Pair of Linear Equations in Two Variables



SOLUTIONS

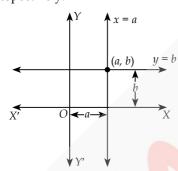
1. (a): Let *x* be the number of boys and *y* be the number of girls in the class.

According to the question,

$$12x + 6y = 900$$
 ...(i)
and $10x + 5y = 900$...(ii)

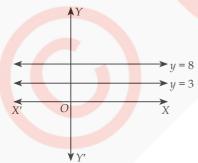
: (i) and (ii) are the required algebraic representation of given situation.

2. (d): x = a and y = b represents lines parallel to y axis and x axis respectively.



 \therefore Graphically, x = a and y = b represents lines intersecting each other at (a, b).

3. (d): Clearly, y = 3 and y = 8 represents two parallel lines.



:. Given pair of equations has no solution.

4. (a): For coincident lines,

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

5. (d) : Since, x = 2 and y = 3 is a solution of

2x - 3y + a = 0 and 2x + 3y - b + 2 = 0

$$\therefore$$
 2(2) - 3(3) + $a = 0 \Rightarrow a = 5$...(i)

and $2(2) + 3(3) - b + 2 = 0 \implies b = 15$...(ii)

From (i) and (ii), we get 3a = b

6. (a): Lines are parallel when $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

.. Another linear equation in two variables can be 6x + 8y + k = 0, where k is constant not equal to -16.

 \therefore Another linear equation can be 6x + 8y - 12 = 0

7. (b): Given equations are

4x + 6y - 9 = 0 and 2x + 3y - 6 = 0

Here, $a_1 = 4$, $b_1 = 6$, $c_1 = -9$ and $a_2 = 2$, $b_2 = 3$, $c_2 = -6$

$$\therefore \frac{a_1}{a_2} = \frac{4}{2} = 2, \frac{b_1}{b_2} = \frac{6}{3} = 2 \text{ and } \frac{c_1}{c_2} = \frac{-9}{-6} = \frac{3}{2}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

.. System of equations has no solution.

8. (c): Since,
$$\frac{2}{4} = \frac{3}{6} = \frac{-9}{-18} = \frac{1}{2}$$

:. System of linear equations is consistent with infinitely many solutions.

9. (a): Given,
$$2x - 3y = 5$$
 ...(i)

$$3x - 2y = 10$$
 ...(ii)

Adding (i) and (ii), we have

$$5x - 5y = 15 \Rightarrow x - y = 3$$

10. (d): For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \Rightarrow \quad \frac{k}{12} \neq \frac{3}{k}$$

$$\Rightarrow k^2 \neq 36 \Rightarrow k \neq \pm 6$$

Thus, k can have any real value except k = 6 and k = -6 for unique solution.

11. Given equations are x + 2y - 8 = 0 and 2x + 4y - 16 = 0Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -8$ and $a_2 = 2$, $b_2 = 4$, $c_2 = -16$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

Since,
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

:. System of equations has infinitely many solutions.

12. Condition for lines to be parallel is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Longrightarrow \frac{2}{k} = \frac{-3}{-9} \neq \frac{-9}{-18}$$

Now,
$$\frac{2}{k} = \frac{-3}{-9} = \frac{1}{3} \implies k = 6$$

13. Condition for lines to be inconsistent is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Longrightarrow \frac{1}{3} = \frac{3}{k} \neq \frac{-4}{12}$$

Now,
$$\frac{1}{3} = \frac{3}{k} \Rightarrow k = 9$$
.

14. For no solution, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{k}{12} = \frac{3}{k} \neq -\frac{(k-2)}{-k}$$

Now,
$$\frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$$

 $k = \pm 6$ also satisfies the last two terms.

15. For coincident lines, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \implies \frac{5}{15} = \frac{7}{21} = \frac{3}{k}$$

Now,
$$\frac{7}{21} = \frac{3}{k} \implies k = 9$$
.

16. For no solution, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-k} \implies k \neq 10$$

So, the system of equations has no solution for every real value of k except when k = 10.

17. For infinitely many solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \implies \frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

$$\implies \frac{2}{k+2} = \frac{1}{2} \implies 4 = k+2 \implies k = 2$$

k = 2 also satisfies the last two terms.

18. For unique solution, we have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \Rightarrow \quad \frac{4}{2} \neq \frac{p}{2} \quad \Rightarrow \quad p \neq 4$$

 \therefore p can have any real value except p = 4.

19. Given, x + 2y = 9...(i)

$$x - y = 6 \qquad \qquad \dots(x)$$

Multiplying (ii) by 2, we have

$$2x - 2y = 12$$
 ...(iii)

Adding (i) and (iii), we have $3x = 21 \Rightarrow x = 7$ Put x = 7 in (ii), we get y = 1.

20. For infinitely many solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \implies \frac{k-1}{k+1} = \frac{-1}{1-k} = \frac{-5}{-3k-1}$$

$$\implies \frac{-1}{1-k} = \frac{5}{3k+1} \implies -3k-1 = 5-5k \implies 2k = 6 \implies k = 3$$

21. Let the original length and breadth of the lawn are x m and y m respectively.

Then, perimeter of lawn = 2(x + y)

According to the question,

$$2(x + y) = 54 \implies x + y - 27 = 0$$
 ...(i)

Also,
$$2\left[\frac{3}{5}x + \frac{4}{5}y\right] = 36$$

$$\Rightarrow 3x + 4y = 18 \times 5 = 90 \Rightarrow 3x + 4y - 90 = 0$$
 ...(ii)

∴ (i) and (ii) is the required algebraic representation of the given situation.

22. Given pair of linear equations are:

$$2x - 3y + 15 = 0$$
 ...(i)

$$3x - 5 = 0$$
 ...(ii)

From (ii), we have, x = 5/3

Substituting $x = \frac{5}{3}$ in (i), we get

$$2\left(\frac{5}{3}\right) - 3y + 15 = 0 \implies \frac{10}{3} - 3y + 15 = 0$$

$$\Rightarrow$$
 $-3y = -15 - \frac{10}{3} \Rightarrow -3y = \frac{-45 - 10}{3} \Rightarrow y = \frac{55}{9}$

$$\therefore \quad x = \frac{5}{3}, \ y = \frac{55}{9}$$

23. Let the numbers be x and y.

According to the question,

$$\frac{x}{y} = \frac{3}{4} \Rightarrow x = \frac{3y}{4}$$
and
$$\frac{x+6}{y+6} = \frac{7}{8} \Rightarrow 8x + 48 = 7y + 42$$
 ...(i)

$$\Rightarrow 8x - 7y = -6 \qquad \dots (ii)$$

Using (i) in (ii), we get

$$8\left(\frac{3y}{4}\right) - 7y = -6 \Rightarrow 6y - 7y = -6 \Rightarrow y = 6$$

Putting *y* = 6 in (i), we have
$$x = \frac{3y}{4} = \frac{3}{4} \times 6 = 4.5$$

Hence, the numbers are 4.5 and 6.

24. Given pair of linear equations are

$$3x - y = 5$$
 ...(i) and $5x - y = 11$...(ii)

Subtracting (ii) from (i), we get

$$3x - y - (5x - y) = 5 - 11$$

$$\Rightarrow$$
 $-2x = -6 \Rightarrow x = 3$

Substituting the value of x in (i), we get

$$3 \times 3 - y = 5 \implies -y = 5 - 9 \implies y = 4$$

Hence, x = 3 and y = 4 is the required solution.

25. Let the tens digit of a number be x and ones digit be y.

The number be 10x + y.

According to the question,

$$\frac{10x + y}{x + y} = 7 \implies 10x + y = 7x + 7y$$

$$\Rightarrow 3x - 6y = 0 \implies x - 2y = 0 \qquad \dots(i)$$

and 10x + y - 27 = 10y + x

$$\Rightarrow 9x - 9y = 27 \Rightarrow x - y = 3$$
 ...(ii)

Subtracting (i) from (ii), we get y = 3

From (ii), $x - 3 = 3 \implies x = 6$

Required number is 63.

26. We know that for a pair of linear equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$

has no solution if
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

For no solution, we have

$$\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{2}{5} \implies \frac{3k+1}{k^2+1} = \frac{3}{k-2}$$
 and $\frac{3}{k-2} \neq \frac{2}{5}$

$$\Rightarrow (k-2)(3k+1) = 3(k^2+1) \Rightarrow 3k^2+k-6k-2 = 3k^2+3$$

\Rightarrow -5k = 5 \Rightarrow k = -1

Also, k = -1 satisfy last two terms.

27. Let the cost of a chair be $\stackrel{?}{\stackrel{?}{$}} x$ and the cost of a table be $\stackrel{?}{\stackrel{?}{$}} y$. Then, according to the question,

$$3x + 2y = 7100$$
 ...(ii)

Multiply (i) by 2 and (ii) by 3, we get

$$4x + 6y = 11300$$
 ...(iii)

$$9x + 6y = 21300$$
 ...(iv)

Subtracting (iii) from (iv), we get $5x = 10000 \implies x = 2000$

Putting the value of x in (i), we get

$$2 \times 2000 + 3y = 5650$$

$$\Rightarrow$$
 3y = 5650 - 4000 \Rightarrow 3y = 1650

$$\Rightarrow y = \frac{1650}{3} = 550$$

Hence, the cost of a chair is ₹ 2000 and cost of a table is ₹ 550.

28. Let the number of rows be x and number of students in each row be y. Then, total number of students in the class = xy.

According to question (y + 3)(x - 1) = xy

:.
$$xy + 3x - y - 3 = xy$$
 or $3x - y = 3$...(i)

Also,
$$(y - 3)(x + 2) = xy$$

$$\therefore xy - 3x + 2y - 6 = xy \text{ or } -3x + 2y = 6$$
 ...(ii)

On adding (i) and (ii), we get y = 9

Put y = 9 in (i), we get

$$3x - 9 = 3 \implies 3x = 12 \implies x = 4$$

- \therefore Number of students in class = $xy = 4 \times 9 = 36$
- **29.** Since $BC \parallel DE$ and $BE \parallel CD$ with $BC \perp CD$ and BCDE is a rectangle
- :. Opposite sides are equal

$$\Rightarrow BE = CD \Rightarrow x + y = 5$$
 ...(i)

Since perimeter of *ABCDE* is 21.

$$\Rightarrow$$
 AB + BC + CD + DE + EA = 21

$$\Rightarrow$$
 3 + (x - y) + (x + y) + (x - y) + 3 = 21

$$\Rightarrow$$
 6 + 3x - y = 21 \Rightarrow 3x - y = 15 ...(ii)

On adding (i) and (ii), we get $4x = 20 \implies x = 5$

Putting x = 5 in (i), we get y = 0.

30. Let the constant expenditure be $\not\in x$ and consumption of wheat be y quintals.

Then total expenditure = $x + y \times \text{Rate per quintal}$

According to the question,

x + 250y = 1000 ...(i) and x + 240y = 980 ...(ii)

Subtracting (ii) from (i), we get $10 y = 20 \Rightarrow y = 2$

Now, substituting y = 2 in (i), we get

$$x + 250(2) = 1000$$

$$\Rightarrow$$
 $x = 1000 - 500 = 500$

- ∴ Total expenses when the price of wheat is ₹ 350 per quintal = $x + 350y = 500 + 350 \times 2 = 500 + 700 = ₹ 1200$
- 31. Let fare from *A* to *B* be $\not\in x$ and face from *A* to *C* be $\not\in y$. Then, according to the question,

$$2x + 3y = 795$$
 ...(i)

$$3x + 5y = 1300$$
 ...(ii)

Multiplying (i) by 3 and (ii) by 2, we get

$$6x + 9y = 2385$$
 ...(iii)

$$6x + 10y = 2600$$
 ...(iv)

Subtracting (iii) from (iv), we get y = 215

Putting y = 215 in (i), we get

$$2x + 3(215) = 795$$

$$\Rightarrow$$
 2x = 150 \Rightarrow x = 75

Hence, fare from *A* to *B* is $\stackrel{?}{\sim}$ 75 and fare from *A* to *C* is $\stackrel{?}{\sim}$ 215.

32. Let the digits at ten's and unit place be x and y respectively. Then, required number = 10x + y.

Also, number obtained by reversing the digit = 10y + x

According to the question, we have

$$x = 2y + 2$$
 ...(i)

and
$$10y + x = 3(x + y) + 5$$

$$\Rightarrow 10y + x = 3x + 3y + 5 \text{ or } -2x + 7y = 5 \qquad \dots(ii)$$

Using (i) in (ii), we get -2(2y + 2) + 7y = 5

$$\Rightarrow$$
 $-4y-4+7y=5$ \Rightarrow $3y=9$ \Rightarrow $y=3$

Putting y = 3 in (i), we have x = 2(3) + 2 = 8

- \therefore Required number = 10(8) + 3 = 83
- 33. Let man's starting salary and his fixed annual increment be $\stackrel{?}{\scriptstyle \times} x$ and $\stackrel{?}{\scriptstyle \times} y$ respectively.

According to the question,
$$x + 4y = 15000$$
 ...(i)

and
$$x + 10y = 18000$$
 ...(ii)

So, (i) and (ii) represents a pair of linear equations in two variables of given situation.

Subtracting (i) from (ii), we get $6y = 3000 \implies y = 500$

Putting y = 500 in (i), we get

[Given]

$$x + 4(500) = 15000 \Rightarrow x + 2000 = 15000 \Rightarrow x = 13000$$

Hence, man's starting salary is ₹ 13000 and his fixed annual increment is ₹ 500.

34. Given equations are 2ax + 3by - (a + 2b) = 0 ...(i)

and
$$3ax + 2by - (2a + b) = 0$$
 ...(ii)

Solving (i) and (ii) by cross multiplication method, we have

$$\frac{x}{3b} - (a+2b) = \frac{y}{-(a+2b)} = \frac{1}{2a} = \frac{1}{2a}$$

$$2b - (2a+b) - (2a+b) = 3a$$

$$\Rightarrow \frac{x}{-3b(2a+b)+2b(a+2b)} = \frac{y}{-3a(a+2b)+2a(2a+b)}$$
$$= \frac{1}{-3a(a+2b)+2a(2a+b)}$$

$$\Rightarrow \frac{x}{-6ab - 3b^2 + 2ab + 4b^2} = \frac{y}{-3a^2 - 6ab + 4a^2 + 2ab} = \frac{1}{-5ab}$$

$$\Rightarrow \frac{x}{b^2 - 4ab} = \frac{y}{a^2 - 4ab} = -\frac{1}{5ab}$$

$$\Rightarrow \frac{x}{b(b-4a)} = \frac{-1}{5ab} \text{ and } \frac{y}{a(a-4b)} = \frac{-1}{5ab}$$

$$\Rightarrow x = \frac{-b(b-4a)}{5ab}$$
 and $y = \frac{-a(a-4b)}{5ab}$

$$\Rightarrow x = \frac{4a - b}{5a}$$
 and $y = \frac{4b - a}{5b}$

35. The given system of equations is 2x - 3y - 6 = 0 ...(i) and x + y = 1

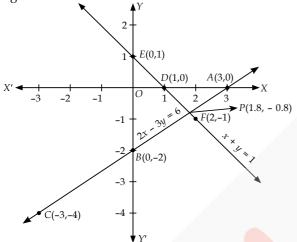
Table of solutions for (i) is:

х	3	0	-3
у	0	-2	-4

Also, table of solutions for (ii) is:

х	1	0	2
у	0	1	-1

Plotting the points on the graph paper and joining them, we get



Clearly from the graph, we see that equations given by (i) and (ii) are intersect each other at point P(1.8, -0.8)and hence, they have a unique solution given by x = 1.8, y = -0.8.

36. The given system of equations is 3x - 2y - 1 = 0...(i) and 2x - 3y + 6 = 0 ...(ii)

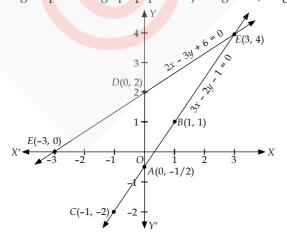
Table of solutions for (i) is:

	()		
x	0	1	-1
y	-1/2	1	-2

Table of solutions for (ii) is:

х	0	3	- 3
у	2	4	0

Plotting the points on graph paper and joining them, we get



From the graph, we see that the two lines represented by (i) and (ii) intersect each other at point E(3, 4).

Hence, x = 3 and y = 4 is the required solution.

37. Let speed of x be p km/hr and speed of y be q km/hr.

Time taken by
$$x = \frac{30}{p}$$
 $\left[\because \text{ Time} = \frac{\text{Distance}}{\text{Speed}}\right]$
Time taken by $y = \frac{30}{q}$

According to the question,
$$\frac{30}{n} - \frac{30}{a} = 3$$

If speed of x is doubled then it becomes 2p.

Then,
$$\frac{30}{q} - \frac{30}{2p} = \frac{3}{2}$$
 ...(ii)

...(i)

Putting $\frac{1}{p} = a$, $\frac{1}{q} = b$ in (i), we get

$$\Rightarrow 10a - 10b = 1$$

$$\Rightarrow a - b = \frac{1}{10}$$
 ...(iii)

From (ii),
$$-15a + 30b = \frac{3}{2}$$

$$\Rightarrow 5a - 10b = \frac{-1}{2}$$

$$\Rightarrow a - 2b = \frac{-1}{10}$$
 ...(iv)

Subtracting (iii) from (iv), we get $b = \frac{2}{10} = \frac{1}{5}$

$$\Rightarrow \frac{1}{q} = \frac{1}{5} \Rightarrow q = 5$$

Using $b = \frac{1}{5}$ in (iii), we get

$$a = \frac{1}{5} + \frac{1}{10} = \frac{3}{10} \implies \frac{1}{p} = \frac{3}{10}$$
$$\implies p = \frac{10}{3} = 3\frac{1}{3}$$

 \therefore Speed of x and y are $3\frac{1}{3}$ km/hr and 5 km/hr respectively.

38. Let the speed of the train be x km/hr and the speed of the bus be y km/hr.

Case I: When mala travels 80 km by train and remaining 220 km by bus.

Then,
$$\frac{80}{x} + \frac{220}{y} = 4$$

 $\Rightarrow \frac{20}{x} + \frac{55}{y} = 1$...(i)

Case II: When mala travels 100 km by train and remaining 200 km by bus.

Then,
$$\frac{100}{x} + \frac{200}{y} = 250 \,\text{min.} = \frac{250}{60} \,\text{hr}$$

$$\Rightarrow \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

...(iii)

$$\Rightarrow \frac{4}{x} + \frac{8}{y} = \frac{1}{6} \qquad \dots (ii)$$

Putting
$$\frac{1}{x} = u$$
 and $\frac{1}{y} = v$ in (i) and (ii), we get

$$20u + 55v = 1$$

$$4u + 8v = \frac{1}{6}$$

$$\Rightarrow 24u + 48v = 1 \qquad \dots (iv)$$

Multiplying (iii) by 6 and (iv) by 5, we get

$$120u + 330v = 6 ...(v)$$

$$120u + 240v = 5$$
 ...(vi)

Subtracting (vi) from (v), we get

$$90v = 1 \implies v = \frac{1}{90}$$

Putting $v = \frac{1}{90}$ in (iii), we get

$$20u + 55\left(\frac{1}{90}\right) = 1$$

$$\Rightarrow 20u = 1 - \frac{11}{18} \Rightarrow 20u = \frac{7}{18} \Rightarrow u = \frac{7}{18} \times \frac{1}{20} = \frac{7}{360}$$

Now,
$$u = \frac{1}{y} = \frac{1}{90} \implies y = 90 \text{ and}$$

$$v = \frac{1}{x} = \frac{7}{360} \implies x = \frac{360}{7}$$

Hence, the speed of the train is $\frac{360}{7}$ km/hr and the speed of the bus is 90 km/hr.

39. Let x units and y units be the length and breadth of rectangle respectively.

Then, its area = xy sq. units

According to the question,

$$(x-5)(y+2) = xy - 80$$

$$\Rightarrow xy + 2x - 5y - 10 = xy - 80 \Rightarrow 2x - 5y = -70$$
and $(x + 10)(y - 5) = xy + 50$...(i)

...(iii)

$$\Rightarrow xy - 5x + 10y - 50 = xy + 50$$

\Rightarrow -5x + 10y = 100 or $x - 2y = -20$

$$\Rightarrow -5x + 10y = 100 \text{ or } x - 2y = -20 \qquad ...(ii)$$
From (i), $x = \frac{5y - 70}{2}$...(iii)

Substituting the value of x from (iii) in (ii), we get

$$\left(\frac{5y-70}{2}\right) - 2y = -20 \implies 5y - 70 - 4y = -40 \implies y = 30$$

Substituting the value of y = 30 in (iii), we get

$$x = \frac{5(30) - 70}{2} = \frac{80}{2} = 40$$

Hence, length = 40 units, breadth = 30 units.

$$\therefore \text{ Perimeter of the rectangle} = 2(l+b)$$

$$= 2(40 + 30) = 2(70) = 140$$
 units.

40. Comparing the given system of equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 2$$
, $b_1 = -3$, $c_1 = -7$ and

$$a_2 = (a + b), b_2 = -(a + b - 3), c_2 = -(4a + b)$$

For infinite solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \implies \frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$$
Taking $\frac{2}{a+b} = \frac{3}{(a+b-3)}$

Taking
$$\frac{2}{a+b} = \frac{3}{(a+b-3)}$$

$$\Rightarrow$$
 2a + 2b - 6 = 3a + 3b \Rightarrow a + b + 6 = 0 ...(i)

and
$$\frac{3}{a+b-3} = \frac{7}{4a+b}$$

$$\Rightarrow 12a + 3b = 7a + 7b - 21 \Rightarrow 5a - 4b + 21 = 0 \qquad ...(ii)$$

From (i),
$$a = -(6 + b)$$

Using (iii) in (ii), we get
$$5[-(6+b)] - 4b + 21 = 0$$

$$\Rightarrow$$
 -30 - 5b - 4b + 21 = 0

$$\Rightarrow -9b = 9 \Rightarrow b = -1$$

From (iii),
$$a = -(6 - 1) = -5$$

:
$$a = -5$$
 and $b = -1$

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