

Quadratic Equations

EXAM DRILL

SOLUTIONS

1. (b) : Given α and β be roots of the equation $kx^2 + bx + c = 0$.

$$\text{We have, } \alpha = \frac{-b + \sqrt{b^2 - 12c}}{6} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 12c}}{6}$$

$$\therefore 2k = 6 \Rightarrow k = 3$$

2. (b) : We have, $9x^2 + 3px + 4 = 0$

Here, $a = 9$, $b = 3p$ and $c = 4$.

$$\therefore D = b^2 - 4ac = (3p)^2 - 4(9)(4) = 9p^2 - 144$$

The equation has real and equal roots, so $D = 0$

$$\Rightarrow 9p^2 - 144 = 0 \Rightarrow p^2 = \frac{144}{9} \Rightarrow p^2 = 16$$

$$\Rightarrow p = \pm 4$$

3. (d) : We have, $m^2x^2 + 2mcx = (a^2 - c^2) - x^2$

$$\Rightarrow (m^2 + 1)x^2 + 2mcx - a^2 + c^2 = 0$$

Here, $A = m^2 + 1$, $B = 2mc$ and $c = -a^2 + c^2$.

$$\begin{aligned} \therefore D &= B^2 - 4AC = 4m^2c^2 - 4(m^2 + 1)(c^2 - a^2) \\ &= 4m^2c^2 - 4m^2c^2 + 4a^2m^2 - 4c^2 + 4a^2 = 4(a^2 - c^2 + a^2m^2) \end{aligned}$$

Since, the equation has equal roots, so $D = 0$

$$\Rightarrow 4(a^2 - c^2 + a^2m^2) = 0 \Rightarrow c^2 = a^2(1 + m^2)$$

4. (b) : Let $x = \sqrt{20} + \sqrt{20} + \sqrt{20} + \dots \infty \Rightarrow x = \sqrt{20 + x}$

Squaring on both sides, we get

$$x^2 = 20 + x \Rightarrow x^2 - x - 20 = 0$$

$$\Rightarrow (x - 5)(x + 4) = 0 \Rightarrow x = 5 \text{ or } x = -4$$

But x is a positive quantity.

$$\therefore x = 5$$

5. (b) : Given, $a^2x^2 - (a^2b^2 + 1)x + b^2 = 0$

$$\Rightarrow a^2x^2 - a^2b^2x - x + b^2 = 0 \Rightarrow a^2x(x - b^2) - 1(x - b^2) = 0$$

$$\Rightarrow (a^2x - 1)(x - b^2) = 0$$

$$\Rightarrow a^2x - 1 = 0 \text{ or } x - b^2 = 0 \Rightarrow x = 1/a^2 \text{ or } x = b^2$$

$\therefore 1/a^2, b^2$ are the required roots.

6. (b) : We have, $21x^2 - 2x + 1/21 = 0$

$$\Rightarrow 441x^2 - 42x + 1 = 0$$

Here, $a = 441$, $b = -42$ and $c = 1$.

$$\therefore D = b^2 - 4ac = (-42)^2 - 4(441)(1) = 1764 - 1764 = 0$$

Hence, both roots are real and repeated.

7. We have, $x(x + 2c) = -ab \Rightarrow x^2 + 2cx + ab = 0$... (i)

(i) has real and unequal roots, so $D = b^2 - 4ac > 0$

$$\Rightarrow 4c^2 - 4ab > 0 \Rightarrow c^2 > ab$$

Also, we have $x^2 - 2(a + b)x + 2c^2 + a^2 + b^2 = 0$... (ii)

Here, $D = 4(a + b)^2 - 4(2c^2 + a^2 + b^2)$

$$= 4(a^2 + b^2 + 2ab - 2c^2 - a^2 - b^2) = 8(ab - c^2) < 0 \quad [\because c^2 > ab]$$

So, (ii) has no real roots.

8. For equal roots, discriminant = 0

$$\therefore (k + 1)^2 - 4(k + 4)(1) = 0$$

$$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0 \Rightarrow k^2 - 2k - 15 = 0$$

$$\Rightarrow (k - 5)(k + 3) = 0 \Rightarrow k = 5 \text{ or } k = -3$$

9. Given, $x = 1$ is root of the given equation, so it will satisfy the given equation.

$$\therefore a(1)^2 - 5(a - 1) \times 1 - 1 = 0$$

$$\Rightarrow a - 5a + 5 - 1 = 0 \Rightarrow -4a = -4 \Rightarrow a = \frac{-4}{-4} = 1$$

10. We have, $p^2q^2x^2 - q^2x - p^2x + 1 = 0$

$$\Rightarrow q^2x(p^2x - 1) - 1(p^2x - 1) = 0$$

$$\Rightarrow (p^2x - 1)(q^2x - 1) = 0 \Rightarrow x = \frac{1}{p^2} \text{ or } x = \frac{1}{q^2}$$

11. Let the numbers be x and $(x + 4)$.

According to the question, $x(x + 4) = 45$

$$\Rightarrow x^2 + 4x - 45 = 0 \Rightarrow x^2 + 9x - 5x - 45 = 0$$

$$\Rightarrow x(x + 9) - 5(x + 9) = 0$$

$$\Rightarrow (x + 9)(x - 5) = 0 \Rightarrow x + 9 = 0 \text{ or } x - 5 = 0$$

$$\Rightarrow x = -9 \text{ or } x = 5$$

If $x = -9$, numbers are $-9, -9 + 4$ i.e., $-9, -5$

If $x = 5$, numbers are $5, 5 + 4$ i.e., $5, 9$

12. Let the number be x .

According to question, $x + 2x^2 = 21$

$$\Rightarrow 2x^2 + x - 21 = 0 \Rightarrow 2x^2 - 6x + 7x - 21 = 0$$

$$\Rightarrow 2x(x - 3) + 7(x - 3) = 0$$

$$\Rightarrow (x - 3)(2x + 7) = 0 \Rightarrow x = 3 \text{ or } x = \frac{-7}{2}$$

13. The given quadratic equation is $3x^2 + 7x + k = 0$... (i)

Here, $a = 3$, $b = 7$ and $c = k$.

$$\therefore D = b^2 - 4ac = (7)^2 - 4(3)(k) = 49 - 12k$$

\therefore Equation (i) has real and equal roots, so $D = 0$.

$$\Rightarrow 49 - 12k = 0 \Rightarrow 12k = 49 \Rightarrow k = \frac{49}{12}$$

14. The given quadratic equation is

$$x(x - 4) + p = 0 \Rightarrow x^2 - 4x + p = 0$$

Here, $a = 1$, $b = -4$ and $c = p$.

For real and equal roots : $D = b^2 - 4ac = 0$

$$\Rightarrow (-4)^2 - 4(1)(p) = 0$$

$$\Rightarrow 16 - 4p = 0 \Rightarrow 4p = 16 \Rightarrow p = 4$$

15. Since 2 is a root of the equation $x^2 + kx + 12 = 0$.

$$\therefore (2)^2 + k(2) + 12 = 0 \Rightarrow 4 + 2k + 12 = 0 \Rightarrow 2k + 16 = 0$$

$$\Rightarrow k = -16/2 \Rightarrow k = -8$$

Putting $k = -8$ in the equation $x^2 + kx + q = 0$, we get

$$x^2 - 8x + q = 0 \quad \dots(i)$$

The equation (i) will have equal roots, if discriminant = 0

$$\Rightarrow (-8)^2 - 4(1)q = 0$$

$$\Rightarrow 64 - 4q = 0 \Rightarrow q = 64/4 \Rightarrow q = 16$$

16. We have, $x^2 - x + 2 = 0$

Here, $a = 1$, $b = -1$ and $c = 2$

$$\therefore D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$$

\therefore The given quadratic equation does not have real roots.

17. Let $\triangle ABC$ is the given triangle.

Let base, $BC = x$ cm, then altitude, $AB = (x + 8)$ cm

By Pythagoras theorem, we have

$$(AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (x + 8)^2 + x^2 = 40^2$$

$$\Rightarrow x^2 + 64 + 16x + x^2 = 1600$$

$$\Rightarrow 2x^2 + 16x - 1536 = 0$$

$$\Rightarrow x^2 + 8x - 768 = 0$$

$$\Rightarrow x^2 + 32x - 24x - 768 = 0 \Rightarrow x(x + 32) - 24(x + 32) = 0$$

$$\Rightarrow (x + 32)(x - 24) = 0 \Rightarrow x = -32 \text{ or } x = 24$$

But side of a triangle can't be negative.

$$\therefore x = 24$$

18. Let the first part be x , then the second part will be $12 - x$.

According to the given condition,

$$x^2 + (12 - x)^2 = 74 \Rightarrow x^2 + 144 + x^2 - 24x - 74 = 0$$

$$\Rightarrow 2x^2 - 24x + 70 = 0 \Rightarrow x^2 - 12x + 35 = 0$$

$$\Rightarrow x^2 - 7x - 5x + 35 = 0 \Rightarrow x(x - 7) - 5(x - 7) = 0$$

$$\Rightarrow (x - 7)(x - 5) = 0 \Rightarrow x - 7 = 0 \text{ or } x - 5 = 0$$

$$\Rightarrow x = 7 \text{ or } x = 5$$

\therefore Two parts of 12 are 7 and 5.

19. Let one number be x , then other number will be $x - 7$.

According to question, $x(x - 7) = 408 \Rightarrow x^2 - 7x - 408 = 0$

$$\Rightarrow x^2 - 24x + 17x - 408 = 0 \Rightarrow x(x - 24) + 17(x - 24) = 0$$

$$\Rightarrow (x - 24)(x + 17) = 0 \Rightarrow x = 24 \text{ or } x = -17 \text{ (rejected)}$$

Thus, one number is 24 and other number is 17.

Sum of numbers = $24 + 17 = 41$

20. Given, $4x^2 - 2(c + 1)x + (c + 4) = 0$

Here, $A = 4$, $B = -2(c + 1)$ and $C = c + 4$

Now, $D = B^2 - 4AC$

$$= \{-2(c + 1)\}^2 - 4 \times 4 \times (c + 4) = 4(c^2 + 2c + 1) - 16(c + 4)$$

$$= 4c^2 + 8c + 4 - 16c - 64 = 4c^2 - 8c - 60$$

For equal roots, $D = 0$

$$\therefore 4c^2 - 8c - 60 = 0 \Rightarrow c^2 - 2c - 15 = 0$$

$$\Rightarrow (c + 3)(c - 5) = 0 \Rightarrow c = -3 \text{ or } c = 5$$

$$21. \text{ Given, } \frac{1}{2a + b + 2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\Rightarrow \frac{1}{2a + b + 2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$

$$\Rightarrow \frac{2x - 2a - b - 2x}{2x(2a + b + 2x)} = \frac{b + 2a}{2ab}$$

$$\Rightarrow \frac{-(2a + b)}{2x(2a + b + 2x)} = \frac{b + 2a}{2ab} \Rightarrow \frac{-1}{x(2a + b + 2x)} = \frac{1}{ab}$$

$$\Rightarrow 2x^2 + 2ax + bx + ab = 0 \Rightarrow 2x(x + a) + b(x + a) = 0$$

$$\Rightarrow (x + a)(2x + b) = 0 \Rightarrow x = -a \text{ or } x = \frac{-b}{2}$$

$$22. \text{ Given, } \frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2} \Rightarrow \frac{6x-6-2x}{x(x-1)} = \frac{1}{x-2}$$

$$\Rightarrow \frac{4x-6}{x^2-x} = \frac{1}{x-2} \Rightarrow 4x^2 - 6x - 8x + 12 = x^2 - x$$

$$\Rightarrow 4x^2 - 14x + 12 = x^2 - x$$

$$\Rightarrow 3x^2 - 13x + 12 = 0 \Rightarrow 3x^2 - 9x - 4x + 12 = 0$$

$$\Rightarrow 3x(x - 3) - 4(x - 3) = 0 \Rightarrow (x - 3)(3x - 4) = 0$$

$$\Rightarrow x - 3 = 0 \text{ or } 3x - 4 = 0$$

$$\Rightarrow x = 3 \text{ or } x = 4/3$$

23. Let the number of persons in 1st condition is x and in 2nd condition is $(x + 15)$.

Amount to be divided = ₹ 6500

$$\text{According to the question, } \frac{6500}{x} - \frac{6500}{x+15} = 30$$

$$\Rightarrow \frac{6500x + 97500 - 6500x}{x(x+15)} = \frac{30}{1}$$

$$\Rightarrow 30x^2 + 450x = 97500 \Rightarrow 30x^2 + 450x - 97500 = 0$$

$$\Rightarrow x^2 + 15x - 3250 = 0 \Rightarrow x^2 + 65x - 50x - 3250 = 0$$

$$\Rightarrow x(x + 65) - 50(x + 65) = 0 \Rightarrow (x + 65)(x - 50) = 0$$

$$\Rightarrow x + 65 = 0 \text{ or } x - 50 = 0 \Rightarrow x = -65 \text{ or } x = 50$$

\therefore Number of persons cannot be negative

\therefore Original number of persons = 50.

24. Let the length of one side of garden be x m and other side be y m. Then,

$$x + y + x = 30$$

$$\Rightarrow y = 30 - 2x \quad \dots(i)$$

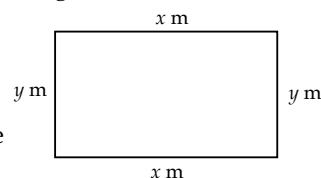
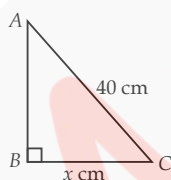
Given, area of the vegetable

garden = 100 m^2

$$\Rightarrow xy = 100$$

$$\Rightarrow x(30 - 2x) = 100$$

[Using (i)]



$$\begin{aligned} \Rightarrow 30x - 2x^2 &= 100 \Rightarrow 15x - x^2 = 50 \\ \Rightarrow x^2 - 15x + 50 &= 0 \Rightarrow x^2 - 10x - 5x + 50 = 0 \\ \Rightarrow (x - 10)(x - 5) &= 0 \Rightarrow x = 5 \text{ or } 10 \end{aligned}$$

When $x = 5$, then $y = 30 - 2 \times 5 = 20$ [Using (i)]

When $x = 10$, then $y = 30 - 2 \times 10 = 10$ [Using (i)]

Hence, the dimensions of the vegetable garden are 5 m and 20 m or 10 m and 10 m.

25. Let x be the total number of students of the class.

Number of students opted for visiting an old age home $= \frac{3}{8}x$.

Number of students opted for having a nature walk = 16.

Number of students opted for tree plantation in the school $= \sqrt{x}$.

According to the given condition,

$$\begin{aligned} \frac{3}{8}x &= 16 + \sqrt{x} \Rightarrow 3x = 128 + 8\sqrt{x} \\ \Rightarrow 3y^2 &= 128 + 8y, \text{ where } \sqrt{x} = y \\ \Rightarrow 3y^2 - 8y - 128 &= 0 \Rightarrow 3y^2 - 24y + 16y - 128 = 0 \\ \Rightarrow 3y(y - 8) + 16(y - 8) &= 0 \Rightarrow (y - 8)(3y + 16) = 0 \\ \Rightarrow y - 8 &= 0 \text{ or } 3y + 16 = 0 \\ \Rightarrow y &= 8 \text{ or } y = -\frac{16}{3} \Rightarrow \sqrt{x} = 8 \left[\because \sqrt{x} \neq -\frac{16}{3} \right] \\ \Rightarrow x &= 64 \end{aligned}$$

Hence, the total number of students of the class is 64.

26. Let the numerator of the fraction = x

Then denominator of the fraction = $2x + 1$

$$\therefore \text{Fraction} = \frac{x}{2x+1} \text{ and its reciprocal} = \frac{2x+1}{x}$$

According to given condition, $\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21}$

$$\begin{aligned} \Rightarrow \frac{x^2 + 4x^2 + 1 + 4x}{2x^2 + x} &= \frac{58}{21} \Rightarrow \frac{5x^2 + 1 + 4x}{2x^2 + x} = \frac{58}{21} \\ \Rightarrow 116x^2 + 58x &= 105x^2 + 84x + 21 \\ \Rightarrow 116x^2 + 58x - 105x^2 - 84x - 21 &= 0 \\ \Rightarrow 11x^2 - 26x - 21 &= 0 \Rightarrow 11x^2 - 33x + 7x - 21 = 0 \\ \Rightarrow 11x(x - 3) + 7(x - 3) &= 0 \Rightarrow (x - 3)(11x + 7) = 0 \\ \Rightarrow x - 3 &= 0 \text{ or } 11x + 7 = 0 \Rightarrow x = 3 \text{ or } x = -7/11 \\ \therefore x &= 3 \quad (\text{Neglecting negative value}) \\ \therefore \text{Fraction} &= \frac{x}{2x+1} = \frac{3}{6+1} = \frac{3}{7} \end{aligned}$$

27. Let the original price of the toy = ₹ x

Then the reduced price of the toy = ₹ $(x - 2)$

According to the question,

$$\frac{360}{x-2} - \frac{360}{x} = 2 \left(\because \text{Number of toys} = \frac{\text{Total amount}}{\text{Price of 1 toy}} \right)$$

$$\Rightarrow \frac{360x - 360x + 720}{x(x-2)} = 2 \Rightarrow \frac{720}{x(x-2)} = \frac{2}{1}$$

$$\begin{aligned} \Rightarrow x(x-2) &= 360 \\ \Rightarrow x^2 - 2x - 360 &= 0 \Rightarrow x^2 - 20x + 18x - 360 = 0 \\ \Rightarrow x(x-20) + 18(x-20) &= 0 \Rightarrow (x-20)(x+18) = 0 \\ \Rightarrow x-20 &= 0 \text{ or } x+18 = 0 \\ \Rightarrow x &= 20 \text{ or } x = -18 \\ \therefore x &= 20 \quad [\because \text{Price cannot be negative}] \end{aligned}$$

\therefore Original price of the toy = ₹ 20

28. Given, $(2p+1)x^2 - (7p+2)x + (7p-3) = 0$... (i)

$$\begin{aligned} \therefore \text{Roots are equal. } \therefore D &= 0 \\ \Rightarrow (- (7p+2))^2 - 4(2p+1)(7p-3) &= 0 \\ \Rightarrow 49p^2 + 4 + 28p - 4(14p^2 + 7p - 6p - 3) &= 0 \\ \Rightarrow 49p^2 + 28p + 4 - 56p^2 - 4p + 12 &= 0 \\ \Rightarrow 7p^2 - 24p - 16 &= 0 \Rightarrow 7p^2 + 4p - 28p - 16 = 0 \\ \Rightarrow p(7p+4) - 4(7p+4) &= 0 \Rightarrow (p-4)(7p+4) = 0 \\ \Rightarrow p &= 4 \text{ or } p = -\frac{4}{7} \end{aligned}$$

When $p = 4$, (i) becomes $9x^2 - 30x + 25 = 0$

$$\begin{aligned} \Rightarrow (3x)^2 - 2(3x)(5) + (5)^2 &= 0 \\ \Rightarrow (3x-5)^2 &= 0 \Rightarrow x = \frac{5}{3}, \frac{5}{3} \\ \text{When } p &= -\frac{4}{7}, \text{ (i) becomes} \end{aligned}$$

$$\begin{aligned} \frac{-x^2}{7} + 2x - 7 &= 0 \Rightarrow x^2 - 14x + 49 = 0 \\ \Rightarrow (x-7)^2 &= 0 \Rightarrow x = 7, 7 \end{aligned}$$

Thus, equal roots of given equation are either $5/3$ or 7 .

29. Let the denominator of the fraction = x

\therefore Numerator of the fraction = $x - 4$

$$\Rightarrow \text{Fraction} = \frac{x-4}{x}$$

According to question,

$$\begin{aligned} \frac{x-4}{x+1} &= \frac{x-4}{x} - \frac{1}{18} \Rightarrow \frac{x-4}{x} - \frac{x-4}{x+1} = \frac{1}{18} \\ \Rightarrow (x-4) \left[\frac{1}{x} - \frac{1}{x+1} \right] &= \frac{1}{18} \Rightarrow (x-4) \left[\frac{x+1-x}{x(x+1)} \right] = \frac{1}{18} \\ \Rightarrow 18(x-4) &= x(x+1) \Rightarrow 18x - 72 = x^2 + x \\ \Rightarrow x^2 - 17x + 72 &= 0 \Rightarrow x^2 - 9x - 8x + 72 = 0 \\ \Rightarrow x(x-9) - 8(x-9) &= 0 \Rightarrow (x-8)(x-9) = 0 \\ \Rightarrow x &= 8 \text{ or } x = 9 \end{aligned}$$

But $x = 8$ is not possible $\therefore x = 9$

Hence, the fraction $\frac{x-4}{x}$ is $\frac{5}{9}$.

30. Let breadth of rectangular park = x m

Then, length of rectangular park = $(x + 3)$ m

Now, area of rectangular park = $x(x + 3) = (x^2 + 3x)m^2$

Given, base of triangular park = Breadth of the rectangular park

\therefore Base of triangular park = x m

and also it is given that altitude of triangular park = 12 m

\therefore Area of triangular park = $\frac{1}{2} \times x \times 12 = 6x m^2$

According to the question,

Area of rectangular park = 4 + Area of triangular park

$$\Rightarrow x^2 + 3x = 4 + 6x \Rightarrow x^2 + 3x - 6x - 4 = 0$$

$$\Rightarrow x^2 - 3x - 4 = 0 \Rightarrow x^2 - 4x + x - 4 = 0$$

$$\Rightarrow x(x - 4) + 1(x - 4) = 0 \Rightarrow (x - 4)(x + 1) = 0$$

$$\Rightarrow x - 4 = 0 \text{ or } x + 1 = 0 \Rightarrow x = 4 \text{ or } x = -1$$

Since, breadth cannot be negative.

$$\therefore x = 4$$

Hence, breadth of the rectangular park = 4 m

and length of the rectangular park = $x + 3 = 4 + 3 = 7$ m.

31. Let the length of piece of cloth = x m

Increased length of piece of cloth = $(x + 5)$ m

Total cost of piece of cloth = ₹ 200

According to the question,

$$\frac{200}{x} - \frac{200}{x+5} = 2 \quad \left[\because \text{Rate per metre} = \frac{\text{Total cost}}{\text{Length}} \right]$$

$$\Rightarrow \frac{200x + 1000 - 200x}{x(x+5)} = 2$$

$$\Rightarrow 1000 = 2x^2 + 10x \Rightarrow 2x^2 + 10x - 1000 = 0$$

$$\Rightarrow x^2 + 5x - 500 = 0 \Rightarrow x^2 + 25x - 20x - 500 = 0$$

$$\Rightarrow x(x + 25) - 20(x + 25) = 0 \Rightarrow (x + 25)(x - 20) = 0$$

$$\Rightarrow x + 25 = 0 \text{ or } x - 20 = 0 \Rightarrow x = -25 \text{ or } x = 20$$

But, length can never be negative.

$$\therefore \text{Length of cloth} = 20 \text{ m}$$

$$\text{and rate per metre} = ₹ \frac{200}{20} = ₹ 10.$$

32. Let the number of students in the group in the beginning be x .

Total internet service charges for x students = ₹ 4800

$$\therefore \text{Internet service charges for each student} = ₹ \frac{4800}{x}$$

It is given that 4 more students join the group.

\therefore The number of students in group for internet service = $(x + 4)$

Now, the internet service charges for each student = ₹ $\frac{4800}{x+4}$

$$\text{According to question, } \frac{4800}{x} - \frac{4800}{x+4} = 200$$

$$\Rightarrow \frac{4800x + 19200 - 4800x}{x(x+4)} = 200$$

$$\Rightarrow 19200 = 200(x^2 + 4x) \Rightarrow 96 = x^2 + 4x$$

$$\Rightarrow x^2 + 4x - 96 = 0 \Rightarrow x^2 + 12x - 8x - 96 = 0$$

$$\Rightarrow x(x + 12) - 8(x + 12) = 0 \Rightarrow (x - 8)(x + 12) = 0$$

$$\Rightarrow x - 8 = 0 \text{ or } x + 12 = 0 \Rightarrow x = 8 \text{ or } x = -12$$

But number of students cannot be negative

$$\therefore x = 8$$

Hence, the number of students in the group in the beginning is 8.

33. Let the speed of the train be x km/hour.

When the speed is 9 km/hour more, then the new speed of the train is $(x + 9)$ km/hour.

Time taken by the train with speed x km/hour for a journey of 180 km = $\frac{180}{x}$ hours

Time taken by the train with new speed $(x + 9)$ km/hour for a journey of 180 km = $\frac{180}{(x+9)}$ hours

$$\text{According to the question, } \frac{180}{x} - \frac{180}{x+9} = 1$$

$$\Rightarrow 180 \left[\frac{1}{x} - \frac{1}{x+9} \right] = 1 \Rightarrow 180 \left[\frac{x+9-x}{x(x+9)} \right] = 1$$

$$\Rightarrow 180 \times 9 = x(x + 9) \Rightarrow x^2 + 9x - 1620 = 0$$

$$\Rightarrow x^2 + 45x - 36x - 1620 = 0 \Rightarrow x(x + 45) - 36(x + 45) = 0$$

$$\Rightarrow (x + 45)(x - 36) = 0 \Rightarrow x + 45 = 0 \text{ or } x - 36 = 0$$

$$\Rightarrow x = -45 \text{ or } x = 36$$

But, speed can't be negative.

$$\therefore x = 36$$

Hence, the uniform speed of the train is 36 km/hour.

34. Let x and y be the sides of two squares, respectively such that $x > y$, where x is the side of the first square and y is the side of the second square.

\therefore Area of the first square + Area of the second square = 640 m²

$$\Rightarrow x^2 + y^2 = 640 \quad \dots(i)$$

Again, it is given that the difference of their perimeters = 64 m

$$\Rightarrow 4x - 4y = 64 \Rightarrow x = 16 + y \quad \dots(ii)$$

From (i) and (ii), we have, $(16 + y)^2 + y^2 = 640$

$$\Rightarrow 256 + y^2 + 32y + y^2 = 640 \Rightarrow 2y^2 + 32y - 384 = 0$$

$$\Rightarrow y^2 + 16y - 192 = 0 \Rightarrow y^2 + 24y - 8y - 192 = 0$$

$$\Rightarrow y(y + 24) - 8(y + 24) = 0 \Rightarrow (y + 24)(y - 8) = 0$$

$$\Rightarrow y + 24 = 0 \text{ or } y - 8 = 0 \Rightarrow y = -24 \text{ or } y = 8$$

But, side of a square can't be negative. $\therefore y = 8$

When $y = 8$, then from (ii), we get $x = 16 + 8 = 24$.

Hence, the sides of the two squares are 24 m and 8 m respectively.

35. Let the speed of Deccan Queen = x km/hr

and speed of other train = $(x - 20)$ km/hr

$$\text{Time taken by Deccan Queen} = \frac{192}{x} \text{ hr}$$

$$\text{and time taken by other train} = \frac{192}{(x-20)} \text{ hr}$$

$$\text{According to the question, } \frac{192}{(x-20)} - \frac{192}{x} = \frac{48}{60} \text{ or } \frac{4}{5}$$

$$\Rightarrow \frac{192x - 192x + 3840}{x(x-20)} = \frac{4}{5}$$

$$\Rightarrow 5(3840) = 4x(x-20) \Rightarrow 19200 = 4x^2 - 80x$$

$$\Rightarrow 4x^2 - 80x - 19200 = 0 \Rightarrow x^2 - 20x - 4800 = 0$$

$$\Rightarrow x^2 - 80x + 60x - 4800 = 0 \Rightarrow x(x-80) + 60(x-80) = 0$$

$$\Rightarrow (x-80)(x+60) = 0 \Rightarrow x-80 = 0 \text{ or } x+60 = 0$$

$$\Rightarrow x = 80 \text{ or } x = -60$$

As speed can never be negative. $\therefore x = 80$

\therefore Speed of Deccan Queen = 80 km/hr.

36. Let the usual speed of the plane be x km/hr.

\therefore Time taken to travel 1500 km at x km/hr

$$= \frac{1500}{x} \text{ hour}$$

Increased speed of the plane = $(x+250)$ km/hr

\therefore Time taken to travel 1500 km at $(x+250)$ km/hr

$$= \frac{1500}{x+250} \text{ hour}$$

According to question,

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60} \Rightarrow 1500 \left(\frac{x+250-x}{x(x+250)} \right) = \frac{1}{2}$$

$$\Rightarrow 2 \times 1500 \times 250 = x^2 + 250x$$

$$\Rightarrow x^2 + 250x - 750000 = 0$$

$$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$$

$$\Rightarrow x(x+1000) - 750(x+1000) = 0$$

$$\Rightarrow (x+1000)(x-750) = 0$$

$$\Rightarrow x = 750 \text{ or } x = -1000 \text{ (But speed can't be negative)}$$

$$\therefore x = 750$$

Hence, usual speed of the plane is 750 km/hr.

