Polynomials

CHAPTER 2

EXAM DRILL

SOLUTIONS

1. (a) : Since α, β are the zeroes of $2x^2 + 6x - 6$, we have

$\therefore\quad \alpha + \beta = \frac{-6}{2} = -3$ and $\alpha\beta = \frac{-6}{2} = -3.$

Hence, $\alpha + \beta = \alpha\beta$

2. (b) : Product of zeroes $= \frac{a}{r} \cdot a \cdot ar = -1$

$\left[\because \text{Product of zeroes in cubic polynomial} = \frac{-\text{Constant term}}{\text{Coefficient of } x^3}\right]$

$\Rightarrow\quad a^3 = -1 \quad\Rightarrow\quad a = -1$

3. (b) : Let $f(x) = px^3 + x^2 - 2x + q$

Since $(x + 1)$ and $(x - 1)$ are factors of $f(x) = px^3 + x^2 - 2x + q$

$\therefore\quad f(1) = 0$ and $f(-1) = 0$

Now, $f(1) = p + 1 - 2 + q = p + q - 1 = 0$

$\Rightarrow\quad p + q = 1$ ...(i)

$f(-1) = 0 \Rightarrow -p + 1 + 2 + q = 0$

$\Rightarrow\quad -p + q = -3$ ...(ii)

Solving (i) and (ii), we get $p = 2$ and $q = -1$

4. (a) : Since α, β and γ are the zeroes of $x^3 + px^2 + qx + r$

$\therefore\quad \alpha + \beta + \gamma = -p$ and $\alpha\beta\gamma = -r$

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{-p}{-r} = \frac{p}{r}$$

5. (d) : Here, dividend $= -x^3 + 8x^2 + 2x + 5$ and divisor $= x^2 + 2x + 6$

$\therefore$ Coefficient of first term of quotient

$$= \frac{\text{Coefficient of first term of dividend}}{\text{Coefficient of first term of divisor}} = -1$$

6. Let $p(x) = (k - 1)x^2 + kx + 1$

Given that, one of the zeroes is -3, then $p(-3) = 0$

$\Rightarrow\quad (k - 1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow\quad 9(k - 1) - 3k + 1 = 0 \Rightarrow 6k - 8 = 0 \Rightarrow k = 4/3.$

7. On dividing $2x^2 - 18x - 45$ by $x - 16$, we have

$$\begin{array}{r|l} & 2x + 14 \\ \hline x - 16 & 2x^2 - 18x - 45 \\ & \underline{\overset{(-)}{2x^2} \overset{(+)}{- 32x}} \\ & 14x - 45 \\ & \underline{\overset{(-)}{14x} \overset{(+)}{- 224}} \\ & 179 \end{array}$$

$\therefore$ 179 should be subtracted from $2x^2 - 18x - 45$, so that 16 is zero of resulting polynomial.

8. Sum of zeroes $= \alpha + \beta = -\left(\frac{-6}{2}\right) = 3$

Product of zeroes $= \alpha\beta = 7/2$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (3)^2 - 2\left(\frac{7}{2}\right) = 9 - 7 = 2$

9. On dividing $2x^3 + 3x^2 - 8x - 12$ by $x^2 - 4$, we have

$$\begin{array}{r|l} & 2x + 3 \\ \hline x^2 - 4 & 2x^3 + 3x^2 - 8x - 12 \\ & \underline{\overset{(-)}{2x^3} \qquad \overset{(+)}{- 8x}} \\ & 3x^2 + 0 - 12 \\ & \underline{\overset{(-)}{3x^2} \qquad \overset{(+)}{- 12}} \\ & 0 \end{array}$$

Thus, the quotient is $2x + 3$ and remainder = 0.

10. Let $p(x) = 2ax^3 + 3x^2 + 5x + 2$

Let α, β and γ be the zeroes of $p(x)$, where $\alpha = 0$.

We know that, sum of product of zeroes taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{5}{2}$

$\Rightarrow\quad 0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{5}{2} \Rightarrow \beta\gamma = \frac{5}{2}$

Hence, product of other two zeroes $= \frac{5}{2}$

11. Given, α and β are the zeroes of the polynomial, $f(x) = x^2 - 19x + k$

$\therefore\quad \alpha + \beta = 19$ ...(i)

and $\alpha\beta = k$ ...(ii)

Also, $\alpha - \beta = 7$ (Given) ...(iii)

Adding (i) and (iii), we get $2\alpha = 26 \Rightarrow \alpha = 13$

From (i), $\beta = 6$

Now, $\alpha\beta = k \Rightarrow 13 \times 6 = k \Rightarrow k = 78$

12. Given that, sum of zeroes $(S) = -\frac{3}{2\sqrt{5}}$

and product of zeroes $(P) = -\frac{1}{2}$

$\therefore$ Required quadratic polynomial is

$$p(x) = x^2 - Sx + P = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2} = 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

Using factorisation method,

$$p(x) = 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}$$

$$= \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = (2x + \sqrt{5})(\sqrt{5}x - 1)$$

Hence, the zeroes of $p(x)$ are $-\frac{\sqrt{5}}{2}$ and $\frac{1}{\sqrt{5}}$.

13. On dividing $x^3 - ax^2 + 6 - a$ by $x - a$, we have

$$\begin{array}{r|l} & x^2 \\ \hline x - a & x^3 - ax^2 + 6 - a \\ & x^3 - ax^2 \\ & \underset{(-)}{} \quad \underset{(+)}{} \\ \hline & 6 - a \\ \hline \end{array}$$

$\therefore$ Quotient $= x^2$ and remainder $= 6 - a$

14. Let $p(x)$ be the required polynomial.
$g(x) = -2x^2 + 3x - 2$ and $q(x) = x - 3$ and $r(x) = 4$
So, by division algorithm, we have $p(x) = g(x) \cdot q(x) + r(x)$

$\Rightarrow \quad p(x) = (-2x^2 + 3x - 2)(x - 3) + 4$
$= -2x^3 + 3x^2 - 2x + 6x^2 - 9x + 6 + 4$
$= -2x^3 + 9x^2 - 11x + 10$

15. Let $p(x) = x^2 - 12x + 35$
For zeroes, put $p(x) = 0$
$\Rightarrow \quad x^2 - 12x + 35 = 0 \Rightarrow x^2 - 5x - 7x + 35 = 0$
$\Rightarrow \quad x(x - 5) - 7(x - 5) = 0$
$\Rightarrow \quad (x - 5)(x - 7) = 0$
$\Rightarrow \quad x - 5 = 0$ or $x - 7 = 0$
$\Rightarrow \quad x = 5$ or $x = 7$
Zeroes of $p(x)$ are 5 and 7.

Sum of zeroes $= 5 + 7 = 12 = \dfrac{-(12)}{1} = \dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= 5 \times 7 = 35 = \dfrac{35}{1} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

16. Given quadratic polynomial is $x^2 - 8x + 7$.
Let α and β be its roots.
$\therefore \quad \alpha + \beta = 8$ ...(i)
$\alpha\beta = 7$ ...(ii)

$\therefore \quad \alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(8)^2 - 4\times 7}$ [Using (i) and (ii)]

$= \sqrt{64 - 28} = \sqrt{36} = 6$

$\Rightarrow \quad \alpha - \beta = 6$ ...(iii)
Adding (i) and (iii), we get $2\alpha = 14 \Rightarrow \alpha = 7$
From (i), we get $\beta = 8 - \alpha = 8 - 7 = 1$

17. Given polynomial, $f(x) = 25P^2 - 15P + 2$
Since, α and β are roots of $f(x)$

$\therefore \quad \alpha + \beta = \dfrac{15}{25} = \dfrac{3}{5}$ and $\alpha\beta = \dfrac{2}{25}$

Sum of zeroes of the required polynomial

$$= \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{\alpha+\beta}{2\alpha\beta} = \frac{\left(\frac{3}{5}\right)}{2\left(\frac{2}{25}\right)} = \frac{\frac{3}{5}}{\frac{4}{25}} = \frac{3}{5}\times\frac{25}{4} = \frac{15}{4}$$

and product of zeroes of the required polynomial

$$= \left(\frac{1}{2\alpha}\right)\left(\frac{1}{2\beta}\right) = \frac{1}{4\alpha\beta} = \frac{1}{4\left(\frac{2}{25}\right)} = \frac{1}{\frac{8}{25}} = \frac{25}{8}$$

Hence, the required quadratic polynomial is

$x^2 - \dfrac{15}{4}x + \dfrac{25}{8}$ or $\dfrac{1}{8}(8x^2 - 30x + 25)$.

18. Let α and β be the zeroes of the polynomial $f(x) = ax^2 + bx + c$. Then, $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

Let S and P denotes respectively the sum and product of the zeroes of a polynomial, whose zeroes are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

Then, $S = \dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{-\frac{b}{a}}{\frac{c}{a}} = -\dfrac{b}{c}$

and $P = \dfrac{1}{\alpha}\times\dfrac{1}{\beta} = \dfrac{1}{\alpha\beta} = \dfrac{1}{\frac{c}{a}} = \dfrac{a}{c}$

Hence, the required polynomial $g(x)$ is given by

$g(x) = k(x^2 - Sx + P) = k\left(x^2 + \dfrac{bx}{c} + \dfrac{a}{c}\right)$, where k is any non-zero constant.

19. Let the cubic polynomial be $ax^3 + bx^2 + cx + d$ and its zeroes are α, β and γ.

Then, $\alpha + \beta + \gamma = 5 = -\dfrac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = -11 = \dfrac{c}{a}$

and $\alpha\beta\gamma = -23 = -\dfrac{d}{a}$

If $a = 1$, then $b = -5$, $c = -11$ and $d = 23$
So, cubic polynomial is $x^3 - 5x^2 - 11x + 23$.

20. We have,

$$\begin{array}{r|l} & 7x^2 + x + 5 \\ \hline 2x - 1 & 14x^3 - 5x^2 + 9x - 1 \\ & 14x^3 - 7x^2 \\ & (-) \quad (+) \\ \hline & \quad 2x^2 + 9x - 1 \\ & \quad 2x^2 - x \\ & \quad (-) \quad (+) \\ \hline & \qquad 10x - 1 \\ & \qquad 10x - 5 \\ & \qquad (-) \quad (+) \\ \hline & \qquad\quad 4 \\ \hline \end{array}$$

$\therefore$ Quotient, $q(x) = 7x^2 + x + 5$ and remainder, $r(x) = 4$

21. On dividing $3x^3 + x^2 + 2x + 5$ by $x^2 + 2x + 1$, we get

$$\begin{array}{r|l} & 3x - 5 \\ \hline x^2 + 2x + 1 & 3x^3 + x^2 + 2x + 5 \\ & 3x^3 + 6x^2 + 3x \\ & (-) \quad (-) \quad (-) \\ \hline & \quad -5x^2 - x + 5 \\ & \quad -5x^2 - 10x - 5 \\ & \quad (+) \quad (+) \quad (+) \\ \hline & \qquad 9x + 10 \\ \hline \end{array}$$

$\therefore$ Quotient $= 3x - 5$ and remainder $= 9x + 10$

22. We have,

$$\begin{array}{r|l} & 2x^2 - x - 1 \\ x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 9 \\ & 2x^4 - 8x^3 + 2x^2 \\ & (-)\quad (+)\quad (-) \\ \hline & -x^3 + 3x^2 + 3x - 9 \\ & -x^3 + 4x^2 - x \\ & (+)\quad (-)\quad (+) \\ \hline & -x^2 + 4x - 9 \\ & -x^2 + 4x - 1 \\ & (+)\quad (-)\quad (+) \\ \hline & -8 \end{array}$$

$\therefore$ Quotient = $2x^2 - x - 1$ and remainder = –8.

23. On dividing $6x^4 + 9x^3 + 17x^2 + 23x + 10$ by $2x^2 + 3x + 1$, we have

$$\begin{array}{r|l} & 3x^2 + 7 \\ 2x^2 + 3x + 1 & 6x^4 + 9x^3 + 17x^2 + 23x + 10 \\ & 6x^4 + 9x^3 + 3x^2 \\ & (-)\quad (-)\quad (-) \\ \hline & +14x^2 + 23x + 10 \\ & +14x^2 + 21x + 7 \\ & (-)\quad (-)\quad (-) \\ \hline & 2x + 3 \end{array}$$

But given remainder is $ax + b$

$\therefore$ $ax + b = 2x + 3$

$\Rightarrow$ $a = 2$ and $b = 3$ [On comparing like terms]

24. We have, $f(x) = abx^2 + (b^2 - ac)x - bc$

$= abx^2 + b^2x - acx - bc = bx(ax + b) - c(ax + b)$

$= (ax + b)(bx - c)$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow$ $(ax + b)(bx - c) = 0$

$\Rightarrow$ $ax + b = 0$ or $bx - c = 0$

$\Rightarrow$ $x = -\frac{b}{a}$ or $x = \frac{c}{b}$

Thus, the zeroes of $f(x)$ are : $\alpha = -\frac{b}{a}$ and $\beta = \frac{c}{b}$

Now, $\alpha + \beta = -\frac{b}{a} + \frac{c}{b} = \frac{ac - b^2}{ab}$ and $\alpha\beta = -\frac{b}{a} \times \frac{c}{b} = -\frac{c}{a}$

Also, $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\left(\frac{b^2 - ac}{ab}\right) = \frac{ac - b^2}{ab}$

and, $\frac{\text{Constant term}}{\text{Coefficient of } x^2} = -\frac{bc}{ab} = -\frac{c}{a}$

Hence, sum of the zeroes = $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and, product of the zeroes = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

25. Let α, β and γ be the zeros of polynomial $f(x) = x^3 - 5x^2 - 16x + 80$, such that $\alpha + \beta = 0$.

Then, sum of the zeroes = $-\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow$ $\alpha + \beta + \gamma = -\left(-\frac{5}{1}\right) \Rightarrow \gamma = 5$ [$\because$ $\alpha + \beta = 0$]

Also, product of the zeroes = $-\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

$\Rightarrow$ $\alpha\beta\gamma = -\frac{80}{1}$ [$\because$ $\gamma = 5$]

$\Rightarrow$ $5\alpha\beta = -80$ $\Rightarrow$ $\alpha\beta = -16$

$\Rightarrow$ $-\alpha^2 = -16$ [$\because$ $\alpha + \beta = 0$ $\therefore$ $\beta = -\alpha$]

$\Rightarrow$ $\alpha = \pm 4$

Now, $\alpha + \beta = 0$ and $\alpha = \pm 4$ $\Rightarrow$ $\beta = \mp 4$

Hence, the zeroes are 4, –4 and 5.

26. Since, α and β are the zeroes of the quadratic polynomial, $f(x) = kx^2 + 4x + 4$

$\therefore$ $\alpha + \beta = -\frac{4}{k}$ and $\alpha\beta = \frac{4}{k}$

Now, $\alpha^2 + \beta^2 = 24 \Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 24$

$\Rightarrow$ $\left(-\frac{4}{k}\right)^2 - 2 \times \frac{4}{k} = 24 \Rightarrow \frac{16}{k^2} - \frac{8}{k} = 24$

$\Rightarrow$ $16 - 8k = 24k^2 \Rightarrow 3k^2 + k - 2 = 0$

$\Rightarrow$ $3k^2 + 3k - 2k - 2 = 0 \Rightarrow 3k(k + 1) - 2(k + 1)$

$\Rightarrow$ $(k + 1)(3k - 2) = 0 \Rightarrow k + 1 = 0$ or $3k - 2 = 0$

$\Rightarrow$ $k = -1$ or $k = 2/3$

Hence, $k = -1$ or $k = 2/3$

27. Let α and β be the zeroes of required quadratic polynomial.

Then, $\alpha + \beta = \frac{5 + \sqrt{2}}{5 - \sqrt{2}} + \frac{5 - \sqrt{2}}{5 + \sqrt{2}} = \frac{(5 + \sqrt{2})^2 + (5 - \sqrt{2})^2}{(5 - \sqrt{2})(5 + \sqrt{2})}$

$= \frac{25 + 2 + 10\sqrt{2} + 25 + 2 - 10\sqrt{2}}{5^2 - (\sqrt{2})^2} = \frac{54}{23}$

Also, $\alpha\beta = \frac{5 + \sqrt{2}}{5 - \sqrt{2}} \times \frac{5 - \sqrt{2}}{5 + \sqrt{2}} = 1$

So, required polynomial is given by

x^2 – (sum of zeroes)x + product of zeroes = $x^2 - \frac{54}{23}x + 1$

Since, zeroes of $x^2 - \frac{54}{23}x + 1$ is same as $23x^2 - 54x + 23$

$\therefore$ Required polynomial is $23x^2 - 54x + 23$.

28. It is given that α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$.

$\therefore$ $\alpha + \beta = -\left(-\frac{5}{2}\right) = \frac{5}{2}$ and $\alpha\beta = \frac{7}{2}$

Let S and P denotes respectively the sum and product of zeroes of the required polynomial.

Then, $S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5 \times \frac{5}{2} = \frac{25}{2}$

and, $P = (2\alpha + 3\beta)(3\alpha + 2\beta) = 6(\alpha^2 + \beta^2) + 13\alpha\beta$

$= 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta = 6(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta$

$= 6(\alpha + \beta)^2 + \alpha\beta = 6\left(\frac{5}{2}\right)^2 + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = 41$

Hence, the required polynomial $g(x)$ is given by

$g(x) = k(x^2 - Sx + P) = k\left(x^2 - \frac{25}{2}x + 41\right)$, where k is any non-zero real number.

29. Let $f(x) = kx^2 + 41x + 42$

Given, product of zeroes = 7

$\Rightarrow \quad 42/k = 7 \Rightarrow 42 = 7k$

$\Rightarrow \quad k = 6$

Putting $k = 6$ in polynomial

$p(x) = (k - 4)x^2 + (k + 1)x + 5$, we get

$p(x) = (6 - 4)x^2 + (6 + 1)x + 5$

$\Rightarrow \quad p(x) = 2x^2 + 7x + 5$

For zeroes of $p(x)$, put $2x^2 + 7x + 5 = 0$

$2x^2 + 5x + 2x + 5 = 0$

$\Rightarrow \quad x(2x + 5) + 1(2x + 5) = 0$

$\Rightarrow \quad (x + 1)(2x + 5) = 0$

$\Rightarrow \quad x = -1, x = -5/2$

$\therefore$ zeroes are -1 and $-5/2$.

30. Let α and β be the zeroes of the polynomial

$p(x) = x^2 + 2kx + k$

$\therefore \quad \alpha + \beta = -2k/1 = -2k \qquad \ldots(i)$

and $\alpha\beta = k/1 = k \qquad \ldots(ii)$

Also, $\alpha = \beta$ (given)

From (i), $\alpha + \alpha = -2k$

$\Rightarrow \quad 2\alpha = -2k \Rightarrow \alpha = -k \qquad \ldots(iii)$

From (ii), $\alpha \cdot \alpha = k$

$\Rightarrow \quad \alpha^2 = k \Rightarrow (-k)^2 = k \qquad$ [Using (iii)]

$\Rightarrow \quad k^2 - k = 0 \Rightarrow k(k - 1) = 0$

$\Rightarrow \quad k = 0$ or $k = 1$

So, the quadratic polynomial $p(x)$ will have equal zeroes at $k = 0$ and $k = 1$.

$\therefore$ $p(x)$ can have equal zeroes for some odd integer $k > 0$.

31. Let $p(x) = 6x^4 + 8x^3 - 5x^2 + ax + b$ and $g(x) = 2x^2 - 5$

On dividing $p(x)$ by $g(x)$, we have

$$\begin{array}{r|l} & 3x^2 + 4x + 5 \\ 2x^2 - 5 & 6x^4 + 8x^3 - 5x^2 + ax + b \\ & 6x^4 \qquad - 15x^2 \\ & (-) \qquad\quad (+) \\ \hline & 8x^3 + 10x^2 + ax + b \\ & 8x^3 \qquad - 20x \\ & (-) \qquad (+) \\ \hline & 10x^2 + (20 + a)x + b \\ & 10x^2 \qquad\qquad - 25 \\ & (-) \qquad\qquad (+) \\ \hline & (20 + a)x + (b + 25) \end{array}$$

$\because$ $p(x)$ is completely divisible by $g(x)$.

$\therefore \quad (20 + a)x + (b + 25) = 0$

$\Rightarrow \quad 20 + a = 0$ and $b + 25 = 0$

$\Rightarrow \quad a = -20$ and $b = -25$.

32. Given, α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$.

$$\therefore \quad \alpha + \beta = -\frac{\text{Coefficient of } s}{\text{Coefficient of } s^2} = \frac{-(-6)}{3} = \frac{6}{3} = 2$$

$$\text{and } \alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } s^2} = \frac{4}{3}$$

$$\text{Now, } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

$[\because a^2 + b^2 = (a + b)^2 - 2ab]$

$$= \frac{(2)^2 - 2(4/3)}{4/3} + 2\left(\frac{2}{4/3}\right) + 3 \times \frac{4}{3}$$

$[\because \alpha + \beta = 2 \text{ and } \alpha\beta = 4/3]$

$$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 2 \times 2 \times \frac{3}{4} + 4 = \frac{12 - 8}{3} \times \frac{3}{4} + 3 + 4$$

$$= \frac{4}{3} \times \frac{3}{4} + 7 = 1 + 7 = 8$$

33. Let the zeroes of the given polynomial $ax^2 + bx + b$ be $m\alpha$ and $n\alpha$.

$$\therefore \quad \text{Sum of zeroes, } m\alpha + n\alpha = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a}$$

$$\Rightarrow \quad \alpha(m + n) = \frac{-b}{a} \Rightarrow \alpha = \frac{-b}{a(m + n)} \qquad \ldots(i)$$

$$\text{and product of zeroes, } m\alpha \times n\alpha = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{b}{a}$$

$$\Rightarrow \quad mn\alpha^2 = \frac{b}{a} \Rightarrow mn\left[\frac{b^2}{a^2(m + n)^2}\right] = \frac{b}{a} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad \frac{mnb}{a(m + n)^2} = 1 \Rightarrow \frac{mn}{(m + n)^2} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{mn}{m^2 + 2mn + n^2} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{1}{\frac{m^2}{mn} + \frac{2mn}{mn} + \frac{n^2}{mn}} = \frac{a}{b}$$

[Dividing numerator and denominator of LHS by mn]

$$\Rightarrow \quad \frac{1}{\frac{m}{n} + 2 + \frac{n}{m}} = \frac{a}{b} \Rightarrow \frac{m}{n} + 2 + \frac{n}{m} = \frac{b}{a}$$

$$\Rightarrow \quad \left(\sqrt{\frac{m}{n}}\right)^2 + 2\sqrt{\frac{m}{n}}\sqrt{\frac{n}{m}} + \left(\sqrt{\frac{n}{m}}\right)^2 = \frac{b}{a} \qquad \left[\because 1 = \sqrt{\frac{m}{n}}\sqrt{\frac{n}{m}}\right]$$

$$\Rightarrow \quad \left(\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}\right)^2 = \frac{b}{a}$$

$$\therefore \quad \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \sqrt{\frac{b}{a}}$$

[Here, we take a positive square root, because values of $\sqrt{\frac{m}{n}}$ and $\sqrt{\frac{n}{m}}$ are always positive.]

34. Let $p(x) = 4x^4 - 2x^3 - 6x^2 + x - 5$

If we subtract $ax + b$ from $p(x)$, then resulting polynomial $f(x)$ should be exactly divisible by $2x^2 + x - 2$

$\therefore \quad f(x) = 4x^4 - 2x^3 - 6x^2 + x - 5 - (ax + b)$

$\quad\quad\quad = 4x^4 - 2x^3 - 6x^2 + (1 - a)x - (5 + b)$

Now, $f(x)$ must be completely divisible by $2x^2 + x - 2$.

$$\begin{array}{r|l} & 2x^2 - 2x \\ 2x^2 + x - 2 & 4x^4 - 2x^3 - 6x^2 + (1 - a)x - (5 + b) \\ & 4x^4 + 2x^3 - 4x^2 \\ & (-) \quad (-) \quad (+) \\ \hline & -4x^3 - 2x^2 + (1 - a)x - (5 + b) \\ & -4x^3 - 2x^2 + 4x \\ & (+) \quad (+) \quad (-) \\ \hline & (1 - a - 4)x - (5 + b) \end{array}$$

Since, remainder must be equal to 0.

$\therefore \quad (1 - a - 4)x - (5 + b) = 0$

$\Rightarrow \quad -[(a + 3)x + (5 + b)] = 0$

$\Rightarrow \quad (a + 3)x + (5 + b) = 0$

$\Rightarrow \quad a + 3 = 0$ and $5 + b = 0 \Rightarrow a = -3$ and $b = -5$

$\therefore$ Required number to be subtracted $= ax + b = -3x - 5$.

35. Here, dividend, $p(x) = x^3 - 3x^2 - 3x - 3$,

quotient, $q(x) = x^2 - 4x - 2$ and remainder, $r(x) = 3x - 1$

We know that, $p(x) = g(x)\, q(x) + r(x)$

$\Rightarrow \quad x^3 - 3x^2 - 3x - 3 = g(x) \cdot (x^2 - 4x - 2) + (3x - 1)$

$\Rightarrow \quad x^3 - 3x^2 - 3x - 3 - (3x - 1) = g(x)\, (x^2 - 4x - 2)$

$$\Rightarrow \quad g(x) = \frac{x^3 - 3x^2 - 3x - 3 - 3x + 1}{x^2 - 4x - 2} = \frac{x^3 - 3x^2 - 6x - 2}{x^2 - 4x - 2}$$

Now, we divide $(x^3 - 3x^2 - 6x - 2)$ by $(x^2 - 4x - 2)$

$$\begin{array}{r|l} & x + 1 \\ x^2 - 4x - 2 & x^3 - 3x^2 - 6x - 2 \\ & x^3 - 4x^2 - 2x \\ & (-) \quad (+) \quad (+) \\ \hline & x^2 - 4x - 2 \\ & x^2 - 4x - 2 \\ & (-) \quad (+) \quad (+) \\ \hline & 0 \end{array}$$

So, $g(x) = x + 1$.

36. Let $p(x) = x^3 - 3\sqrt{5}\,x^2 - 5x + 15\sqrt{5}$

Given, $x - \sqrt{5}$ is a factor of $p(x)$.

For other factors, we divide

$x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5}$ by $x - \sqrt{5}$.

$$\begin{array}{r|l} & x^2 - 2\sqrt{5}\,x - 15 \\ x - \sqrt{5} & x^3 - 3\sqrt{5}\,x^2 - 5x + 15\sqrt{5} \\ & x^3 - \sqrt{5}\,x^2 \\ & (-) \quad (+) \\ \hline & -2\sqrt{5}\,x^2 - 5x + 15\sqrt{5} \\ & -2\sqrt{5}\,x^2 + 10x \\ & (+) \quad (-) \\ \hline & -15x + 15\sqrt{5} \\ & -15x + 15\sqrt{5} \\ & (+) \quad (-) \\ \hline & 0 \end{array}$$

For other zeroes, Put $x^2 - 2\sqrt{5}x - 15 = 0$

$\Rightarrow \quad x^2 - 3\sqrt{5}x + \sqrt{5}x - 15 = 0$

$\Rightarrow \quad x(x - 3\sqrt{5}) + \sqrt{5}(x - 3\sqrt{5}) = 0$

$\Rightarrow \quad (x - 3\sqrt{5})(x + \sqrt{5}) = 0 \Rightarrow x = 3\sqrt{5}, x = -\sqrt{5}$

$\therefore \quad$ All zeroes of $p(x)$ are $\sqrt{5}, 3\sqrt{5}$ and $-\sqrt{5}$.