CHAPTER 6

Triangles

EXAM DRILL

SOLUTIONS

1. **(a) :** We have $\Delta ABC \sim \Delta DEF$

$\therefore \quad \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

Given, $\angle A = 47°, \angle E = 83°$

$\therefore \quad \angle B = 83°$

Now, in ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 47° - 83° = 50°$

2. **(a) :** We have, $\Delta ABC \sim \Delta XYZ$

$$\Rightarrow \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} \Rightarrow \frac{4}{x} = \frac{6}{7.2} = \frac{5}{6} \Rightarrow x = \frac{4 \times 7.2}{6}$$

$\Rightarrow \quad x = 4.8$ cm

3. **(d) :** In triangle CAB, if DE divides CA and CB in the same ratio, then $DE \parallel AB$.

$$\therefore \frac{CD}{DA} = \frac{CE}{EB} \Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$\Rightarrow \quad 3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

$\Rightarrow \quad 6x = 12 \Rightarrow x = 2$

4. **(b) :** We have $\Delta ABC \sim \Delta DEF$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{5}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

5. **(c) :** Let ΔABC and ΔDEF be the similar triangles and AM and DN are their corresponding altitudes.

$\therefore \quad AM : DN = 4 : 9$

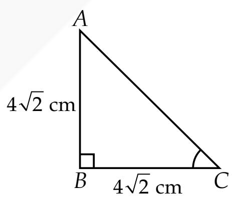
$$\text{Now, } \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AM^2}{DN^2}$$

[$\because$ The ratio of the area of two similar triangles is equal to the ratio of square of their corresponding altitudes]

$$= \left(\frac{AM}{DN}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

6. **(b) :** Let ΔABC be the right isosceles triangle right angled at B.

$\therefore \quad BC = AB = 4\sqrt{2}$ cm



In ΔABC, by Pythagoras theorem

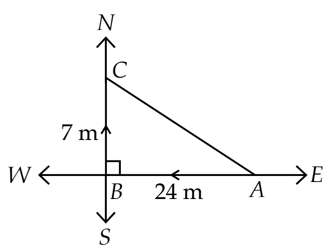
$AC^2 = AB^2 + BC^2$

$= (4\sqrt{2})^2 + (4\sqrt{2})^2 = 32 + 32 = 64$

$\Rightarrow \quad AC = 8$ cm

7. **(c) :** Let A be the starting point and C be the final point.

In right ΔABC, by Pythagoras theorem



$AC^2 = AB^2 + BC^2$

$= 24^2 + 7^2 = 576 + 49 = 62$

$\Rightarrow \quad AC = 25$ m

8. In triangle ABC, $DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By B.P.T.]}$$

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1} \Rightarrow x^2 - x = x^2 - 4 \Rightarrow x = 4$$

9. We have, $\Delta ABE \cong \Delta ACD$

$\therefore \quad AB = AC$ and $AD = AE$ [By C.P.C.T.] ...(i)

Now, in ΔADE and ΔABC,

$\angle A = \angle A$ [Common]

$\frac{AB}{AD} = \frac{AC}{AE}$ [Using (i)]

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By SAS similarity criterion]

10. Let ΔABC and ΔDEF be two similar triangles such that $AB = 9$ cm.

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$$

[$\because$ Ratio of corresponding sides of similar triangles is equal to the ratio of their perimeters]

$$\Rightarrow \frac{9}{DE} = \frac{36}{48} \Rightarrow DE = 12 \text{ cm}$$

11. Basic proportionality theorem : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

12. SAS similarity criterion : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

13. Pythagoras theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

14. In ΔACF, $BP \parallel CF$

$$\therefore \frac{AB}{BC} = \frac{AP}{PF} \quad \text{[By B.P.T.]}$$

$$\Rightarrow \frac{2}{8-2} = \frac{AP}{PF} \Rightarrow \frac{AP}{PF} = \frac{1}{3} \quad \text{...(i)}$$

In ΔAEF, $DP \parallel EF$

$$\therefore \frac{AD}{DE} = \frac{AP}{PF} \quad \text{[By B.P.T.]}$$

$$\Rightarrow \frac{AD}{DE} = \frac{1}{3} \quad \text{[Using (i)]}$$

15. In ΔABD and ΔASR, $RS \parallel DB$

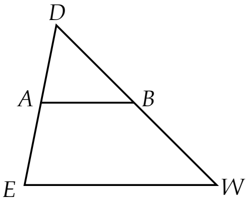
$\therefore \quad \angle ABD = \angle ASR$ [Corresponding angles]

$\angle A = \angle A$ [Common]

$\therefore \quad \Delta ABD \sim \Delta ASR$ [By AA similarity criterion]

$$\Rightarrow \frac{AB}{AS} = \frac{AD}{AR} = \frac{BD}{RS} \Rightarrow \frac{3+3}{3} = \frac{x}{y} \Rightarrow x = 2y$$

16. In ΔDEW, $AB \parallel EW$,



$\therefore \quad \frac{DA}{AE} = \frac{DB}{BW}$ [By B.P.T.]

$\Rightarrow \quad \frac{DA}{DE - AD} = \frac{DB}{DW - DB}$

$\Rightarrow \quad \frac{4}{12 - 4} = \frac{DB}{24 - DB}$

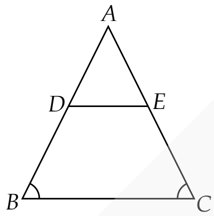
[$DA = 4$ cm, $DE = 12$ cm, $DW = 24$ cm]

$\Rightarrow \quad \frac{4}{8} = \frac{DB}{24 - DB} \Rightarrow \frac{1}{2} = \frac{DB}{24 - DB}$

$\Rightarrow \quad 24 - DB = 2DB \Rightarrow 24 = 3DB$

$\Rightarrow \quad DB = 24/3 = 8$ cm

17. In ΔABC, we have



$\angle B = \angle C \quad \Rightarrow \quad AC = AB$

$\Rightarrow \quad AE + EC = AD + DB$

$\Rightarrow \quad AE + CE = AD + CE$

[$\because BD = CE$]

$\Rightarrow \quad AE = AD$

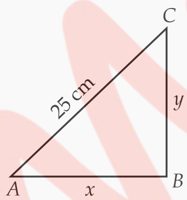
Thus, we have

$AD = AE$ and $BD = CE$

$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE}$

$\Rightarrow \quad DE \parallel BC$ [By the converse of B.P.T.]

18. Let ABC be a right triangle right angled at B. Let $AB = x$ and $BC = y$.



In ΔABC, by Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$x^2 + y^2 = 25^2$ ...(i)

Now, $x + y + 25 = 60$

[$\because$ Perimeter of triangle is 60 cm]

$\Rightarrow \quad x + y = 35$

$\Rightarrow \quad (x + y)^2 = 35^2$ [On squaring both sides]

$\Rightarrow \quad x^2 + y^2 + 2xy = 35^2$

$\Rightarrow \quad 25^2 + 2xy = 35^2$ [Using (i)]

$\Rightarrow \quad 2xy = 35^2 - 25^2$

$\Rightarrow \quad 2xy = (35 + 25)(35 - 25) \Rightarrow \quad 2xy = 60 \times 10$

$\Rightarrow \quad xy = 300 \quad \Rightarrow \frac{1}{2}xy = 150$

$\Rightarrow \quad$ Area of $\Delta ABC = 150$ cm^2

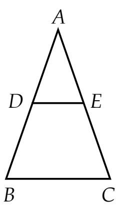
19. In ΔADE and ΔABC, $DE \parallel BC$

$\Rightarrow \quad \angle ADE = \angle ABC$ and $\angle AED = \angle ACB$

[Corresponding angles]

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By AA similarity criterion]

$\Rightarrow \quad \frac{AB}{AD} = \frac{BC}{DE}$ ...(i)



Given, $\frac{AD}{DB} = \frac{2}{3} \Rightarrow \frac{DB}{AD} = \frac{3}{2}$

$\Rightarrow \quad \frac{DB}{AD} + 1 = \frac{3}{2} + 1 \Rightarrow \frac{DB + AD}{AD} = \frac{3+2}{2}$

$\Rightarrow \quad \frac{AB}{AD} = \frac{5}{2}$ ...(ii)

From (i) and (ii), we get $\frac{BC}{DE} = \frac{5}{2}$

20. Since, $AB \parallel DC$

$\therefore \quad \angle OAB = \angle OCQ$ [Alternate angles]

and $\angle APO = \angle OQC$ [Alternate angles]

Now, in ΔOAP and ΔOCQ,

$\angle OAP = \angle OCQ$ [Proved above]

$\angle APO = \angle OQC$ [Proved above]

$\angle AOP = \angle QOC$ [Vertically opposite angles]

$\therefore \quad \Delta OAP \sim \Delta OCQ$ [By AAA similarity criterion]

$\Rightarrow \quad \frac{OA}{OC} = \frac{OP}{OQ} = \frac{AP}{CQ}$

$\Rightarrow \quad OA \cdot CQ = OC \cdot AP$

21. In ΔABC, $\angle A + \angle B + \angle C = 180°$

[By angle sum property]

$\Rightarrow \quad \angle A = 180° - 30° - 20° = 130°$

Also, $\frac{DE}{AC} = \frac{7}{63} = \frac{1}{9}$ and $\frac{EF}{AB} = \frac{5}{45} = \frac{1}{9}$

Now, in ΔABC and ΔEFD,

$\angle A = \angle E = 130°$

$\frac{DE}{AC} = \frac{EF}{AB}$

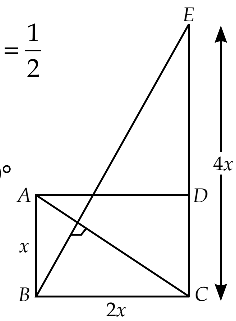
$\therefore \quad \Delta ABC \sim \Delta EFD$ [By SAS similarity criterion]

$\Rightarrow \quad \angle A = \angle E, \angle B = \angle F, \angle C = \angle D$

$\therefore \quad \angle D = 20°$ and $\angle F = 30°$.

22. Let $AB = x$

$\Rightarrow \quad BC = 2x$ and $CE = 4x$



Now, in ΔABC and ΔBCE, $\frac{AB}{BC} = \frac{x}{2x} = \frac{1}{2}$

$\frac{BC}{CE} = \frac{2x}{4x} = \frac{1}{2}$

$\therefore \quad \frac{AB}{BC} = \frac{BC}{CE} = \frac{1}{2}$ and $\angle B = \angle C = 90°$

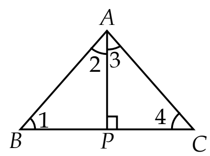
$\therefore \quad \Delta ABC \sim \Delta BCE$

[By SAS similarity criterion]

$\therefore \quad \frac{AB}{BC} = \frac{BC}{CE} = \frac{AC}{BE}$

$\Rightarrow \quad \frac{AC}{BE} = \frac{1}{2}$ or $AC : BE = 1 : 2$

23. In ΔABC, $AP \perp BC$



and $AC^2 = BC^2 - AB^2$

$\Rightarrow \quad BC^2 = AB^2 + AC^2$

$\therefore \quad$ By the converse of Pythagoras theorem, ΔABC is a right triangle right angled at A.

$\Rightarrow \quad \angle 2 + \angle 3 = 90°$ ...(i)

Also, $\angle 1 + \angle 2 = 90°$ ...(ii) [$\because \angle APB = 90°$]

$\Rightarrow \quad \angle 1 = \angle 3$ [Using (i) and (ii)]

Similarly, $\angle 2 = \angle 4$

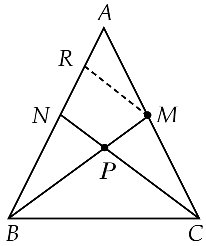
Now, in ΔBAP and ΔACP

$\angle 1 = \angle 3$ and $\angle 2 = \angle 4$

$\therefore \quad \Delta BAP \sim \Delta ACP$ [By AA similarity criterion]

$\Rightarrow \quad \frac{BP}{AP} = \frac{AP}{CP} \Rightarrow AP^2 = BP \times CP \Rightarrow PA^2 = PB \times CP$

24. Given, $AM : MC = 3 : 4$, $BP : PM = 3 : 2$ and $BN = 12$ cm
Draw MR parallel to CN which meets AB at the point R.
In ΔBMR, $PN \parallel MR$



$\therefore \quad \frac{BN}{NR} = \frac{BP}{PM}$ [By B.P.T.]

$\Rightarrow \quad \frac{12}{NR} = \frac{3}{2} \Rightarrow NR = \frac{12 \times 2}{3} = 8$ cm

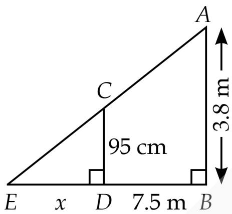
In ΔANC, $RM \parallel NC$

$\therefore \quad \frac{AR}{RN} = \frac{AM}{MC}$ [By B.P.T.]

$\Rightarrow \quad \frac{AR}{8} = \frac{3}{4} \Rightarrow AR = \frac{3 \times 8}{4} = 6$ cm

$\therefore \quad AN = AR + RN = 6 + 8 = 14$ cm

25. Let AB be the lamp post and CD be the boy after walking 5 seconds. Let $DE = x$ m be the length of his shadow such that $BD = 1.5 \times 5 = 7.5$ m.
In ΔABE and ΔCDE,



$\angle B = \angle D$ [Each equals 90°]
$\angle E = \angle E$ [Common]
$\therefore \quad \Delta ABE \sim \Delta CDE$ [By AA similarity criterion]

$\Rightarrow \quad \frac{BE}{DE} = \frac{AB}{CD} \quad \Rightarrow \frac{7.5 + x}{x} = \frac{3.8}{0.95}$

[$\because$ $AB = 3.8$ m, $CD = 95$ cm $= 0.95$ m and $BE = BD + DE = (7.5 + x)$m]

$\Rightarrow \quad 7.125 + 0.95x = 3.8x \Rightarrow 7.125 = 3.8x - 0.95x$
$\Rightarrow \quad 7.125 = 2.85x \Rightarrow x = 7.125 \div 2.85 \Rightarrow x = 2.5$
Hence, the length of his shadow after 5 seconds is 2.5 m.

26. Since, $\Delta NSQ \cong \Delta MTR$
$\therefore \quad \angle SQN = \angle TRM$
$\Rightarrow \quad \angle Q = \angle R$
In ΔPQR,
$\angle P + \angle Q + \angle R = 180°$ [By angle sum property]
$\Rightarrow \quad \angle Q + \angle Q = 180° - \angle P$

$\Rightarrow \angle Q = \frac{1}{2}(180° - \angle P) \Rightarrow \angle Q = \angle R = 90° - \frac{1}{2}\angle P$ ...(i)

Again, in ΔPST, $\angle 1 = \angle 2$ [Given]
and $\angle P + \angle 1 + \angle 2 = 180°$ [By angle sum property]
$\Rightarrow \quad \angle 1 + \angle 1 = 180° - \angle P$

$\Rightarrow \quad \angle 1 = \frac{1}{2}(180° - \angle P)$

$\Rightarrow \quad \angle 1 = \angle 2 = 90° - \frac{1}{2}\angle P$ ...(ii)

Now, in ΔPTS and ΔPRQ
$\angle 1 = \angle Q$ [From (i) and (ii)]
$\angle P = \angle P$ [Common]
$\therefore \quad \Delta PTS \sim \Delta PRQ$ [By AA similarity criterion]

27. Let $CD = 4x$ and $DA = 3x$
Then $CA = CD + DA = 4x + 3x = 7x$
In ΔABC and ΔAED, we have
$\angle ABC = \angle AED$ [Corresponding angles]
$\angle ACB = \angle ADE$ [Corresponding angles]
$\therefore \quad \Delta ABC \sim \Delta AED$ [By AA similarity criterion]

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta AED)} = \frac{(CA)^2}{(DA)^2}$

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta ABC) - ar(\text{quad}.\,BCDE)} = \frac{(7x)^2}{(3x)^2} = \frac{49}{9}$

$\Rightarrow \quad 9\,ar(\Delta ABC) = 49\,ar(\Delta ABC) - 49\,ar(\text{quad}.\,BCDE)$
$\Rightarrow \quad 49\,ar(\text{quad}.\,BCDE) = 40\,ar(\Delta ABC)$

$\Rightarrow \quad \frac{ar(\text{quad}.\,BCDE)}{ar(\Delta ABC)} = \frac{40}{49}$

Hence, ar (quad. $BCDE$) : $ar(\Delta ABC) = 40 : 49$.

28. We have,
$ar(\Delta BXY) = 2ar(\text{quad}.\,ACYX)$
$\Rightarrow \quad ar(\Delta BXY = 2[ar(\Delta BAC) - ar(\Delta BXY)]$
$\Rightarrow \quad ar(\Delta BXY) = 2ar(\Delta BAC) - 2\,ar(\Delta BXY)$
$\Rightarrow \quad 3ar(\Delta BXY) = 2ar(\Delta BAC)$

$\Rightarrow \quad \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{2}{3}$ ...(i)

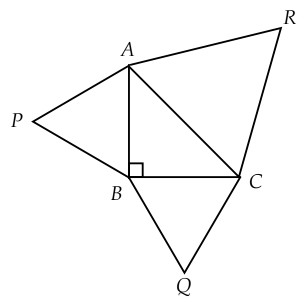
In ΔBXY and ΔBAC,
$\angle B = \angle B$ [Common]
$\therefore \quad XY \parallel AC \Rightarrow \angle BXY = \angle BAC$ [Corresponding angles]
$\therefore \quad \Delta BXY \sim \Delta BAC$ [By AA similarity criterion]

$\Rightarrow \quad \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{BX^2}{BA^2} \Rightarrow \frac{2}{3} = \frac{BX^2}{BA^2}$

$\Rightarrow \quad \frac{BX}{BA} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow 1 - \frac{BX}{BA} = 1 - \frac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \quad \frac{BA - BX}{BA} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}} \Rightarrow \frac{AX}{AB} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}} = \frac{3 - \sqrt{6}}{3}$

29. Given, a right triangle ABC right angled at B. Equilateral triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.
Since, triangles PAB, QBC and RAC are equilateral. Therefore, they are equiangular and hence similar.



$\therefore \quad \frac{\text{Area of } \Delta PAB}{\text{Area of } \Delta RAC} + \frac{\text{Area of } \Delta QBC}{\text{Area of } \Delta RAC} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$

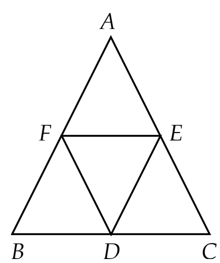
$= \frac{AB^2 + BC^2}{AC^2} = \frac{AC^2}{AC^2}$

[$\because$ ΔABC is a right triangle with $\angle B = 90°$
$\therefore AC^2 = AB^2 + BC^2$]

$\Rightarrow \quad \frac{\text{Area of } \Delta PAB + \text{Area of } \Delta QBC}{\text{Area of } \Delta RAC} = 1$

$\Rightarrow \quad$ Area of ΔPAB + Area of ΔQBC = Area of ΔRAC

30. Given, ΔABC in which D, E and F are the mid-points of sides BC, CA and AB respectively.
Since, F and E are mid-points of AB and AC respectively.



$\therefore \quad FE \parallel BC$
$\Rightarrow \quad \angle AFE = \angle B$ [Corresponding angles]
Thus, in ΔAFE and ΔABC, we have
$\angle AFE = \angle B$ [Proved above]
and $\angle A = \angle A$ [Common]

$\therefore \quad \Delta AFE \sim \Delta ABC$ [By AA similarity criterion]

Similarly, we have

$\Delta FBD \sim \Delta ABC$ and $\Delta EDC \sim \Delta ABC$.

Now, we shall prove that $\Delta DEF \sim \Delta ABC$.

Clearly, $ED \parallel AF$ and $DF \parallel EA$.

$\therefore \quad AFDE$ is a parallelogram.

$\Rightarrow \quad \angle EDF = \angle A$

[$\because$ Opposite angles of a parallelogram are equal]

Similarly, $BDEF$ is a parallelogram.

$\therefore \quad \angle DEF = \angle B$

Thus, in ΔDEF and ΔABC, we have

$\angle EDF = \angle A$ and $\angle DEF = \angle B$

$\therefore \quad \Delta DEF \sim \Delta ABC$ [By AA similarity criterion]

Thus, each one of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

31. In ΔDFG and ΔDAB,

$AB \parallel FE \Rightarrow \angle 1 = \angle 2$ [Corresponding angles]

$\angle FDG = \angle ADB$ [Common]

$\therefore \quad \Delta DFG \sim \Delta DAB$ [By AA similarity criterion]

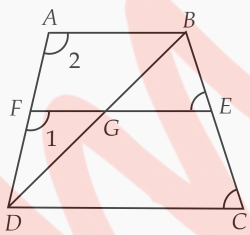
$$\Rightarrow \quad \frac{DF}{DA} = \frac{FG}{AB} \quad \text{...(i)}$$

In trapezium $ABCD$, we have

$EF \parallel AB \parallel DC$

$$\therefore \quad \frac{AF}{DF} = \frac{BE}{EC} \Rightarrow \frac{AF}{DF} = \frac{3}{4} \quad \left[\because \frac{BE}{EC} = \frac{3}{4}\text{(given)}\right]$$

$$\Rightarrow \quad \frac{AF}{DF} + 1 = \frac{3}{4} + 1$$



$$\Rightarrow \quad \frac{AF + DF}{DF} = \frac{3+4}{4}$$

$$\Rightarrow \quad \frac{AD}{DF} = \frac{7}{4} \Rightarrow \frac{DF}{AD} = \frac{4}{7} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{FG}{AB} = \frac{4}{7} \Rightarrow FG = \frac{4}{7}AB \quad \text{...(iii)}$$

In ΔBEG and ΔBCD,

$EF \parallel CD \Rightarrow \angle BEG = \angle BCD$ [Corresponding angles]

$\angle B = \angle B$ [Common]

$\therefore \quad \Delta BEG \sim \Delta BCD$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{BE}{BC} = \frac{EG}{CD} \Rightarrow \frac{3}{7} = \frac{EG}{CD}$$

$$\left[\because \frac{BE}{EC} = \frac{3}{4} \Rightarrow \frac{EC}{BE} = \frac{4}{3} \Rightarrow \frac{EC}{BE} + 1 = \frac{4}{3} + 1 \Rightarrow \frac{BC}{BE} = \frac{7}{3}\right]$$

$$\Rightarrow \quad EG = \frac{3}{7}CD \Rightarrow EG = \frac{3}{7} \times 2AB \quad [\because CD = 2AB \text{ (Given)}]$$

$$\Rightarrow \quad EG = \frac{6}{7}AB \quad \text{...(iv)}$$

Adding (iii) and (iv), we get

$$FG + EG = \frac{4}{7}AB + \frac{6}{7}AB \Rightarrow FE = \frac{10}{7}AB$$

$\Rightarrow \quad 7\,FE = 10\,AB$

32. We have $\dfrac{XP}{PY} = \dfrac{XQ}{QZ}$

$\therefore \quad PQ \parallel YZ$ [By converse of B.P.T.]

In ΔXPQ and ΔXYZ

$\angle XPQ = \angle XYZ$ [Corresponding angles]

$\angle X = \angle X$ [Common]

$\therefore \quad \Delta XPQ \sim \Delta XYZ$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{ar(\Delta XPQ)}{ar(\Delta XYZ)} = \frac{(XP)^2}{(XY)^2} = \frac{(XQ)^2}{(XZ)^2} = \frac{(PQ)^2}{(YZ)^2} \quad \text{...(i)}$$

[$\because$ The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides]

$$\text{Now, } \frac{XP}{PY} = \frac{XQ}{QZ} = \frac{3}{1} \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{PY}{XP} = \frac{QZ}{XQ} = \frac{1}{3} \Rightarrow \frac{PY}{XP} + 1 = \frac{QZ}{XQ} + 1 = \frac{1}{3} + 1$$

$$\Rightarrow \quad \frac{PY + XP}{XP} = \frac{QZ + XQ}{XQ} = \frac{1+3}{3}$$

$$\Rightarrow \quad \frac{XY}{XP} = \frac{XZ}{XQ} = \frac{4}{3} \Rightarrow \frac{XP}{XY} = \frac{XQ}{XZ} = \frac{3}{4} \quad \text{...(ii)}$$

From (i) and (ii), we have

$$\frac{ar(\Delta XPQ)}{ar(\Delta XYZ)} = \left(\frac{XP}{XY}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$\Rightarrow \quad ar(\Delta XPQ) = \frac{9}{16} \times ar(\Delta XYZ)$$

$$\Rightarrow \quad ar(\Delta XPQ) = \frac{9}{16} \times 32 \quad [\therefore ar(\Delta XYZ) = 32 \text{ cm}^2 \text{ (Given)}]$$

$\Rightarrow \quad ar(\Delta XPQ) = 18 \text{ cm}^2$

Now, $ar(\text{quad. } PYZQ) = ar(\Delta XYZ) - ar(\Delta XPQ)$

$= (32 - 18) \text{ cm}^2 = 14 \text{ cm}^2$

33. In ΔABC, we have $DE \parallel BC$

$\Rightarrow \quad \angle ADE = \angle ABC$ and $\angle AED = \angle ACB$

[Corresponding angles]

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC} \quad \text{...(i)}$$

$$\text{Now, } \frac{AD}{DB} = \frac{5}{4} \Rightarrow \frac{DB}{AD} = \frac{4}{5} \Rightarrow \frac{DB}{AD} + 1 = \frac{4}{5} + 1$$

$$\Rightarrow \quad \frac{DB + AD}{AD} = \frac{4+5}{5} \Rightarrow \frac{AB}{AD} = \frac{9}{5}$$

$$\Rightarrow \quad \frac{AD}{AB} = \frac{5}{9}$$

$$\Rightarrow \quad \frac{DE}{BC} = \frac{5}{9} \quad \text{[Using (i)]}$$

In ΔDEF and ΔCBF, we have

$\angle DFE = \angle CFB$ [Vertically opposite angles]

$\angle DEF = \angle FBC$ [Alternate angles]

$\therefore \quad \Delta DEF \sim \Delta CBF$ [By AA similarity criterion]

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta CBF)} = \frac{DE^2}{BC^2} = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$$

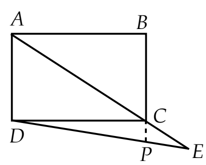
34. Given, $AB = 8$ cm and $BC = 6$ cm

$\therefore \quad AC = \sqrt{8^2 + 6^2} = 10$ cm

Also, $AC : CE = 2 : 1$ [Given]

Produce BC to meet DE at the point P.

Now $AD \parallel BC \Rightarrow AD \parallel CP$



In ΔECP and ΔEAD,

$\angle E = \angle E$ [Common]

$\angle ECP = \angle EAD$ [Corresponding angles]

$\therefore \quad \Delta ECP \sim \Delta EAD$ [By AA similarity criterion]

$\Rightarrow \quad \frac{CP}{AD} = \frac{CE}{AE} \Rightarrow \frac{CP}{6} = \frac{1}{3} \quad \left(\because \frac{CE}{AC} = \frac{1}{2} \Rightarrow \frac{CE}{AE} = \frac{1}{3}\right)$

$\Rightarrow \quad CP = 2$ cm

In right ΔCPD, by Pythagoras theroem

$$DP = \sqrt{CD^2 + CP^2} = \sqrt{64+4} = \sqrt{68} = 2\sqrt{17}\text{cm}$$

Now, in ΔEAD, $CP \parallel AD$

and $\frac{CE}{AC} = \frac{1}{2} \Rightarrow \frac{PE}{PD} = \frac{1}{2}$

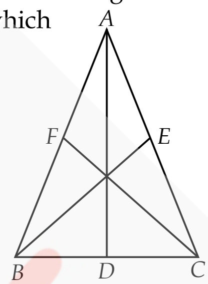
$\therefore \quad PE = \frac{1}{2} \times PD = \frac{1}{2} \times 2\sqrt{17} = \sqrt{17}\text{cm}$

So, $DE = DP + PE = 2\sqrt{17} + \sqrt{17} = 3\sqrt{17}\text{cm}$

35. Given, ΔABC in which AD, BE and CF are three medians.

Since, in any triangle, the sum of the squares of any two sides is equal to twice the sum of square of half of the third side and the square of the median bisecting it.

Therefore, taking AD as the median which bisects side BC, we have



$AB^2 + AC^2 = 2[AD^2 + BD^2]$

$\Rightarrow \quad AB^2 + AC^2 = 2\left[AD^2 + \left(\frac{1}{2}BC\right)^2\right]$

$\Rightarrow \quad AB^2 + AC^2 = 2\left[AD^2 + \frac{1}{4}BC^2\right]$

$\Rightarrow \quad AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$

$\Rightarrow \quad 2(AB^2 + AC^2) = 4AD^2 + BC^2$ ...(i)

Similarly, by taking BE and CF respectively as the medians, we get

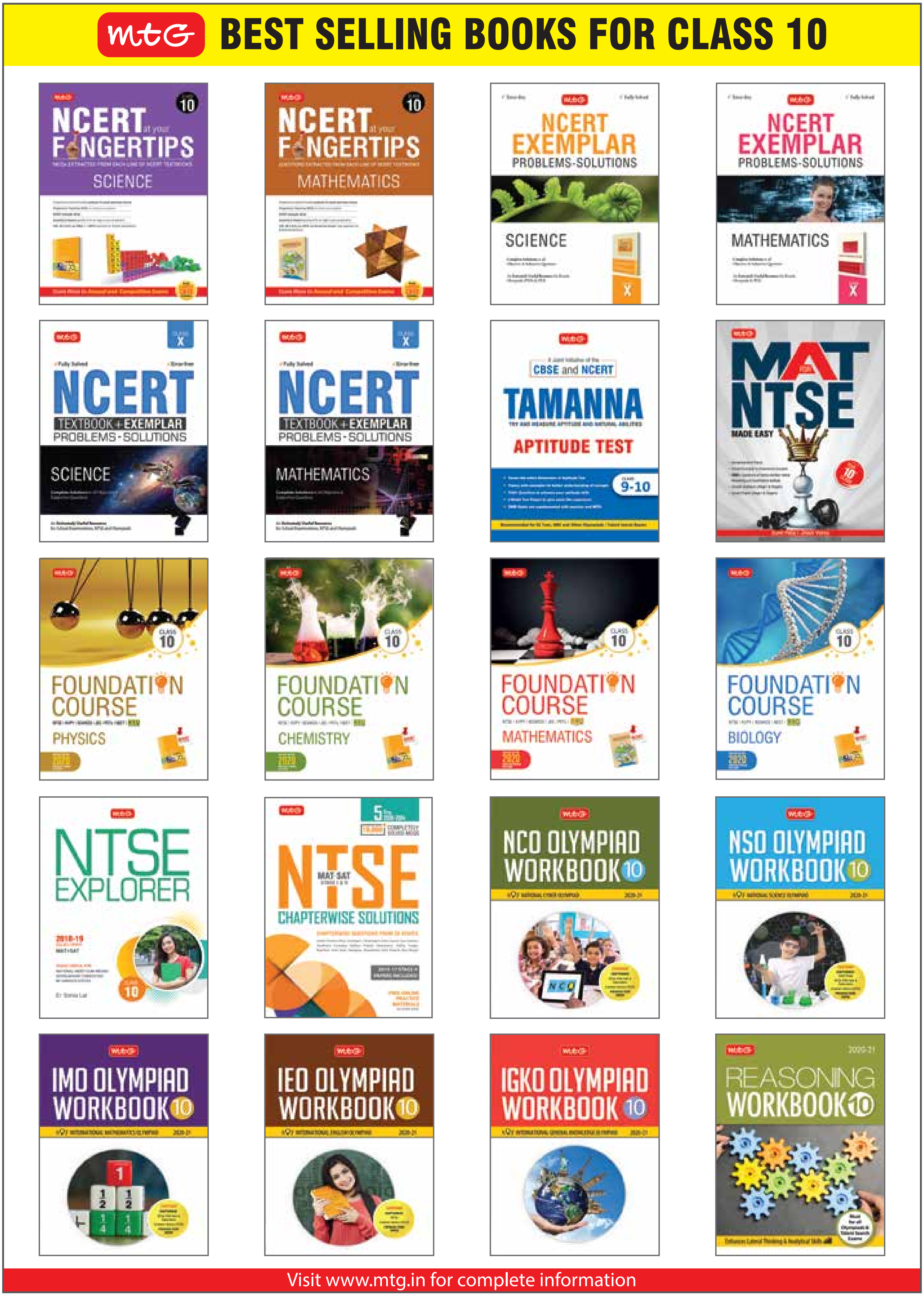
$2(AB^2 + BC^2) = 4BE^2 + AC^2$ ...(ii)

and $2(AC^2 + BC^2) = 4CF^2 + AB^2$ ...(iii)

Adding (i), (ii) and (iii), we get

$4(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2) + BC^2 + AC^2 + AB^2$

$\Rightarrow \quad 3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$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