

Coordinate Geometry

EXAM DRILL

SOLUTIONS

1. (b) : Distance between $A(0, 6)$ and $B(0, -2)$,
 $AB = \sqrt{(0-0)^2 + (-2-6)^2} = \sqrt{0 + (-8)^2} = \sqrt{8^2} = 8$ units

2. (a) : Let $A(4, p)$ and $B(1, 0)$ be the given points.
 Given, $AB = 5$ units

$$\Rightarrow \left(\sqrt{(1-4)^2 + (0-p)^2} \right)^2 = (5)^2$$

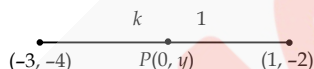
$$\Rightarrow 9 + p^2 = 25 \Rightarrow p^2 = 16 \Rightarrow p = \pm 4$$

3. (a) : Let $A(4, 7)$ be the given point. Suppose the perpendicular from A meet the y -axis at B .
 So, the coordinates of B is $(0, 7)$.

$$\therefore AB = \sqrt{(4-0)^2 + (7-7)^2} = \sqrt{16+0} = 4 \text{ units}$$

4. (d) : Let a point on x -axis be $(x_1, 0)$, then its distance from the point $(2, 3) = \sqrt{(x_1-2)^2 + 9} = c$
 $\Rightarrow (x_1-2)^2 = c^2 - 9 \Rightarrow x_1 - 2 = \sqrt{c^2 - 9}$
 But $c < 3 \Rightarrow c^2 - 9 < 0 \therefore$ No real point exist.

5. (c) : Let the point $P(0, y)$ on the y -axis divides the line segment joining the points $(-3, -4)$ and $(1, -2)$ in the ratio $k : 1$.



Using section formula, we have $(0, y) = \left(\frac{k-3}{k+1}, \frac{-2k-4}{k+1} \right)$

$$\Rightarrow \frac{k-3}{k+1} = 0 \Rightarrow k-3=0 \Rightarrow k=3$$

Hence, the required ratio is $3 : 1$.

6. (c) : Let $A(3, -5)$, $B(-7, 4)$ and $C(10, -k)$ be the given points.

Given, centroid of $\triangle ABC = (k, -1)$

$$\Rightarrow \left(\frac{3-7+10}{3}, \frac{-5+4-k}{3} \right) = (k, -1)$$

$$\Rightarrow \left(\frac{6}{3}, \frac{-1-k}{3} \right) = (k, -1) \Rightarrow \frac{6}{3} = k \text{ and } \frac{-1-k}{3} = -1$$

$$\Rightarrow k=2 \text{ and } k=3-1=2$$

7. Let $A(0, 0)$, $B(2, 0)$, $C(0, 3)$ and $D(x, y)$ be the vertices of rectangle $ABCD$.

Since, diagonals of rectangle bisect each other.

$\therefore$ Mid-point of $AC =$ Mid-point of BD

$$\Rightarrow \left(\frac{0+0}{2}, \frac{0+3}{2} \right) = \left(\frac{2+x}{2}, \frac{0+y}{2} \right)$$

$$\Rightarrow \left(0, \frac{3}{2} \right) = \left(\frac{2+x}{2}, \frac{y}{2} \right)$$

$$\Rightarrow \frac{2+x}{2} = 0 \text{ and } \frac{y}{2} = \frac{3}{2}$$

$$\Rightarrow x = -2 \text{ and } y = 3$$

8. Coordinate of the centroid G of $\triangle ABC$

$$= \left(\frac{-1+0-5}{3}, \frac{3+4+2}{3} \right) = \left(\frac{-6}{3}, \frac{9}{3} \right) = (-2, 3)$$

Since, G lies on the median, $x - 2y + k = 0$

$$\therefore -2 - 2(3) + k = 0 \Rightarrow -2 - 6 + k = 0 \Rightarrow k = 8$$

9. Given, $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear.

$\therefore$ Area of $\triangle ABC = 0$

$$\Rightarrow \frac{1}{2} [x(-4+5) + (-3)(-5-2) + 7(2+4)] = 0$$

$$\Rightarrow \frac{1}{2} [x + (-3)(-7) + 7(6)] = 0 \Rightarrow \frac{1}{2} [x + 21 + 42] = 0$$

$$\Rightarrow x + 63 = 0 \Rightarrow x = -63$$

10. Required distance

$$= \sqrt{[c+a-(b+c)]^2 + [a+b-(c+a)]^2}$$

$$= \sqrt{[c+a-b-c]^2 + [a+b-c-a]^2}$$

$$= \sqrt{(a-b)^2 + (b-c)^2} = \sqrt{a^2 + 2b^2 + c^2 - 2ab - 2bc} \text{ units}$$

11. Here, point A lies on Y -axis, so its abscissa is zero and given its ordinate is 5, therefore, its coordinates are $A(0, 5)$.

$$\text{Now, } AB = \sqrt{(-5-0)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29} \text{ units}$$

12. Diameter of circle, $d = \sqrt{(2-24)^2 + (23-1)^2}$

$$= \sqrt{(-22)^2 + (22)^2} = \sqrt{(22)^2(1+1)} = 22\sqrt{2} \text{ units}$$

$$\therefore \text{Radius of a circle, } r = \frac{d}{2} = \frac{22\sqrt{2}}{2} = 11\sqrt{2} \text{ units}$$

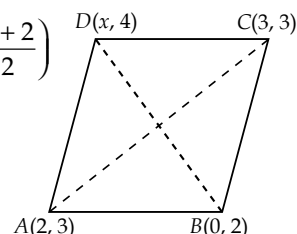
13. As the diagonals of parallelogram bisect each other.

$\therefore$ Mid-point of $AC =$ Mid-point of BD

$$\Rightarrow \left(\frac{2+3}{2}, \frac{3+3}{2} \right) = \left(\frac{x+0}{2}, \frac{4+2}{2} \right)$$

$$\Rightarrow \left(\frac{5}{2}, 3 \right) = \left(\frac{x}{2}, 3 \right)$$

$$\Rightarrow \frac{x}{2} = \frac{5}{2} \Rightarrow x = 5$$



14. Let coordinates of P and Q be $(x, 0)$ and $(0, y)$ respectively.

Let $M(-2, -6)$ be the mid-point of PQ .

∴ By mid-point formula, we have

$$-2 = \frac{x+0}{2} \text{ and } -6 = \frac{0+y}{2} \Rightarrow -4 = x \text{ and } -12 = y$$

∴ Points are $P(-4, 0)$ and $Q(0, -12)$.

15. Let $A(1, 2)$, $B(4, 3)$ and $C(6, 6)$ and $D(x, y)$ be the vertices of parallelogram.

Since diagonals AC and BD bisect each other.

∴ Mid-point of BD = Mid-point of AC

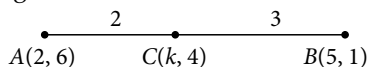
$$\Rightarrow \left(\frac{4+x}{2}, \frac{3+y}{2} \right) = \left(\frac{1+6}{2}, \frac{2+6}{2} \right)$$

$$\Rightarrow \left(\frac{4+x}{2}, \frac{3+y}{2} \right) = \left(\frac{7}{2}, \frac{8}{2} \right)$$

$$\Rightarrow 4+x=7 \text{ and } 3+y=8 \Rightarrow x=3 \text{ and } y=5$$

∴ Coordinates of D are $(3, 5)$.

16. We have, $A(2, 6)$, $B(5, 1)$ and $C(k, 4)$ divides the given line segment in the ratio $2:3$.



Using section formula, we have

$$(k, 4) = \left(\frac{2 \times 5 + 3 \times 2}{2+3}, \frac{2 \times 1 + 3 \times 6}{2+3} \right) = \left(\frac{16}{5}, \frac{20}{5} \right) \Rightarrow k = \frac{16}{5}$$

17. Let $A(0, 6)$, $B(8, 12)$ and $C(8, 0)$ be the vertices of triangle.

∴ Coordinates of centroid of triangle

$$= \left(\frac{0+8+8}{3}, \frac{6+12+0}{3} \right) = \left(\frac{16}{3}, 6 \right)$$

18. We have, $P(x, y)$, $A(5, 1)$ and $B(-1, 5)$

Given, $AP = BP \Rightarrow AP^2 = BP^2$

$$\Rightarrow (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$$

$$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 25 - 10y$$

$$\Rightarrow -10x - 2y = 2x - 10y \Rightarrow -12x = -8y \Rightarrow 3x = 2y$$

19. Here, $AB = \sqrt{(4-1)^2 + (5-2)^2}$ units

$$= \sqrt{18} \text{ units} = 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(-1-4)^2 + (0-5)^2} \text{ units}$$

$$= \sqrt{50} \text{ units} = 5\sqrt{2} \text{ units}$$

$$\text{and } AC = \sqrt{(-1-1)^2 + (0-2)^2} \text{ units}$$

$$= \sqrt{4+4} \text{ units} = 2\sqrt{2} \text{ units}$$

$$\text{Now, } AB + AC = (3\sqrt{2} + 2\sqrt{2}) \text{ units} = 5\sqrt{2} \text{ units} = BC$$

Hence, A , B and C lie on a straight line. In other words, A , B , C are collinear.

20. Let $A(6, -2)$ and $B(-2, y)$ be the given points.

Length of the line segment $AB = 10$ units (Given)

$$\Rightarrow \sqrt{(-2-6)^2 + (y+2)^2} = 10$$

$$\Rightarrow \sqrt{64 + y^2 + 4y + 4} = 10$$

$$\Rightarrow y^2 + 4y + 68 = 100 \Rightarrow y^2 + 4y - 32 = 0$$

$$\Rightarrow y^2 + 8y - 4y - 32 = 0 \Rightarrow y(y+8) - 4(y+8) = 0$$

$$\Rightarrow (y+8)(y-4) = 0 \Rightarrow y = 4 \text{ or } y = -8$$

So, ordinate of B will be 4 or -8 .

21. The given points are $A(0, 2)$, $B(3, p)$ and $C(p, 5)$.

Since, A is equidistant from B and C .

$$\therefore AB = AC \Rightarrow AB^2 = AC^2$$

$$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$$

$$\Rightarrow 9 + p^2 + 4 - 4p = p^2 + 9 \Rightarrow 4 - 4p = 0$$

$$\Rightarrow 4p = 4 \Rightarrow p = 1$$

22. Let the coordinates of B be (x, y) .

Using section formula, we have

coordinates of C are $A(2, 7)$ $C(-2, 4)$ $B(x, y)$

$$\left(\frac{4x+5(2)}{4+5}, \frac{4y+5(7)}{4+5} \right) = \left(\frac{4x+10}{9}, \frac{4y+35}{9} \right)$$

Also, the coordinates of C are $(-2, 4)$.

(Given)

$$\therefore \frac{4x+10}{9} = -2 \text{ and } \frac{4y+35}{9} = 4$$

$$\Rightarrow 4x+10 = -18 \text{ and } 4y+35 = 36$$

$$\Rightarrow 4x = -18 - 10 \text{ and } 4y = 36 - 35$$

$$\Rightarrow 4x = -28 \text{ and } 4y = 1$$

$$\Rightarrow x = -7 \text{ and } y = \frac{1}{4}$$

23. Let $A(a, a^2)$, $B(b, b^2)$ and $C(0, 0)$

For the points A , B and C to be collinear, area of $\triangle ABC$ must be zero.

$$\text{Now, area of } \triangle ABC = \frac{1}{2}[a(b^2-0) + b(0-a^2) + 0(a^2-b^2)]$$

$$= \frac{1}{2}(ab^2 - ba^2) = \frac{ab}{2}(b-a) \neq 0 \quad [\text{Since, } a \neq b \neq 0]$$

∴ A , B and C are not collinear.

24. Let $A(t, t-2)$, $B(t+2, t+2)$, $C(t+3, t)$

be the given points.

∴ Area of triangle ABC

$$= \frac{1}{2}[t(t+2-t) + (t+2)(t-t+2) + (t+3)(t-2-t-2)]$$

$$= \frac{1}{2}[2t + 2t + 4 - 4t - 12] = \frac{1}{2}(-8)$$

$$= 4 \text{ sq. units} \quad [\because \text{Area of triangle can't be negative}]$$

∴ Area of triangle is independent of t .

25. Given, points $A(-1, -4)$, $B(b, c)$ and $C(5, -1)$ are collinear.

∴ Area of $\triangle ABC = 0$

$$\Rightarrow \frac{1}{2}[-1(c-(-1)) + b(-1-(-4)) + 5(-4-c)] = 0$$

$$\Rightarrow -c - 1 - b + 4b - 20 - 5c = 0$$

$$\Rightarrow 3b - 6c = 21 \Rightarrow b - 2c = 7$$

...(i)

Also, given that $2b + c = 4$

$$\Rightarrow 2(7+2c) + c = 4$$

(From (i))

$$\therefore 14 + 4c + c = 4$$

$$\Rightarrow 5c = -10 \Rightarrow c = -2$$

Put $c = -2$ in (i), we get $b = 7 + 2c = 7 - 4 = 3$

Hence, we have $b = 3$, $c = -2$

26. Using distance formula, we have

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

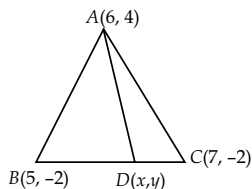
$$AC = \sqrt{(6-7)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$BC = \sqrt{(5-7)^2 + (-2+2)^2} = \sqrt{4+0} = 2 \text{ units}$$

Now, $AB = AC \neq BC$.

So, $\triangle ABC$ is an isosceles triangle.

Let $D(x, y)$ be the mid-point of BC .



Using mid-point formula, we have

$$(x, y) = \left(\frac{5+7}{2}, \frac{-2-2}{2} \right) = \left(\frac{12}{2}, \frac{-4}{2} \right) = (6, -2)$$

$\therefore$ Coordinates of D are $(6, -2)$

$$\begin{aligned} \therefore \text{Length of median, } AD &= \sqrt{(6-6)^2 + (-2-4)^2} \\ &= \sqrt{0+36} = 6 \text{ units} \end{aligned}$$

27. Since C is equidistant from A and B .

$$\therefore AC = CB \Rightarrow AC^2 = CB^2$$

$$\Rightarrow (3+2)^2 + (-1-3)^2 = (x+2)^2 + (8-3)^2$$

$$\Rightarrow 25 + 16 = x^2 + 4 + 4x + 25$$

$$\Rightarrow x^2 + 4x - 12 = 0 \Rightarrow x^2 + 6x - 2x - 12 = 0$$

$$\Rightarrow x(x+6) - 2(x+6) = 0 \Rightarrow (x+6)(x-2) = 0$$

$$\Rightarrow x = -6 \text{ or } x = 2$$

Now, using distance formula

$$BC = \sqrt{(-6+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

$$\text{or } \sqrt{(2+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

$$AB = \sqrt{(-6-3)^2 + (8+1)^2} = \sqrt{81+81} = 9\sqrt{2} \text{ units}$$

$$\text{or } \sqrt{(2-3)^2 + (8+1)^2} = \sqrt{1+81} = \sqrt{82} \text{ units}$$

28. Let the given points are $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$.

We know, diagonals of a parallelogram bisect each other.

$\therefore$ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{-2+4}{2}, \frac{1+b}{2} \right) = \left(\frac{a+1}{2}, \frac{0+2}{2} \right)$$

$$\Rightarrow \left(\frac{2}{2}, \frac{1+b}{2} \right) = \left(\frac{a+1}{2}, \frac{2}{2} \right) \Rightarrow \frac{2}{2} = \frac{a+1}{2}$$

$$\Rightarrow 1+a=2 \Rightarrow a=1 \text{ and } \frac{1+b}{2} = \frac{2}{2}$$

$$\Rightarrow b+1=2 \Rightarrow b=1$$

Hence, $a=1, b=1$

Now, $AB = CD$ and $BC = AD$

($\because$ Opposite sides of a parallelogram are equal)

$$\therefore AB = CD = \sqrt{(-2-1)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$\text{and } BC = AD = \sqrt{(1-4)^2 + (0-1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

29. $\therefore O$ is the mid-point of the base BC .

$\therefore$ Coordinates of point B are $(0, 3)$

$$\text{So, } BC = \sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{6^2} = 6 \text{ units.}$$

Let the coordinates of point A be $(x, 0)$.

$$\begin{aligned} \therefore AB &= \sqrt{(0-x)^2 + (3-0)^2} \\ &= \sqrt{x^2+9} \end{aligned}$$

Also, $AB = BC$

($\because \triangle ABC$ is an equilateral triangle)

$$\Rightarrow \sqrt{x^2+9} = 6 \Rightarrow x^2+9 = 36$$

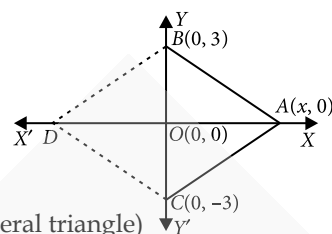
$$\Rightarrow x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

$$\therefore \text{Coordinates of point } A = (x, 0) = (3\sqrt{3}, 0)$$

Since, $BACD$ is a rhombus.

$$\therefore AB = AC = CD = DB$$

$$\therefore \text{Coordinates of point } D = (-3\sqrt{3}, 0)$$



30. We have $ABCD$ is a rectangle, where AC and BD are its diagonal.

$$\text{Now, } AC = \sqrt{[6-(-4)]^2 + (3-5)^2}$$

$$= \sqrt{(10)^2 + (-2)^2}$$

$$= \sqrt{100+4} = \sqrt{104} \text{ units}$$

$$BD = \sqrt{(-4-6)^2 + (3-5)^2} = \sqrt{(-10)^2 + (-2)^2}$$

$$= \sqrt{100+4} = \sqrt{104} \text{ units}$$

$$\Rightarrow AC = BD$$

Hence, diagonals of rectangle $ABCD$ are equal.

Let O is the mid-point of both AC and BD .

Using mid-point formula, we have

$$\text{coordinates of } O \text{ from } AC = \left(\frac{6+(-4)}{2}, \frac{3+5}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{8}{2} \right) = (1, 4)$$

$$\text{Coordinates of } O \text{ from } BD = \left(\frac{-4+6}{2}, \frac{3+5}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{8}{2} \right) = (1, 4)$$

$\Rightarrow AC$ and BD bisect each other at O .

31. Let D , E and F be the mid-points of the sides AC , BC and AB respectively.

Then the coordinates of D are

$$\left(\frac{-3+4}{2}, \frac{0+1}{2} \right) = \left(\frac{1}{2}, \frac{1}{2} \right)$$

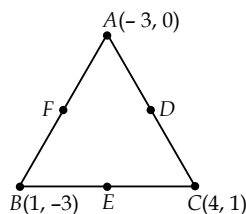
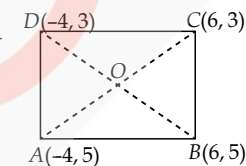
Coordinates of E are

$$\left(\frac{1+4}{2}, \frac{-3+1}{2} \right) = \left(\frac{5}{2}, \frac{-2}{2} \right) = \left(\frac{5}{2}, -1 \right)$$

$$\text{Coordinates of } F \text{ are } \left(\frac{-3+1}{2}, \frac{0-3}{2} \right) = \left(\frac{-2}{2}, \frac{-3}{2} \right) = \left(-1, \frac{-3}{2} \right)$$

Using distance formula, lengths of medians are

$$\therefore AE = \sqrt{\left[\frac{5}{2} - (-3) \right]^2 + (-1-0)^2}$$



$$= \sqrt{\left(\frac{11}{2}\right)^2 + 1} = \sqrt{\frac{121}{4} + 1} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2} \text{ units}$$

$$BD = \sqrt{\left(\frac{1}{2} - 1\right)^2 + \left[\frac{1}{2} - (-3)\right]^2} = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{49}{4}} = \sqrt{\frac{50}{4}} = \frac{\sqrt{50}}{2} \text{ units}$$

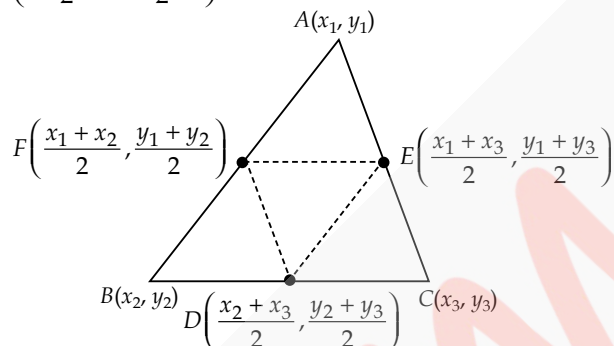
$$CF = \sqrt{(-1-4)^2 + \left(\frac{-3}{2} - 1\right)^2} = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{100+25}{4}}$$

$$= \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2} \text{ units}$$

32. Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of $\triangle ABC$. Then, the coordinates of D , E and F are

$$\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right), \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right) \text{ and}$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \text{ respectively.}$$



$$\Delta_1 = \text{Area of } \triangle ABC$$

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Delta_2 = \text{Area of } \triangle DEF$$

$$= \frac{1}{2} \left[\left(\frac{x_2 + x_3}{2} \right) \left(\frac{y_1 + y_3}{2} - \frac{y_1 + y_2}{2} \right) + \left(\frac{x_1 + x_3}{2} \right) \left(\frac{y_1 + y_2}{2} - \frac{y_2 + y_3}{2} \right) + \left(\frac{x_1 + x_2}{2} \right) \left(\frac{y_2 + y_3}{2} - \frac{y_1 + y_3}{2} \right) \right]$$

$$\Rightarrow \Delta_2 = \frac{1}{8} [(x_2 + x_3)(y_3 - y_2) + (x_1 + x_3)(y_1 - y_3) + (x_1 + x_2)(y_2 - y_1)]$$

$$\Rightarrow \Delta_2 = \frac{1}{8} [x_1(y_1 - y_3 + y_2 - y_1) + x_2(y_3 - y_2 + y_2 - y_1) + x_3(y_3 - y_2 + y_1 - y_3)]$$

$$\Rightarrow \Delta_2 = \frac{1}{8} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow \Delta_2 = \frac{1}{4} (\text{Area of } \triangle ABC) = \frac{1}{4} \Delta_1$$

$$\text{Hence, Area of } \triangle DEF = \frac{1}{4} (\text{Area of } \triangle ABC)$$

33. We have, $A(1, 2)$, $B(7, -2)$, $C(12, 3)$ and $D(7, 22)$

Area of quadrilateral $ABCD$

$$= \text{Area of } \triangle ABC + \text{Area of } \triangle ACD \dots (i)$$

Now, Area of $\triangle ABC =$

$$\frac{1}{2} [1(-2-3) + 7(3-2) + 12(2+2)]$$

$$= \frac{1}{2} [1(-5) + 7(1) + 12(4)] = \frac{1}{2} [-5 + 7 + 48]$$

$$= \frac{1}{2} [50] = 25 \text{ sq. units}$$

$$\text{Also, area of } \triangle ACD = \frac{1}{2} [1(3-22) + 12(22-2) + 7(2-3)]$$

$$= \frac{1}{2} [1(-19) + 12(20) + 7(-1)] = \frac{1}{2} [-19 + 240 - 7]$$

$$= \frac{1}{2} [214] = 107 \text{ sq. units}$$

From (i), we have

$$\text{Area of quadrilateral } ABCD = (25 + 107) \text{ sq. units} = 132 \text{ sq. units}$$

34. Let the vertices of an equilateral triangle are $A(-6, 5)$, $B(6, 5)$ and $C(x, y)$.

We know that, in an equilateral triangle all three sides are equal.

$$\therefore AB = BC = CA \Rightarrow AB^2 = BC^2 = CA^2$$

Consider, $AB^2 = BC^2$

$$\Rightarrow (6+6)^2 + (5-5)^2 = (x-6)^2 + (y-5)^2$$

$$\Rightarrow 144 + 0 = x^2 + 36 - 12x + y^2 + 25 - 10y$$

$$\Rightarrow x^2 + y^2 - 12x - 10y + 61 = 144$$

$$\Rightarrow x^2 + y^2 - 12x - 10y = 83$$

...(i)

Now, consider $AB^2 = CA^2$

$$\Rightarrow (6+6)^2 + (5-5)^2 = (x+6)^2 + (y-5)^2$$

$$\Rightarrow 144 = 36 + x^2 + 12x + 25 + y^2 - 10y$$

$$\Rightarrow x^2 + y^2 + 12x - 10y + 61 = 144$$

$$\Rightarrow x^2 + y^2 + 12x - 10y = 83$$

...(ii)

From (i) and (ii) we get

$$x^2 + y^2 - 12x - 10y = x^2 + y^2 + 12x - 10y$$

$$\Rightarrow 12x + 12x = 0 \Rightarrow 24x = 0 \Rightarrow x = 0$$

Putting $x = 0$ in (i), we get

$$0 + y^2 - 12(0) - 10y = 83$$

$$\Rightarrow y^2 - 10y - 83 = 0$$

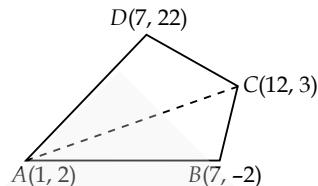
$$\therefore y = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-83)}}{2 \times 1}$$

$$\Rightarrow y = \frac{10 \pm \sqrt{100 + 332}}{2}$$

$$\Rightarrow y = \frac{10 \pm \sqrt{432}}{2}$$

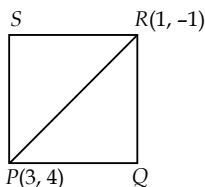
$$\therefore y = \frac{10 + \sqrt{432}}{2} \text{ or } y = \frac{10 - \sqrt{432}}{2}$$

$$\text{Hence, the third vertex is } \left(0, \frac{10 + \sqrt{432}}{2}\right) \text{ or } \left(0, \frac{10 - \sqrt{432}}{2}\right).$$



35. Let PQRS be a square and let P(3, 4) and R(1, -1) be the given opposite angular points.

Let Q(x, y) be the unknown vertex.
Since, all sides of square are equal.



$$\begin{aligned} \therefore PQ &= QR \\ \Rightarrow PQ^2 &= QR^2 \\ \Rightarrow (x-3)^2 + (y-4)^2 &= (x-1)^2 + (y+1)^2 \\ \Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 &= x^2 - 2x + 1 + y^2 + 2y + 1 \\ \Rightarrow -6x - 8y + 25 &= -2x + 2y + 2 \\ \Rightarrow -6x + 2x - 8y - 2y &= 2 - 25 \\ \Rightarrow -4x - 10y &= -23 \Rightarrow 4x + 10y = 23 \quad \dots(i) \\ \Rightarrow x &= \frac{23 - 10y}{4} \end{aligned}$$

In right angled triangle PQR, we have

$$\begin{aligned} PQ^2 + QR^2 &= PR^2 \\ \Rightarrow (x-3)^2 + (y-4)^2 + (x-1)^2 + (y+1)^2 &= (3-1)^2 + (4+1)^2 \\ \Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 + x^2 - 2x + 1 + y^2 + 2y + 1 &= 4 + 25 \\ \Rightarrow 2x^2 + 2y^2 - 8x - 6y + 27 &= 29 \\ \Rightarrow 2x^2 + 2y^2 - 8x - 6y &= 2 \\ \Rightarrow x^2 + y^2 - 4x - 3y - 1 &= 0 \quad \dots(ii) \end{aligned}$$

Substitute the value of x from (i) into (ii), we get

$$\begin{aligned} \left(\frac{23-10y}{4}\right)^2 + y^2 - 4\left(\frac{23-10y}{4}\right) - 3y - 1 &= 0 \\ \Rightarrow \left(\frac{529+100y^2-460y}{16}\right) + y^2 - 23 + 10y - 3y - 1 &= 0 \\ \Rightarrow \left(\frac{529+100y^2-460y}{16}\right) + y^2 + 7y - 24 &= 0 \\ \Rightarrow \frac{529+100y^2-460y+16y^2+112y-384}{16} &= 0 \\ \Rightarrow 116y^2 - 348y + 145 &= 0 \\ \Rightarrow 4y^2 - 12y + 5 &= 0 \quad [\text{On dividing by 29}] \\ \Rightarrow 4y^2 - 10y - 2y + 5 &= 0 \\ \Rightarrow 2y(2y-5) - 1(2y-5) &= 0 \\ \Rightarrow (2y-5)(2y-1) &= 0 \\ \Rightarrow y = \frac{5}{2} \text{ or } y = \frac{1}{2} \end{aligned}$$

On putting $y = \frac{1}{2}$ in (i), we get

$$x = \frac{23 - 10\left(\frac{1}{2}\right)}{4} = \frac{23 - 5}{4} = \frac{18}{4} = \frac{9}{2}$$

On putting $y = \frac{5}{2}$ in (i) we get

$$x = \frac{23 - 10\left(\frac{5}{2}\right)}{4} = \frac{23 - 25}{4} = \frac{-2}{4} = \frac{-1}{2}$$

Hence, the other two vertices are $\left(\frac{9}{2}, \frac{1}{2}\right)$ and $\left(\frac{-1}{2}, \frac{5}{2}\right)$.

36. Given, $\triangle AOB$ is a right angle triangle right angled at O and AB is hypotenuse and C is the mid-point of AB. Let the coordinates of B and A are (0, b) and (a, 0) respectively.

$$\Rightarrow OA = a \text{ and } OB = b$$

So, the coordinates of $C = \left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

$$\begin{aligned} \text{Now, } CO &= \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2} \text{ units} \end{aligned}$$

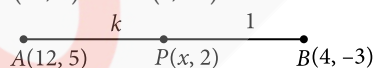
$$\begin{aligned} CA &= \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2} \text{ units} \end{aligned}$$

$$\begin{aligned} CB &= \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \\ &= \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2} \text{ units} \end{aligned}$$

$$\Rightarrow CO = CA = CB$$

Therefore, C is equidistant from the three vertices of $\triangle AOB$.

37. Let the point P(x, 2) divides the line segment joining the points A(12, 5) and B(4, -3) in the ratio k : 1.



Using section formula, we have

$$\text{coordinates of P are } \left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right)$$

Now, the coordinates of P are (x, 2).

(Given)

$$\therefore \frac{4k+12}{k+1} = x \text{ and } \frac{-3k+5}{k+1} = 2$$

$$\Rightarrow -3k+5 = 2k+2 \Rightarrow 5k = 3 \Rightarrow k = 3/5$$

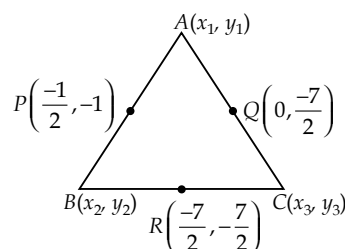
Substituting, $k = \frac{3}{5}$ in $\frac{4k+12}{k+1} = x$, we get

$$x = \frac{4 \times \frac{3}{5} + 12}{\frac{3}{5} + 1} = \frac{12 + 60}{3 + 5} = \frac{72}{8} = 9$$

Thus, the value of x is 9.

Also, the point P divides the line segment joining the points A(12, 5) and B(4, -3) in the ratio $\frac{3}{5} : 1$ i.e., 3 : 5.

38. Let the coordinates of the vertices A, B and C of $\triangle ABC$ be (x_1, y_1) , (x_2, y_2) and (x_3, y_3) respectively.



As, $P\left(-\frac{1}{2}, -1\right)$ is the mid-point of AB.

$$\therefore \frac{x_1 + x_2}{2} = \frac{-1}{2} \text{ and } \frac{y_1 + y_2}{2} = -1$$

$$\Rightarrow x_1 + x_2 = -1 \text{ and } y_1 + y_2 = -2$$

Point R is the mid-point of BC.

$$\therefore \frac{x_2 + x_3}{2} = \frac{-7}{2} \text{ and } \frac{y_2 + y_3}{2} = \frac{-7}{2}$$

$$\Rightarrow x_2 + x_3 = -7 \text{ and } y_2 + y_3 = -7$$

Point Q is the mid-point of AC.

$$\therefore \frac{x_1 + x_3}{2} = 0 \text{ and } \frac{y_1 + y_3}{2} = \frac{-7}{2}$$

$$\Rightarrow x_1 + x_3 = 0 \text{ and } y_1 + y_3 = -7$$

Adding (i), (ii) and (iii), we get

$$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = -1 - 7 + 0 \text{ and } y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = -2 - 7 - 7$$

$$\Rightarrow 2(x_1 + x_2 + x_3) = -8 \text{ and } 2(y_1 + y_2 + y_3) = -16$$

$$\Rightarrow x_1 + x_2 + x_3 = -4 \text{ and } y_1 + y_2 + y_3 = -8 \quad \dots(\text{iv})$$

From (i) and (iv), we get

$$-1 + x_3 = -4 \text{ and } -2 + y_3 = -8$$

$$\Rightarrow x_3 = -3 \text{ and } y_3 = -6$$

... (i) So, coordinates of C are (-3, -6).

From (ii) and (iv) we get

$$x_1 + (-7) = -4 \text{ and } y_1 + (-7) = -8$$

$$\Rightarrow x_1 = 3 \text{ and } y_1 = -1$$

... (ii) $\therefore$ Coordinates of A are (3, -1)

From (iii) and (iv), we get

$$x_2 + 0 = -4 \text{ and } y_2 + (-7) = -8$$

$$\Rightarrow x_2 = -4 \text{ and } y_2 = -1$$

$\therefore$ Coordinates of B are (-4, -1).

$$\begin{aligned} \therefore \text{Area of } \triangle ABC &= \frac{1}{2} [3(-1+6) + (-4)(-6+1) + (-3)(-1+1)] \\ &= \frac{1}{2} [3(5) + (-4)(-5) + 0] \\ &= \frac{1}{2} [15 + 20] = \frac{35}{2} \text{ sq. units.} \end{aligned}$$

