EXAM DRILL

Introduction to Trigonometry

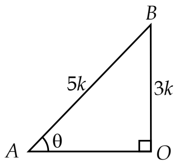
CHAPTER 8

SOLUTIONS

1. **(a) :** Given, $\sin\theta = 3/5$.

Let ΔAOB be the right angled triangle such that $\angle OAB = \theta$.

Assume that $OB = 3k$ units and $AB = 5k$ units.



Then, $OA = \sqrt{AB^2 - OB^2} = \sqrt{25k^2 - 9k^2} = 4k$ units

So, $\tan\theta = \dfrac{OB}{OA} = \dfrac{3}{4}$

2. **(b) :** We have, $\dfrac{\sin A}{\tan A} + \dfrac{\cos A \cot A}{\operatorname{cosec} A}$

$= \dfrac{\sin A}{\frac{\sin A}{\cos A}} + \cos A \times \dfrac{\cos A}{\sin A} \times \sin A$

$\left[\because \tan A = \dfrac{\sin A}{\cos A}, \cot A = \dfrac{\cos A}{\sin A}, \dfrac{1}{\operatorname{cosec} A} = \sin A\right]$

$= \cos A + \cos^2 A$

3. **(b) :** We have, $\sin^2 60° - \sin^2 30°$

$= \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{1}{2}\right)^2 = \dfrac{3}{4} - \dfrac{1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

4. **(a) :** We have, $\sin\theta = \cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \tan\theta = \tan 45°$ $[\because \tan 45° = 1]$

$\Rightarrow \theta = 45°$

5. **(c) :** Given, $\sec(2x + 17)° = \sqrt{2}$

$\Rightarrow \sec(2x + 17)° = \sec 45°$ $[\because \sec 45° = \sqrt{2}\,]$

$\Rightarrow 2x + 17 = 45 \Rightarrow 2x = 45 - 17$

$\Rightarrow 2x = 28 \Rightarrow x = 14$

6. **(b) :** We have, $\cos 1° \cdot \cos 2° \cdot \cos 3° \cdot \cos 4° \cdot \ldots \cos 100°$

$= \cos 1° \cdot \cos 2° \cdot \cos 3° \ldots \cos 90° \ldots \cos 100°$

$= 0$ $[\because \cos 90° = 0]$

7. **(b) :** $\sin(45° + \theta) - \cos(45° - \theta)$

$= \cos[90° - (45° + \theta)] - \cos(45° - \theta)$ $[\because \cos(90° - \theta) = \sin\theta]$

$= \cos(45° - \theta) - \cos(45° - \theta) = 0$

8. We have, $\sec 5A = \operatorname{cosec}(A + 30°)$

$\Rightarrow \sec 5A = \sec[90° - (A + 30°)]$

$\Rightarrow \sec 5A = \sec(60° - A)$

$\Rightarrow 5A = 60° - A \Rightarrow 6A = 60°$

$\Rightarrow A = 10°$

9. Given, $\sin\theta = \dfrac{a}{b}$

$\therefore \cos\theta = \sqrt{1 - \sin^2\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \sqrt{1 - \left(\dfrac{a}{b}\right)^2} = \sqrt{1 - \dfrac{a^2}{b^2}} = \dfrac{\sqrt{b^2 - a^2}}{b}$

10. $\dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta}$

$= \dfrac{1 - \sin\theta + 1 + \sin\theta}{(1 + \sin\theta)(1 - \sin\theta)} = \dfrac{2}{1 - \sin^2\theta} = \dfrac{2}{\cos^2\theta} = 2\sec^2\theta$

11. In a ΔABC, we have $A + B + C = 180°$

$\Rightarrow B + C = 180° - A \Rightarrow \dfrac{B + C}{2} = 90° - \dfrac{A}{2}$

$\Rightarrow \tan\left(\dfrac{B + C}{2}\right) = \tan\left(90° - \dfrac{A}{2}\right)$

$\Rightarrow \tan\left(\dfrac{B + C}{2}\right) = \cot\dfrac{A}{2}$

12. We have, $\tan 3x = \sin 45° \cos 45° + \sin 30°$

$\Rightarrow \tan 3x = \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} + \dfrac{1}{2} = \dfrac{1}{2} + \dfrac{1}{2}$

$\Rightarrow \tan 3x = 1 \Rightarrow \tan 3x = \tan 45°$

$\Rightarrow 3x = 45° \Rightarrow x = 15°$

13. L.H.S. $= \sqrt{\cos\alpha \operatorname{cosec}\beta - \cos\alpha \sin\beta}$

$= \sqrt{\cos\alpha \operatorname{cosec}(90° - \alpha) - \cos\alpha \sin(90° - \alpha)}$

[Given $\alpha + \beta = 90°$]

$= \sqrt{\cos\alpha \sec\alpha - \cos\alpha \cos\alpha} = \sqrt{1 - \cos^2\alpha} = \sin\alpha$ = R.H.S.

$[\because \sin^2\theta + \cos^2\theta = 1]$

14. We have, $\dfrac{1}{x}\left[\dfrac{\sin^2 5° + \sin^2 85°}{\cos^2 5° + \cos^2 85°}\right] - \dfrac{3}{4} = 1$

$\Rightarrow \dfrac{1}{x}\left[\dfrac{\sin^2 5° + \sin^2(90° - 5°)}{\cos^2(90° - 85°) + \cos^2 85°}\right] - \dfrac{3}{4} = 1$

$\Rightarrow \dfrac{1}{x}\left[\dfrac{\sin^2 5° + \cos^2 5°}{\sin^2 85° + \cos^2 85°}\right] - \dfrac{3}{4} = 1$

$[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta]$

$\Rightarrow \dfrac{1}{x} \times 1 - \dfrac{3}{4} = 1 \Rightarrow \dfrac{1}{x} = 1 + \dfrac{3}{4} \Rightarrow \dfrac{1}{x} = \dfrac{7}{4} \Rightarrow x = \dfrac{4}{7}$

15. L.H.S. $= \tan^4\theta + \tan^2\theta = \tan^2\theta(\tan^2\theta + 1)$

$= \tan^2\theta \cdot \sec^2\theta$ $[\because \tan^2\theta + 1 = \sec^2\theta]$

$= (\sec^2\theta - 1) \cdot \sec^2\theta$ $[\because \tan^2\theta = \sec^2\theta - 1]$

$= \sec^4\theta - \sec^2\theta$ = R.H.S.

16. Given, $x = b\sec^3\theta$ and $y = a\tan^3\theta$

$\Rightarrow \sec^3\theta = \dfrac{x}{b}$ and $\tan^3\theta = \dfrac{y}{a}$ ...(i)

Now consider, $\left(\frac{x}{b}\right)^{2/3}-\left(\frac{y}{a}\right)^{2/3}$

$= (\sec^3\theta)^{2/3} - (\tan^3\theta)^{2/3}$ (Using (i))

$= \sec^2\theta - \tan^2\theta = 1$

17. We know that $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow (\sin^2\theta + \cos^2\theta)^3 = 1^3$ [Cubing both the sides]

$\Rightarrow (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta) = 1$

$\Rightarrow \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

18. L.H.S. $= (\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta$

$= [(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) + 1]\operatorname{cosec}^2\theta$

$= (\sin^2\theta - \cos^2\theta + 1)\operatorname{cosec}^2\theta$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= 2\sin^2\theta\operatorname{cosec}^2\theta$ $[\because 1 - \cos^2\theta = \sin^2\theta]$

$= 2 =$ R.H.S.

19. We have, $2(\operatorname{cosec}^2\theta - 1)\tan^2\theta$

$= 2(\cot^2\theta)\tan^2\theta$ $[\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta]$

$= 2$ $[\because \tan\theta\cdot\cot\theta = 1]$

20. L.H.S. $= \frac{\tan^2 A(1+\cot^2 A)}{(1+\tan^2 A)} = \frac{\tan^2 A(\operatorname{cosec}^2 A)}{\sec^2 A}$

$[\because 1 + \cot^2 A = \operatorname{cosec}^2 A ; 1 + \tan^2 A = \sec^2 A]$

$= \frac{\sin^2 A}{\cos^2 A}\cdot\frac{\cos^2 A}{\sin^2 A} = 1 =$ R.H.S.

21. Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \tan 45° = 1\right]$

$\Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45°$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$ $\left[\because \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}\right]$

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

22. Given that, $A = 60°$ and $B = 30°$

$\therefore \cos A = \cos 60° = \frac{1}{2}$; $\cos B = \cos 30° = \frac{\sqrt{3}}{2}$

and $\cos(A + B) = \cos(60° + 30°) = \cos 90° = 0$

Now, $\cos A + \cos B = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2} \neq 0$

$\therefore \cos(A + B) \neq \cos A + \cos B$.

23. L.H.S. $= \operatorname{cosec}^2 60° \sec^2 30° \cos^2 0° \sin 45° \cot^2 60° \tan^2 60°$

$= \left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{2}{\sqrt{3}}\right)^2(1)^2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right)^2(\sqrt{3})^2 = \frac{4}{3}\times\frac{4}{3}\times\frac{1}{\sqrt{2}}\times\frac{1}{3}\times 3$

$= \frac{16}{9}\times\frac{1}{\sqrt{2}} = \frac{8\sqrt{2}}{9} =$ R.H.S.

24. We have, $\sin(A + B) = 1$

$\Rightarrow \sin(A + B) = \sin 90° \Rightarrow A + B = 90°$ ...(i)

Also, $\sin(A - B) = \frac{1}{2}$

$\Rightarrow \sin(A - B) = \sin 30° \Rightarrow A - B = 30°$ ...(ii)

Adding (i) and (ii), we get

$A + B + A - B = 120° \Rightarrow 2A = 120° \Rightarrow A = 60°$

From (i), we have $60° + B = 90° \Rightarrow B = 30°$

25. We have, $\cos 15\theta = \sin 3\theta$

$\Rightarrow \cos 15\theta = \cos(90° - 3\theta) \Rightarrow 15\theta = 90° - 3\theta$

$\Rightarrow 18\theta = 90° \Rightarrow \theta = \frac{90°}{18} = 5°$

$\therefore \cot 9\theta + \tan 9\theta = \cot 45° + \tan 45° = 1 + 1 = 2$

26. Given, $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 2\sin^2\theta - (1 - \sin^2\theta) = 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow 2\sin^2\theta + \sin^2\theta - 1 = 2$

$\Rightarrow 3\sin^2\theta = 3 \Rightarrow \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 = \sin 90°$ $[\because \sin 90° = 1]$

$\therefore \theta = 90°$

27. In ΔOPQ, we have $OQ^2 = OP^2 + PQ^2$

$\Rightarrow (1 + PQ)^2 = OP^2 + PQ^2$ $[\because OQ - PQ = 1]$

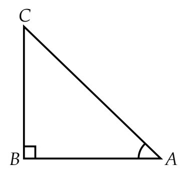
$\Rightarrow 1 + PQ^2 + 2PQ = OP^2 + PQ^2$

$\Rightarrow 1 + 2PQ = 7^2 \Rightarrow 2PQ = 48$

$\Rightarrow PQ = 24$ cm and $OQ = 1 + PQ = 25$ cm

So, $\sin Q = \frac{OP}{OQ} = \frac{7}{25}$ and $\cos Q = \frac{PQ}{OQ} = \frac{24}{25}$

28. In ΔABC, $\tan A = \frac{BC}{AB} = 1$



$\Rightarrow BC = AB$

Let $AB = BC = k$ units

$\therefore$ By Pythagoras theorem, we have

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{(k)^2 + (k)^2} = k\sqrt{2}$ units

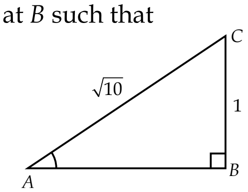
$\therefore \sin A = \frac{BC}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$ and $\cos A = \frac{AB}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$

So, $2\sin A\cos A = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 1$

Hence verified.

29. We have, $\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$

So, we draw a ΔABC, right-angled at B such that $BC = 1$ unit and $AC = \sqrt{10}$ units.



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow (\sqrt{10})^2 = AB^2 + 1^2$

$\Rightarrow AB^2 = 10 - 1 = 9$

$\Rightarrow AB = \sqrt{9} = 3$ units

$\therefore \sin A = \frac{BC}{AC} = \frac{1}{\sqrt{10}}$, $\cos A = \frac{AB}{AC} = \frac{3}{\sqrt{10}}$,

$\tan A = \frac{BC}{AB} = \frac{1}{3}$, $\sec A = \frac{1}{\cos A} = \frac{\sqrt{10}}{3}$

and $\cot A = \frac{1}{\tan A} = \frac{3}{1} = 3$

30. Since, $\tan R = \frac{1}{\sqrt{3}} = \tan 30° \Rightarrow R = 30°$

Again, $\cos P = \frac{1}{2} = \cos 60° \Rightarrow P = 60°$

(i) Now, $\cos P \cos R + \sin P \sin R$

$= \cos 60° \cos 30° + \sin 60° \sin 30°$

$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

(ii) $\cos(P - R) = \cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

From (i) and (ii), we get

$\cos(P - R) = \cos P \cos R + \sin P \sin R$

31. We have, $\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$

$= \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2(90° - 68°) + \cos^2 68°} + \sin^2 63° + \cos 63° \sin(90° - 63°)$

$= \frac{\sin^2 22° + \cos^2 22°}{\sin^2 68° + \cos^2 68°} + \sin^2 63° + \cos 63° \cos 63°$

[$\because \sin(90° - \theta) = \cos\theta$ and $\cos(90° - \theta) = \sin\theta$]

$= \frac{1}{1} + (\sin^2 63° + \cos^2 63°)$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= 1 + 1 = 2$

32. Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Squaring both the sides, we get

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow 1 = 2\cos^2\theta - 2\cos\theta\sin\theta$

$\Rightarrow 1 - 2\cos^2\theta = -2\cos\theta\sin\theta$

$\Rightarrow 1 + 1 - 2\cos^2\theta = 1 - 2\cos\theta\sin\theta$

$\Rightarrow 2 - 2\cos^2\theta = \cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta$

$\Rightarrow 2(1 - \cos^2\theta) = (\cos\theta - \sin\theta)^2$

$\Rightarrow 2\sin^2\theta = (\cos\theta - \sin\theta)^2$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2\sin^2\theta} = \sqrt{2}\sin\theta$

33. L.H.S. $= \frac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta) \cdot \tan(30° - \theta)}$

$= \frac{\cos^2(45° + \theta) + [\sin\{90° - (45° - \theta)\}]^2}{\tan(60° + \theta) \cdot \cot\{90° - (30° - \theta)\}}$

[$\because \sin(90° - \theta) = \cos\theta$ and $\cot(90° - \theta) = \tan\theta$]

$= \frac{\cos^2(45° + \theta) + \sin^2(45° + \theta)}{\tan(60° + \theta) \cdot \cot(60° + \theta)} = \frac{1}{\tan(60° + \theta) \cdot \frac{1}{\tan(60° + \theta)}}$

[$\because \sin^2\theta + \cos^2\theta = 1, \cot\theta = 1/\tan\theta$]

$= 1 =$ R.H.S.

34. L.H.S. $= \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$

$\left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$

$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cdot \cos^2\theta}} = \sqrt{\frac{1}{\sin^2\theta \cdot \cos^2\theta}}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= \frac{1}{\sin\theta \cdot \cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta}$ [$\because 1 = \sin^2\theta + \cos^2\theta$]

$= \frac{\sin^2\theta}{\sin\theta \cdot \cos\theta} + \frac{\cos^2\theta}{\sin\theta \cdot \cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

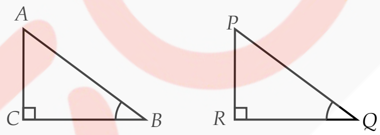
$= \tan\theta + \cot\theta$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

= R.H.S.

35. Let us consider two triangles ABC and PQR, right angled at C and R respectively such that $\sin B = \sin Q$.

We have, $\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$

Then $\frac{AC}{AB} = \frac{PR}{PQ} \Rightarrow \frac{AC}{PR} = \frac{AB}{PQ} = k$ (say) ...(i)



Now, by using Pythagoras theorem, we have

$BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

So, $\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}}$

$= \frac{\sqrt{k^2 PQ^2 - k^2 PR^2}}{\sqrt{PQ^2 - PR^2}} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$ ...(ii)

From (i) and (ii), we have, $\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

So, $\Delta ACB \sim \Delta PRQ$ and therefore, $\angle B = \angle Q$.

36. Given, $\sin(A + C - B) = \frac{\sqrt{3}}{2}$ and $\cot(B + C - A) = \sqrt{3}$

$\Rightarrow \sin(A + C - B) = \sin 60°$ and $\cot(B + C - A) = \cot 30°$

$\Rightarrow A + C - B = 60°$ ...(i)

and $B + C - A = 30°$ ...(ii)

Adding (i) and (ii), we have $2C = 90° \Rightarrow C = 45°$

Putting this value of C in (i), we get $A - B = 15°$ ...(iii)

Also, by angle sum property of a triangle

$A + B + C = 180°$

$\Rightarrow A + B = 135°$ ...(iv)

Adding (iii) and (iv), we have

$2A = 150° \Rightarrow A = 75°$

From (iii), we have $B = 75° - 15° = 60°$

37. We are given that, $\operatorname{cosec}\theta = \frac{17}{12} \Rightarrow \sin\theta = \frac{12}{17}$

$\therefore \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{12}{17}\right)^2} = \frac{\sqrt{145}}{17}$

So, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12}{17} \times \frac{17}{\sqrt{145}} = \frac{12}{\sqrt{145}}$ ...(i)

Now, $\sqrt{\dfrac{(\sec\theta-1)(\sec\theta+1)}{(\text{cosec}\,\theta-1)(\text{cosec}\,\theta+1)}}=\sqrt{\dfrac{(\sec^2\theta-1)}{(\text{cosec}^2\theta-1)}}$

$=\sqrt{\dfrac{\tan^2\theta}{\cot^2\theta}}=\sqrt{\left(\dfrac{\tan\theta}{\cot\theta}\right)^2}=\dfrac{\tan\theta}{\cot\theta}=\tan\theta\times\tan\theta=\tan^2\theta$

$=\left(\dfrac{12}{\sqrt{145}}\right)^2$ [Using (i)]

$=\dfrac{144}{145}$

38. We have,

$\dfrac{\text{cosec}^2(90°-\theta)-\tan^2\theta}{2(\cos^2 37°+\cos^2 53°)}-\dfrac{2\tan^2 30°\sec^2 37°\sin^2 53°}{\text{cosec}^2 63°-\tan^2 27°}$

$=\dfrac{\sec^2\theta-\tan^2\theta}{2(\cos^2(90°-53°)+\cos^2 53°)}-\dfrac{2\left(\dfrac{1}{\sqrt3}\right)^2\left[\dfrac{1}{\cos^2(90°-53°)}\times\sin^2 53°\right]}{\text{cosec}^2(90°-27°)-\tan^2 27°}$

$\left[\because \text{cosec}(90°-\theta)=\sec\theta \text{ and } \tan 30°=\dfrac{1}{\sqrt3}\right]$

$=\dfrac{1}{2(\sin^2 53°+\cos^2 53°)}-\dfrac{2\times\dfrac{1}{3}\left[\dfrac{1}{\sin^2 53°}\times\sin^2 53°\right]}{\sec^2 27°-\tan^2 27°}$

$=\dfrac{1}{2}-\dfrac{2}{3}=\dfrac{3-4}{6}=\dfrac{-1}{6}$

39. We have, $a\sin\theta=b\cos\theta$ ...(i)

Also, $a\sin^3\theta+b\cos^3\theta=\sin\theta\cos\theta$

$\Rightarrow (a\sin\theta)\sin^2\theta+b\cos^3\theta=\sin\theta\cos\theta$

$\Rightarrow b\cos\theta\,.\,\sin^2\theta+b\cos^3\theta=\sin\theta\cos\theta$ [Using (i)]

$\Rightarrow b\cos\theta[\sin^2\theta+\cos^2\theta]=\sin\theta\cos\theta$

$\Rightarrow b\cos\theta\times1=\sin\theta\cos\theta$

$\Rightarrow b=\sin\theta$ ...(ii)

From (i) and (ii), we have

$a\cdot b=b\cos\theta\Rightarrow a=\cos\theta$ ...(iii)

Squaring and adding (ii) and (iii), we get

$a^2+b^2=\sin^2\theta+\cos^2\theta=1$

$\Rightarrow a^2+b^2=1$

Hence proved.

40. Given, $m=\cos A-\sin A$, $n=\cos A+\sin A$

Now, $\dfrac{m}{n}-\dfrac{n}{m}=\dfrac{m^2-n^2}{mn}$

$=\dfrac{(\cos A-\sin A)^2-(\cos A+\sin A)^2}{(\cos A-\sin A)(\cos A+\sin A)}$

$=\dfrac{\cos^2 A+\sin^2 A-2\cos A\sin A-\cos^2 A-\sin^2 A-2\sin A\cos A}{\cos^2 A-\sin^2 A}$

$=\dfrac{-4\sin A\cos A}{\cos^2 A-\sin^2 A}$ ...(i)

Dividing numerator and denominator by $\sin A\cos A$, we get

$\dfrac{-4}{\dfrac{\cos^2 A}{\sin A\cos A}-\dfrac{\sin^2 A}{\sin A\cos A}}=\dfrac{-4}{\cot A-\tan A}$ ...(ii)

From (i) and (ii), we get

$\dfrac{m}{n}-\dfrac{n}{m}=\dfrac{-4\sin A\cos A}{\cos^2 A-\sin^2 A}=\dfrac{-4}{\cot A-\tan A}$

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