 9 |
| 105 - 125 | 13 | 9 + 13 = 22 |
| 125 - 145 | 20 | 22 + 20 = 42 |
| 145 - 165 | 14 | 42 + 14 = 56 |
| 165 - 185 | 8 | 56 + 8 = 64 |
| 185 - 205 | 4 | 64 + 4 = 68 |
| Total | $\Sigma f_i = 68$ | |

$$\text{We have, } n = 68 \Rightarrow \frac{n}{2} = \frac{68}{2} = 34$$

Cumulative frequency just greater than 34 is 42 and corresponding class-interval is 125-145.

\therefore 125-145 is the median class.

So, $l = 125, c.f. = 22, f = 20$ and $h = 20$

$$\begin{aligned}\therefore \text{Median} &= l + \left[\frac{\frac{n}{2} - c.f.}{f} \right] \times h = 125 + \left[\frac{34 - 22}{20} \right] \times 20 \\ &= 125 + \frac{12}{20} \times 20 = 125 + 12 = 137 \text{ units.}\end{aligned}$$

Here, $h = 20$

| Class mark (x_i) | f_i | $u_i = \frac{x_i - 135}{h}$ | $f_i u_i$ |
|----------------------|-------|-----------------------------|-----------|
| 75 | 4 | -3 | -12 |
| 95 | 5 | -2 | -10 |
| 115 | 13 | -1 | -13 |
| 135 = a (let) | 20 | 0 | 0 |
| 155 | 14 | 1 | 14 |
| 175 | 8 | 2 | 16 |

| | | | |
|-------|-------------------|---|----------------------|
| 195 | 4 | 3 | 12 |
| Total | $\Sigma f_i = 68$ | | $\Sigma f_i u_i = 7$ |

$$\therefore \text{Mean, } \bar{x} = a + h \times \left\{ \frac{\Sigma f_i u_i}{\Sigma f_i} \right\}$$

$$= 135 + 20 \times \frac{7}{68} = 135 + 2.05 = 137.05 \text{ units.}$$

Now, we find the mode.

\therefore Class 125-145 has the highest frequency.

\therefore This is the modal class.

So, $h = 20, l = 125, f_1 = 20, f_0 = 13, f_2 = 14$

$$\begin{aligned}\therefore \text{Mode} &= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\ &= 125 + \left[\frac{20 - 13}{2 \times 20 - 13 - 14} \right] \times 20 \\ &= 125 + \frac{140}{13} = 125 + 10.76 = 135.76 \text{ units.}\end{aligned}$$

We observe that the three measures are approximately equal.

2. Cumulative frequency table for the given data can be drawn as:

| Class-interval | Frequency (f_i) | Cumulative frequency (c.f.) |
|----------------|---------------------|-----------------------------|
| 0 - 10 | 5 | 5 |
| 10 - 20 | x | 5 + x |
| 20 - 30 | 20 | 25 + x |
| 30 - 40 | 15 | 40 + x |
| 40 - 50 | y | 40 + x + y |
| 50 - 60 | 5 | 45 + x + y |
| Total | $\Sigma f_i = 60$ | |

Since, median = 28.5, which lies in the interval 20-30.

\therefore Median class is 20-30.

So, $l = 20, h = 10, f = 20, c.f. = 5 + x, n = 60$

$$\therefore \text{Median} = l + \left[\frac{\frac{n}{2} - c.f.}{f} \right] \times h$$

$$\Rightarrow 28.5 = 20 + \left[\frac{30 - (5 + x)}{20} \right] \times 10 \Rightarrow 28.5 = 20 + \frac{25 - x}{2}$$

$$\Rightarrow 57 = 40 + 25 - x \Rightarrow x = 40 + 25 - 57 = 8 \quad \dots (i)$$

$$\text{Also, } 45 + x + y = 60 \Rightarrow 45 + 8 + y = 60 \quad (\text{From (i)})$$

$$\Rightarrow y = 60 - 45 - 8 = 7.$$

Thus, $x = 8, y = 7$

3. The given table is cumulative frequency distribution. We write the frequency distribution as given below :

| Class-interval | Cumulative frequency (c.f.) | Frequency (f_i) |
|----------------|-----------------------------|---------------------|
| 18 - 20 | 2 | 2 |
| 20 - 25 | 6 | 6 - 2 = 4 |
| 25 - 30 | 24 | 24 - 6 = 18 |
| 30 - 35 | 45 | 45 - 24 = 21 |
| 35 - 40 | 78 | 78 - 45 = 33 |
| 40 - 45 | 89 | 89 - 78 = 11 |
| 45 - 50 | 92 | 92 - 89 = 3 |
| 50 - 55 | 98 | 98 - 92 = 6 |
| 55 - 60 | 100 | 100 - 98 = 2 |

We have, $n = 100 \Rightarrow \frac{n}{2} = \frac{100}{2} = 50$

\therefore The cumulative frequency just greater than 50 is 78.

\therefore The median class is 35 - 40.

Now, $l = 35$, $c.f. = 45$, $f = 33$ and $h = 5$

$$\begin{aligned}\therefore \text{Median} &= l + \left[\frac{\frac{n}{2} - c.f.}{f} \right] \times h \\ &= 35 + \left[\frac{50 - 45}{33} \right] \times 5 \\ &= 35 + \frac{5}{33} \times 5 = 35 + \frac{25}{33} = 35 + 0.76 = 35.76\end{aligned}$$

Thus, the median age = 35.76 years.

4. After changing the given table as continuous classes we prepare the cumulative frequency table as follows:

| Length (in mm) | Number of leaves (f_i) | Cumulative frequency (c.f.) |
|----------------|----------------------------|-----------------------------|
| 117.5 - 126.5 | 3 | 3 |
| 126.5 - 135.5 | 5 | 3 + 5 = 8 |
| 135.5 - 144.5 | 9 | 8 + 9 = 17 |
| 144.5 - 153.5 | 12 | 17 + 12 = 29 |
| 153.5 - 162.5 | 5 | 29 + 5 = 34 |
| 162.5 - 171.5 | 4 | 34 + 4 = 38 |
| 171.5 - 180.5 | 2 | 38 + 2 = 40 |
| Total | $\Sigma f_i = 40$ | |

Here, $n = 40 \Rightarrow \frac{n}{2} = \frac{40}{2} = 20$

The cumulative frequency just greater than 20 is 29 and it corresponds to the class 144.5-153.5.

So, 144.5-153.5 is the median class.

We have, $l = 144.5$, $f = 12$, $c.f. = 17$ and $h = 9$

$$\begin{aligned}\therefore \text{Median} &= l + \left[\frac{\frac{n}{2} - c.f.}{f} \right] \times h = 144.5 + \left[\frac{20 - 17}{12} \right] \times 9 \\ &= 144.5 + \frac{3}{12} \times 9 = 144.5 + \frac{9}{4} = 144.5 + 2.25 = 146.75. \\ \therefore \text{Median length of leaves} &= 146.75 \text{ mm.}\end{aligned}$$

5. To compute the median, let us write the cumulative frequency distribution as given :

| Life time (in hours) | Number of lamps (f_i) | Cumulative frequency (c.f.) |
|----------------------|---------------------------|-----------------------------|
| 1500 - 2000 | 14 | 14 |
| 2000 - 2500 | 56 | 14 + 56 = 70 |
| 2500 - 3000 | 60 | 70 + 60 = 130 |
| 3000 - 3500 | 86 | 130 + 86 = 216 |
| 3500 - 4000 | 74 | 216 + 74 = 290 |
| 4000 - 4500 | 62 | 290 + 62 = 352 |
| 4500 - 5000 | 48 | 352 + 48 = 400 |
| Total | $\Sigma f_i = 400$ | |

Here, $n = 400 \Rightarrow \frac{n}{2} = \frac{400}{2} = 200$

Since, the cumulative frequency just greater than 200 is 216 and corresponding interval is 3000 - 3500.

\therefore The median class is 3000-3500 and so, $l = 3000$, $c.f. = 130$, $f = 86$, $h = 500$

$$\begin{aligned}\text{Now, median} &= l + \left[\frac{\frac{n}{2} - c.f.}{f} \right] \times h = 3000 + \left[\frac{200 - 130}{86} \right] \times 500 \\ &= 3000 + \frac{70}{86} \times 500 = 3000 + \frac{35000}{86} \\ &= 3000 + 406.98 = 3406.98\end{aligned}$$

Thus, median life = 3406.98 hours.

6. Median : The cumulative frequency distribution table is as follows:

| Number of letters | Frequency (f_i) | Cumulative Frequency (c.f.) |
|-------------------|---------------------|-----------------------------|
| 1 - 4 | 6 | 6 |
| 4 - 7 | 30 | 6 + 30 = 36 |
| 7 - 10 | 40 | 36 + 40 = 76 |
| 10 - 13 | 16 | 76 + 16 = 92 |
| 13 - 16 | 4 | 92 + 4 = 96 |
| 16 - 19 | 4 | 96 + 4 = 100 |
| Total | $\Sigma f_i = 100$ | |

Here, $n = 100 \Rightarrow \frac{n}{2} = \frac{100}{2} = 50$.

Since, the cumulative frequency just greater than 50 is 76 and corresponding interval is 7-10.

\therefore The class 7-10 is the median class.

We have, $l = 7$, $c.f. = 36$, $f = 40$ and $h = 3$

$$\begin{aligned}\therefore \text{Median} &= l + \left[\frac{\frac{n}{2} - c.f.}{f} \right] \times h = 7 + \left[\frac{50 - 36}{40} \right] \times 3 \\ &= 7 + \frac{14}{40} \times 3 = 7 + \frac{42}{40} = 7 + 1.05 = 8.05\end{aligned}$$

Mean : We have, the following table :

| Class - intervals | Frequency (f_i) | Class mark (x_i) | $f_i x_i$ |
|-------------------|---------------------|----------------------|------------------------|
| 1 - 4 | 6 | 2.5 | 15 |
| 4 - 7 | 30 | 5.5 | 165 |
| 7 - 10 | 40 | 8.5 | 340 |
| 10 - 13 | 16 | 11.5 | 184 |
| 13 - 16 | 4 | 14.5 | 58 |
| 16 - 19 | 4 | 17.5 | 70 |
| Total | $\Sigma f_i = 100$ | | $\Sigma f_i x_i = 832$ |

$$\therefore \text{Mean, } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{832}{100} = 8.32$$

Mode : Since the class 7-10 has the maximum frequency.

\therefore The modal class is 7-10.

So, we have $l = 7$, $h = 3$, $f_1 = 40$, $f_0 = 30$, $f_2 = 16$

$$\begin{aligned}\therefore \text{Mode} &= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h = 7 + \left[\frac{40 - 30}{2 \times 40 - 30 - 16} \right] \times 3 \\ &= 7 + \left(\frac{10}{34} \right) \times 3 = 7 + \frac{30}{34} = 7 + 0.88 = 7.88\end{aligned}$$

7. We have cumulative frequency table as follows:

| Weight (in kg) | Frequency (f_i) | Cumulative Frequency (c.f.) |
|----------------|---------------------|-----------------------------|
| 40 - 45 | 2 | 2 |
| 45 - 50 | 3 | 2 + 3 = 5 |
| 50 - 55 | 8 | 5 + 8 = 13 |
| 55 - 60 | 6 | 13 + 6 = 19 |
| 60 - 65 | 6 | 19 + 6 = 25 |
| 65 - 70 | 3 | 25 + 3 = 28 |
| 70 - 75 | 2 | 28 + 2 = 30 |
| Total | $\Sigma f_i = 30$ | |

Here, $n = 30 \Rightarrow \frac{n}{2} = \frac{30}{2} = 15$

The cumulative frequency just greater than 15 is 19, which corresponds to the class 55-60. So, median class is 55-60 and we have $l = 55$, $f = 6$, $c.f. = 13$ and $h = 5$

$$\begin{aligned} \therefore \text{Median} &= l + \left[\frac{\frac{n}{2} - c.f.}{f} \right] \times h \\ &= 55 + \left[\frac{15 - 13}{6} \right] \times 5 = 55 + \frac{2}{6} \times 5 \\ &= 55 + \frac{10}{6} = 55 + 1.67 = 56.67 \end{aligned}$$

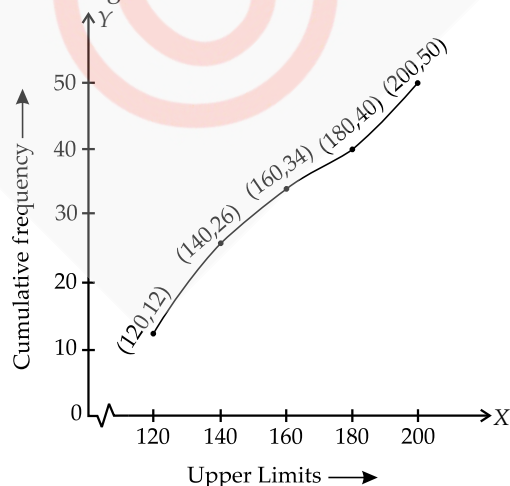
Thus, the required median weight of the students = 56.67 kg.

EXERCISE - 14.4

1. We have the less than type cumulative frequency distribution as follows:

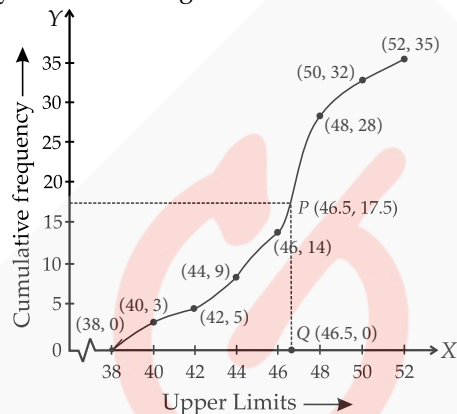
| Daily income (in ₹) | Cumulative frequency |
|---------------------|----------------------|
| Less than 120 | 12 |
| Less than 140 | 12 + 14 = 26 |
| Less than 160 | 26 + 8 = 34 |
| Less than 180 | 34 + 6 = 40 |
| Less than 200 | 40 + 10 = 50 |

Now, we plot the points (120, 12), (140, 26), (160, 34), (180, 40) and (200, 50) on a graph paper and join them by a free hand to get a smooth curve as shown below :



The curve so obtained is called the less than ogive.

2. Here, the values 38, 40, 42, 44, 46, 48, 50 and 52 are the upper limits of the respective class-intervals. We plot the points (38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35) on a graph paper and join them by a free hand to get a smooth curve.



The curve so obtained is the less than type ogive.

Now, $n = 35$

$$\Rightarrow \frac{n}{2} = \frac{35}{2} = 17.5$$

Locate the point 17.5 on y -axis.

From this point (i.e., from 17.5) we draw a line parallel to the x -axis which cuts the curve at P . From this point P , draw a perpendicular to the x -axis, meeting the x -axis at Q . The point Q represents the median of the data which is 46.5.

Verification : To verify the result using the formula, let us make the following table in order to find median using the formula :

| Weight (in kg) | Frequency | Number of students (Cumulative Frequency) |
|----------------|--------------|---|
| Below 38 | 0 | 0 |
| 38 - 40 | 3 - 0 = 3 | 3 |
| 40 - 42 | 5 - 3 = 2 | 5 |
| 42 - 44 | 9 - 5 = 4 | 9 |
| 44 - 46 | 14 - 9 = 5 | 14 |
| 46 - 48 | 28 - 14 = 14 | 28 |
| 48 - 50 | 32 - 28 = 4 | 32 |
| 50 - 52 | 35 - 32 = 3 | 35 |

Here, $n = 35 \Rightarrow \frac{n}{2} = \frac{35}{2} = 17.5$

The cumulative frequency just greater than 17.5 is 28 and corresponding interval is 46-48.

\therefore The median class is 46-48.

So, $l = 46$, $h = 2$, $f = 14$, $c.f. = 14$

$$\therefore \text{Median} = l + \left[\frac{\frac{n}{2} - c.f.}{f} \right] \times h = 46 + \left[\frac{17.5 - 14}{14} \right] \times 2$$

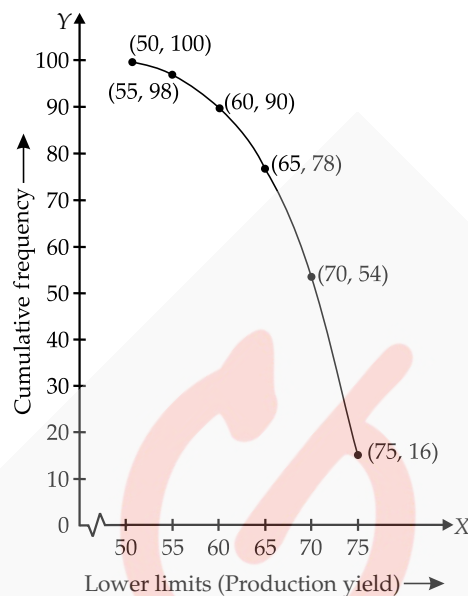
$$= 46 + \frac{3.5}{14} \times 2 = 46 + \frac{1}{2} = 46.5 \text{ kg}$$

Thus, the median = 46.5 kg is verified.

3. For more than type distribution, we have:

| Production yield (in kg/ha) | Number of Farms (Cumulative Frequency) |
|--------------------------------|---|
| More than or equal to 50 | 100 |
| More than or equal to 55 | $100 - 2 = 98$ |
| More than or equal to 60 | $98 - 8 = 90$ |
| More than or equal to 65 | $90 - 12 = 78$ |
| More than or equal to 70 | $78 - 24 = 54$ |
| More than or equal to 75 | $54 - 38 = 16$ |

Now, we plot the points (50, 100), (55, 98), (60, 90), (65, 78), (70, 54) and (75, 16) and join the points with a free hand to get a smooth curve.



The curve so obtained is the 'more than type ogive'.

