Arithmetic Progressions

CHAPTER 5

EXAM DRILL

SOLUTIONS

1. **(c) :** Since, $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in A.P.

$\therefore \frac{1}{x+3}-\frac{1}{x+2}=\frac{1}{x+5}-\frac{1}{x+3}$

$\Rightarrow \frac{x+2-x-3}{(x+3)(x+2)}=\frac{x+3-x-5}{(x+5)(x+3)}$

$\Rightarrow \frac{-1}{(x+3)(x+2)}=\frac{-2}{(x+5)(x+3)} \Rightarrow \frac{-1}{x+2}=\frac{-2}{x+5}$

$\Rightarrow -x-5=-2x-4 \Rightarrow -x+2x=-4+5 \Rightarrow x=1$

2. **(d) :** Given A.P. is $5, \frac{19}{4}, \frac{9}{2}, \frac{17}{4}, ...$

Here, $a = 5$, $d=\frac{19}{4}-5=-\frac{1}{4}$

$\therefore$ 10^{th} term, $a_{10} = a + (10-1)d$

$=5+9\left(-\frac{1}{4}\right)=\frac{20-9}{4}=\frac{11}{4}$

3. **(d) :** Since, alternate terms of an A.P. also forms an A.P.

So, $(x-y)-(x+y)=(2x+3y)-(x-y)$

$\Rightarrow -2y = x + 4y \Rightarrow -2y-4y = x \Rightarrow x = -6y$

4. **(c) :** Given A.P. is 25, 50, 75, 100, ...

Here, $a = 25$, $d = 25$ and $a_k = 1000$

Now, $a_k = 1000$

$\Rightarrow a+(k-1)d = 1000 \Rightarrow 25+(k-1)25 = 1000$

$\Rightarrow 25+25k-25 = 1000 \Rightarrow 25k = 1000 \Rightarrow k = 40$

5. **(d) :** Let d be the same common difference of two A.P.s.

Given, first term of 1^{st} A.P., $a = 8$

First term of 2^{nd} A.P., $a_1 = 3$

Now, 30^{th} term of 1^{st} A.P. $= a + 29d = 8 + 29d$

Also, 30^{th} term of 2^{nd} A.P. $= a_1 + 29d = 3 + 29d$

$\therefore$ Required difference $= (8+29d)-(3+29d) = 5$

6. **(c) :** Given, first term, $a = 5$

common difference, $d = 6$

$\because S_n=\frac{n}{2}[2a+(n-1)d] \Rightarrow S_{40}=\frac{40}{2}[2(5)+(40-1)6]$

$= 20[10 + 234] = 20 \times 244 = 4880$

7. **(b) :** Given, first term, $a = 1$, last term $l = 13$ and $S_n = 42$

We know that, $S_n=\frac{n}{2}[a+l] \Rightarrow 42=\frac{n}{2}[1+13]$

$\Rightarrow 84 = n[14] \Rightarrow n=\frac{84}{14}=6$

8. Given, A.P. is $\sqrt{27}, \sqrt{48}, \sqrt{75},...$

i.e., A.P. is $3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3},...$

Clearly, first term, $a = 3\sqrt{3}$

Second term, $a + d = 4\sqrt{3}$

$\therefore$ Common difference, $d = 4\sqrt{3}-3\sqrt{3}=\sqrt{3}$

9. Since, $\frac{7}{8}, a, 3$ are three consecutive terms of an A.P.

So, $a-\frac{7}{8}=3-a \Rightarrow 2a=3+\frac{7}{8}$

$\Rightarrow 2a=\frac{24+7}{8}=\frac{31}{8} \Rightarrow a=\frac{31}{16}$

10. Given, common difference, $d = -6$

Let a be the first term of the A.P.

Given, $a_9 = 5$

$\Rightarrow a+(9-1)\times(-6) = 5 \Rightarrow a-48 = 5 \Rightarrow a = 53$

Hence, first term of A.P. is 53.

11. Given, common difference, $d = 3$ be the first term of the A.P.

Now, $a_{15}-a_9 = [a+(15-1)d]-[a+(9-1)d]$

$=14d-8d = 6d = 6 \times 3 = 18$ $[\because d = 3]$

12. Let d be the common difference of the A.P.

Given, first term $= a$ and n^{th} term, $a_n = b$

$\Rightarrow a+(n-1)d = b \Rightarrow (n-1)d = b-a \Rightarrow d=\frac{b-a}{n-1}$

13. $\because \frac{1}{yz}, \frac{1}{zx}$ and $\frac{1}{xy}$ are in A.P.

$\Rightarrow \frac{1}{zx}-\frac{1}{yz}=\frac{1}{xy}-\frac{1}{zx} \Rightarrow \frac{y-x}{xyz}=\frac{z-y}{xyz}$

$\Rightarrow y-x=z-y \Rightarrow y=\frac{x+z}{2}$

$\therefore$ x, y and z are in A.P.

14. Given that, $a = 4$ and $d=\frac{4}{3}$.

$\because S_n=\frac{n}{2}[2a+(n-1)d]$

$\therefore S_{22}=\left(\frac{22}{2}\right)\left[(2)(4)+(22-1)\left(\frac{4}{3}\right)\right]=(11)(8+28)=396$

15. Given, $a_n = 2n + 5$

$\therefore a_1 = 2(1)+5 = 7, a_2 = 2(2)+5 = 9,$

$a_3 = 2(3)+5 = 11, a_4 = 2(4)+5 = 13$

$\therefore S_4 = a_1+a_2+a_3+a_4 = 7+9+11+13 = 40$

16. Let a and d are respectively the first term and common difference of the given A.P.

Given, $a_4 = a + 3d = 11$ ...(i)

Also, $a_5 + a_7 = 34$ [Given]

$\Rightarrow [a+4d]+[a+6d] = 34$

$\Rightarrow$ $2a + 10d = 34 \Rightarrow 2(11 - 3d) + 10d = 34$ [Using (i)]

$\Rightarrow$ $22 - 6d + 10d = 34 \Rightarrow 4d = 12 \Rightarrow d = 3$

17. The first 10 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

It is an A.P. with first term, $a = 3$ and common difference, $d = 3$.

$\therefore$ Sum of first 10 multiples of 3 = S_{10}

$= \frac{10}{2}\{2 \times 3 + (10-1) \times 3\}$ $\left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$

$= 5(6 + 27) = 5 \times 33 = 165$

18. Here A.P. is $\sqrt{6}, \sqrt{24}, \sqrt{54}, \sqrt{96}, \ldots$ then

$a = \sqrt{6}, d = \sqrt{24} - \sqrt{6} = 2\sqrt{6} - \sqrt{6} = \sqrt{6}$.

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2\sqrt{6} + (n-1)\sqrt{6}]$

$= \frac{n}{2}\left[\sqrt{6}\,n + \sqrt{6}\right] = \frac{\sqrt{6}n(n+1)}{2}$

19. Here, first term $a = 7$

Common difference, $d = 13 - 7 = 6$

Let the given A.P. contains n terms, then

$a_n = 187$ ($\because$ Given) $\Rightarrow a + (n-1)d = 187$

$\Rightarrow$ $7 + (n-1)6 = 187 \Rightarrow (n-1)6 = 180$

$\Rightarrow$ $n - 1 = 30 \Rightarrow n = 30 + 1 = 31$

Thus, the given A.P. contains 31 terms.

Here $n = 31$ (odd number)

$\therefore$ Middle term $= \frac{1}{2}(n+1)^{\text{th}}$.

$= \frac{1}{2}(31+1)^{\text{th}} = \left(\frac{1}{2} \times 32\right)^{\text{th}} = 16^{\text{th}}$

Hence, middle term, a_{16}

$= a + 15d = 7 + 15 \times 6 = 7 + 90 = 97$

20. The given A.P. is 27, 23, 19,..., –65.

Here, first term, $a = 27$, common difference, $d = 23 - 27 = -4$, last term, $l = -65$

Now, n^{th} term from the end $= l - (n-1)d$

$\therefore$ 11^{th} term from the end $= -65 - (11-1)(-4)$

$= -65 - (10)(-4) = -65 + 40 = -25$

Hence, the 11^{th} term from the end is –25.

21. Given, A.P. is 115, 110, 105, ...

Here, $a = 115$, $d = 110 - 115 = -5$

Let n^{th} term of the given A.P. be the first negative term.

i.e., $a_n < 0 \Rightarrow a + (n-1)d < 0$

$\Rightarrow$ $115 + (n-1)(-5) < 0$

$\Rightarrow$ $115 - 5n + 5 < 0 \Rightarrow 120 - 5n < 0$

$\Rightarrow$ $5n > 120 \Rightarrow n > 24 \Rightarrow n \geq 25$

$\therefore$ 25^{th} term of the given A.P. will be the first negative term.

22. Let $a = 2$ be the first term and d be the common difference of the A.P.

Given, 10^{th} term of the A.P. is 47.

$\therefore$ $a_{10} = 2 + (10-1)d$ $[\because a_n = a + (n-1)d]$

$\Rightarrow$ $47 = 2 + 9d \Rightarrow 9d = 45 \Rightarrow d = 5$

Now, $S_{15} = \frac{15}{2}[2a + (15-1)d]$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= \frac{15}{2}[2 \times 2 + 14 \times 5] = \frac{15}{2}[4 + 70] = \frac{15}{2} \times 74 = 15 \times 37 = 555$

Hence, the sum of 15 terms of the given A.P. is 555.

23. Given A.P. is 3, 7, 11, 15, ...

Here, $a = 3$, $d = 7 - 3 = 4$

Let sum of n terms is 406.

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $406 = \frac{n}{2}[2(3) + (n-1)(4)]$

$\Rightarrow$ $406 = n[1 + 2n] \Rightarrow 2n^2 + n - 406 = 0$

$\Rightarrow$ $2n^2 + 29n - 28n - 406 = 0$

$\Rightarrow$ $n(2n + 29) - 14(2n + 29) = 0 \Rightarrow (n - 14)(2n + 29) = 0$

$\Rightarrow$ $n = 14$ [Since, n can't be a fraction]

24. Given, $S_n = 2n^2 + 3n$

We know that, $a_n = S_n - S_{n-1}$

$\therefore$ $a_{16} = S_{16} - S_{15} = [2(16)^2 + 3(16)] - [2(15)^2 + 3(15)]$

$= [2(256) + 3(16)] - [2(225) + 3(15)]$

$= [512 + 48] - [450 + 45] = 560 - 495 = 65$

25. Sum of all natural numbers from 1 to 1000 which are not divisible by 5 = (Sum of all natural numbers from 1 to 1000, S_n) – (Sum of all natural numbers from 1 to 1000 which are divisible by 5, S_n')

Now, all the natural numbers from 1 to 1000 are 1, 2, 3, …, 1000, which is an A.P. where $a = 1$, $l = 1000$ and $n = 1000$

$\therefore$ $S_n = \frac{n}{2}[a + l]$

$= \frac{1000}{2}[1 + 1000] = 500 \times 1001 = 500500$ ...(i)

Again, all the natural numbers from 1 to 1000 which are divisible by 5 are 5, 10, 15, ..., 1000, which is also an A.P. where, first term, $a = 5$, last term, $l = 1000$ and common difference, $d = 5$

$\because$ $a_n = a + (n-1)d \Rightarrow 1000 = 5 + (n-1)5$

$\Rightarrow$ $5n = 1000 \Rightarrow n = 200$

$\therefore$ $S_n' = \frac{n}{2}[a + l] = \frac{200}{2}[5 + 1000] = 100 \times 1005 = 100500$ ...(ii)

$\therefore$ Required sum $= S_n - S'_n = 500500 - 100500 = 400000$

26. Given, $a = 100$

Let d be the common difference of the A.P.

According to the question,

$100 + (100 + d) + (100 + 2d) + (100 + 3d) + (100 + 4d) + (100 + 5d) = 5[(100 + 6d) + (100 + 7d) + (100 + 8d) + (100 + 9d) + (100 + 10d) + (100 + 11d)]$

$\Rightarrow$ $600 + 15d = 5(600 + 51d)$

$\Rightarrow$ $120 + 3d = 600 + 51d \Rightarrow -48d = 480 \Rightarrow d = -10$

27. Let a and d be the first term and common difference of the A.P.

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

Consider, $S_{10} - S_5 = \frac{10}{2}[2a + 9d] - \frac{5}{2}[2a + 4d]$

$= 5(2a + 9d) - 5(a + 2d)$

$= 5[2a + 9d - a - 2d] = 5(a + 7d)$ ... (i)

Also, $S_{15} = \frac{15}{2}[2a + 14d] = 15(a + 7d) = 3[5(a + 7d)]$

$= 3(S_{10} - S_5)$ (From (i))

28. Let a be the first term and d be the common difference of given A.P. Then

$(p + q)^{th}$ term, $a_{p+q} = a + (p + q - 1)d$ ...(i)

and $(p - q)^{th}$ term, $a_{p-q} = a + (p - q - 1)d$ ...(ii)

$\therefore$ Sum of $(p + q)^{th}$ and $(p - q)^{th}$ terms $= a_{p+q} + a_{p-q}$

$= [a + (p + q - 1)d] + [a + (p - q - 1)d]$ [Using (i) and (ii)]

$= 2a + (p + q - 1 + p - q - 1)d$

$= 2a + (2p - 2)d = 2 \times [a + (p - 1)d] = 2 \times p^{th}$ term of the A.P.

29. Given, A.P. is 8, 10, 12,...

Here, first term, $a = 8$ and common difference $(d) = 10 - 8 = 2$

If the given A.P. has a total 60 terms, then

$a_{60} = a + 59d$ $[\because a_n = a + (n - 1)d]$

$= 8 + 59 \times 2 = 8 + 118 = 126$

Sum of the last 10 terms of the given A.P.

$= a_{51} + a_{52} + ... + a_{60} = (a + 50d) + (a + 51d) + ... + 126$

$= (8 + 100) + (8 + 102) + ... + 126 = 108 + 110 + ... + 126$

$= \frac{10}{2}[108 + 126]$ $\left[\because S_n = \frac{n}{2}(\text{First term} + \text{Last term})\right]$

$= 5 \times 234 = 1170$

30. Let the four parts be $(a - 3d), (a - d), (a + d), (a + 3d)$.

Sum of the numbers = 56

$\Rightarrow (a - 3d) + (a - d) + (a + d) + (a + 3d) = 56$

$\Rightarrow 4a = 56 \Rightarrow a = 14$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$ [Given] $\Rightarrow \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{5}{6}$

$\Rightarrow 6(196 - 9d^2) = 5(196 - d^2)$ $[\because a = 14]$

$\Rightarrow 6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$

$\Rightarrow 49d^2 = 6 \times 196 - 5 \times 196 \Rightarrow 49d^2 = 196$

$\Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

$\therefore$ Required four parts are

$(14 - 3 \times 2), (14 - 2), (14 + 2), (14 + 3 \times 2)$

or $[(14 - 3(-2)], (14 + 2), (14 - 2), [(14 + 3(-2)]$.

i.e., 8, 12, 16, 20 or 20, 16, 12, 8

31. The given sequence is 12000, 16000, 20000, ... , which is an A.P.

Here first term, $a = 12000$, common difference, $d = 4000$, $S_n = 1000000$

Let the man saves ₹ 1000000 in n years.

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 1000000 = \frac{n}{2}[2 \times 12000 + (n-1)4000]$

$\Rightarrow 1000 = \frac{n}{2}[24 + 4n - 4] \Rightarrow 1000 = \frac{n}{2} \times 4(n + 5)$

$\Rightarrow 500 = n^2 + 5n \Rightarrow n^2 + 5n - 500 = 0$

$\Rightarrow n^2 + 25n - 20n - 500 = 0 \Rightarrow (n + 25)(n - 20) = 0$

$\Rightarrow n = 20$ (as n can't be negative)

$\therefore$ Man saves ₹ 1000000 in 20 years.

32. Total amount of ten prizes = ₹1600

Let the value of first prize be ₹ x

According to the question, prizes are

$x, x - 20, x - 40$ … to 9 terms

Here, $a = x$, $d = -20$ and $n = 10$

$\because S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 1600 = \frac{10}{2}[2x + (10-1)(-20)] = 10(x - 90)$

$\Rightarrow 160 = x - 90 \Rightarrow x = 160 + 90 = 250$

Hence, amount of each prize (in ₹) are 250, 230, 210, ..., 70.

33. Let a and d are respectively the first term and common difference of an A.P.:, $a, a + d, a + 2d$,...

Given, 14^{th} term of an A.P. is twice its 8^{th} term.

$\therefore a_{14} = 2a_8 \Rightarrow a + (14 - 1)d = 2[a + (8 - 1)d]$

$\Rightarrow a + 13d = 2a + 14d \Rightarrow 2a - a = (13 - 14)d$

$\Rightarrow a = -d$ ...(i)

Also, $a_6 = -8$ (Given)

$\Rightarrow a + (6 - 1)d = -8 \Rightarrow -d + 5d = -8$ [Using (i)]

$\Rightarrow 4d = -8 \Rightarrow d = -2$

From (i), $a = -(-2) = 2$

Therefore, the A.P. is 2, 2 + (–2), 2 + 2(–2), 2 + 3(–2),...*i.e.*, 2, 0, –2, –4,...

$\because S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{20} = \frac{20}{2}[2 \times 2 + (20-1)(-2)]$

$= 10[4 - 38] = 10(-34) = -340$

34. Here, 8 and 20 are the first term and common difference respectively of an A.P.

$\therefore S_n = \frac{n}{2}[2(8) + (n-1)20] = 8n + 10n^2 - 10n$

$= 10n^2 - 2n$ ...(i)

Also, –30 and 8 are the first term and common difference respectively of another A.P.

$\therefore S_{2n} = \frac{2n}{2}[2(-30) + (2n-1)8]$

$= -60n + 16n^2 - 8n = 16n^2 - 68n$ ...(ii)

According to the question, $S_n = S_{2n}$

$\Rightarrow 16n^2 - 68n = 10n^2 - 2n$ [From (i) and (ii)]

$\Rightarrow 16n^2 - 10n^2 - 68n + 2n = 0$

$\Rightarrow 6n^2 - 66n = 0 \Rightarrow 6n(n - 11) = 0$

$\Rightarrow$ Either $n - 11 = 0$ or $n = 0 \Rightarrow n = 11$ or $n = 0$

$\because n = 0$ is not possible.

Hence, value of n is 11.

35. Consider the sequence, 2, 5, 8, 11, ..., x, which is an A.P.

Here, $a = 2$, $d = 3$, $a_n = x$

$\because a_n = a + (n - 1)d \Rightarrow x = 2 + (n - 1)3$

$\Rightarrow x = 2 + 3n - 3 \Rightarrow x + 1 = 3n \Rightarrow n = \frac{x+1}{3}$

$\because S_n = \frac{n}{2}[a + l] \Rightarrow 345 = \frac{x+1}{3 \times 2}[2 + x]$ [Given, $S_n = 345$]

$\Rightarrow (x + 1)(x + 2) = 2070 \Rightarrow x^2 + 3x - 2068 = 0$

$\Rightarrow \quad x = \frac{-3 \pm \sqrt{9+8272}}{2} = \frac{-3 \pm \sqrt{8281}}{2} = \frac{-3 \pm 91}{2} = 44, -47$

Since, the given A.P. is an increasing A.P. with $a = 2$ and $d = 3$, so x can't be negative.

$\therefore \quad x = 44$

36. Let $a = 8$ years be the first term of the A.P.

i.e., age of the youngest boy participating in a painting competition.

Common difference, d *i.e.*, age difference of the participants = 4 months (given)

$= \frac{4}{12}$ year $= \frac{1}{3}$ year

Let n be the total number of participants in the painting competition and S_n denotes the sum of ages of all the participants. Then, $S_n = 168$ years (given)

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 168 = \frac{n}{2}\left[2 \times 8 + (n-1)\left(\frac{1}{3}\right)\right]$

$\Rightarrow \quad 336 = n\left[16 + (n-1)\left(\frac{1}{3}\right)\right]$

$\Rightarrow \quad 336 \times 3 = n[48 + (n-1)] \Rightarrow 1008 = 48n + n(n-1)$

$\Rightarrow \quad 1008 = 48n + n^2 - n \Rightarrow n^2 + 47n - 1008 = 0$

$\Rightarrow \quad n^2 + 63n - 16n - 1008 = 0 \Rightarrow n(n+63) - 16(n+63) = 0$

$\Rightarrow \quad (n-16)(n+63) = 0$

$\Rightarrow$ Either $n - 16 = 0$ or $n + 63 = 0$

$\Rightarrow$ Either $n = 16$ or $n = -63$

$\Rightarrow$ $n = 16$, rejecting $n = -63$ as n can't be negative.

$\therefore$ Age of eldest participant is a_{16}.

Now, $a_{16} = 8 + (16-1) \times \frac{1}{3}$ $\quad [\because a_n = a + (n-1)d]$

$= 8 + \frac{15}{3} = 8 + 5 = 13$ years

Hence, the total number of participants are 16 and the age of the eldest participant is 13 years.

37. Original cost of house = ₹2200000

Amount paid in cash = ₹400000

Balance to be paid = ₹(2200000 – 400000) = ₹1800000

Amount paid in each installment = ₹100000

$\therefore$ Number of installments = 18

Interest paid with 1st installment $= 1800000 \times \frac{10}{100}$

= ₹ 180000

Interest paid with 2nd installment $= 1700000 \times \frac{10}{100}$

= ₹ 170000

and so on ....

Interest paid with last installment $= 100000 \times \frac{10}{100}$

= ₹ 10000

Total interest paid = (180000 + 170000 + ... + 10000), which is an A.P. with first term, $a = 180000$, last term, $l = 10000$.

$= \frac{18}{2}[180000 + 10000]$ $\quad \left[\because S_n = \frac{n}{2}(a+l)\right]$

= 9[190000] = ₹ 1710000

$\therefore$ Total cost of house for Ronit

= ₹ (2200000 + 1710000) = ₹ 3910000

38. Since, the A.P. consists of 37 terms, so 19th term is the middle term.

Let $a_{19} = a$ and d be the common difference of the A.P.

The A.P. is ; $a - 18d, a - 17d, \ldots, a - d, a, a + d, \ldots, a + 17d, a + 18d$

Sum of the three middle most terms = 225

$\Rightarrow \quad (a - d) + a + (a + d) = 225$

$\Rightarrow \quad 3a = 225 \Rightarrow a = 75$ $\quad$ …(i)

Sum of the three last terms = 429

$\Rightarrow \quad (a + 18d) + (a + 17d) + (a + 16d) = 429$

$\Rightarrow \quad 3a + 51d = 429 \Rightarrow a + 17d = 143$

$\Rightarrow \quad 17d = 143 - a = 143 - 75$ $\quad$ (Using (i))

$\Rightarrow \quad 17d = 68 \Rightarrow d = \frac{68}{17} = 4$

Now, first term = $a - 18d = 75 - 18 \times 4 = 3$

$\therefore$ The A.P. is 3, 7, 11, ..., 147.