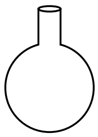
Surface Areas and Volumes

EXAM DRILL

SOLUTIONS

1. (a) : The shape of Surahi is as



, which is combination of sphere and cylinder.

2. (b) : Let the radius of the cone be r cm.

Since, the height and diameter of the base of the largest right circular cone = Edge of the cube.

$\therefore \quad 2r = 8 \text{ cm} \Rightarrow r = 4 \text{ cm}$

3. (a) : The radius of the greatest sphere that can be cut off from the cylinder = 1 cm

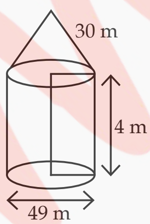
$\therefore \quad$ Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \text{ cm}^3$

4. (c) : Volume of a cone : Volume of a hemisphere : Volume of a cylinder

$= \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h = \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 \quad (\because r = h)$

$= 1 : 2 : 3$

5. (a) : Area of canvas required = Curved surface area of cylinder + Curved surface area of cone



$= 2\pi rh + \pi rl$

$= \pi r[2h + l] = \frac{22}{7} \times \frac{98}{2}[2 \times 4 + 30]$

$= 5852 \text{ m}^2$

6. (b) : Given, diameter of cylinder = 16 cm

$\therefore \quad$ Radius of cylinder (r) = 8 cm

Height of cylinder (h) = 2 cm

Volume of the cylinder $= \pi r^2 h = \pi \cdot (8)^2 \cdot 2 = 128\pi \text{ cm}^3$

Let R be the radius of sphere.

Volume of one sphere $= \frac{\text{Volume of the cylinder}}{12}$

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{128}{12}\pi \Rightarrow R^3 = 8 \Rightarrow R = 2 \text{ cm}$

Hence, radius of each sphere is 2 cm.

7. When, we join two solid hemispheres along their bases of radius r, we get a solid sphere. Also, curved surface area of a hemisphere is $2\pi r^2$.

Hence, the curved surface area of new solid $= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$.

8. Since sphere is recast into right circular cylinder.

$\therefore \quad$ Volume of cylinder = Volume of sphere

$\Rightarrow \quad \pi r^2 h = \pi$

where r and h are radius of base and height of cylinder

$\Rightarrow \quad (0.5)^2 h = 1 \qquad [\because r = 0.5 \text{ cm (Given)}]$

$\Rightarrow \quad \left(\frac{1}{2}\right)^2 h = 1 \Rightarrow h = 4 \text{ cm}$

9. Given, radius of spherical bullet = 1 mm $= \frac{1}{10}$ cm

Radius of cylinder (r) = 4 cm

Height of cylinder (h) = 6 cm

Number of spherical bullets

$= \frac{\text{Volume of cylinder}}{\text{Volume of a spherical bullet}} = \frac{\pi(4)^2 \times 6}{\frac{4}{3}\pi\left(\frac{1}{10}\right)^3}$

$= 4 \times 3 \times 6 \times (10)^3 = 72000$

10. Here, radius of two circular ends of frustum are $r_1 = 20$ cm, $r_2 = 17$ cm and height $h = 4$ cm

Slant height of frustum (l) $= \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{4^2 + (20 - 17)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ cm}$

11. Let the rainfall be x m.

Now, volume of water on roof = volume of cone

$\Rightarrow \quad 44 \times 10 \times x = \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 7$

$\Rightarrow \quad x = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{1}{44} \times \frac{1}{10}$

$\Rightarrow \quad x = \frac{1}{240} \text{ m} = \frac{1}{240} \times 100 \text{ cm} = \frac{5}{12} \text{ cm}$

Hence, required rainfall is 5/12 cm.

12. Let r be radius of sphere.

Since, cone is recast into a sphere.

$\therefore \quad$ Volume of sphere = Volume of cone

$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{1}{3}\pi \times 7 \times 7 \times 28 \quad \Rightarrow \quad r^3 = \frac{7 \times 7 \times 28}{4}$

$\Rightarrow \quad r^3 = 343 \Rightarrow r = 7 \text{ cm}$

$\therefore \quad$ Radius of the sphere = 7 cm

13. Radius of the sphere (r) $= \frac{18}{2} \text{ cm} = 9 \text{ cm}$

Radius of the cylinder (R) $= \frac{36}{2} \text{ cm} = 18 \text{ cm}$

Let us assume that the water level in the cylinder rises by h cm.

After the sphere is completely submerged,

Volume of the sphere = Volume of water raised in the cylinder

$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi R^2 h \quad \Rightarrow \quad \frac{4}{3}\pi(9)^3 = \pi(18)^2 \times h$

$\Rightarrow \quad h = \dfrac{4\times 9\times 9\times 9}{3\times 18\times 18} \quad \Rightarrow \quad h = 3$

Thus, the water level in the cylinder rises by 3 cm.

14. Let h be the height of the frustum of the cone.

Let r_1 and r_2 be radii of the ends of the frustum.

$\therefore \quad r_1 - r_2 = 4$ cm

Slant height (l) = 5 cm

$\because$ The slant height of the frustum of a cone is given by,

$l^2 = h^2 + (r_1 - r_2)^2$

$\Rightarrow \quad h^2 = l^2 - (r_1 - r_2)^2 \quad \Rightarrow \quad h^2 = (5)^2 - (4)^2 = 9$

$\Rightarrow \quad h = 3$ cm

Hence, height of frustum is 3 cm.

15. Volume of one cube = 64 cm^3

$\Rightarrow$ (Edge)^3 = 64 cm^3 $\Rightarrow$ Edge = 4 cm

Length of the cuboid (l) = 5 × Edge = 5 × 4 = 20 cm

breadth (b) = 4 cm and height (h) = 4 cm

$\therefore$ Surface area of cuboid = $2(lb + bh + hl)$

= 2[20 × 4 + 4 × 4 + 4 × 20] = 2 × 176 = 352 cm^2

Volume of the cuboid = $l \times b \times h$

= 20 × 4 × 4 = 320 cm^3

16. Let height of cylinder be h cm and radius be r cm.

Given, $h + r = 37$ cm

Total surface area of cylinder = $2\pi rh + 2\pi r^2$

$\Rightarrow \quad 2\pi r(h + r) = 1628 \Rightarrow 2\pi r(37) = 1628$

$\Rightarrow \quad r = \dfrac{1628\times 7}{2\times 22\times 37} = 7$

$\therefore \quad h + r = 37 \Rightarrow h + 7 = 37 \Rightarrow h = 30$

Hence, volume of cylinder = $\pi r^2 h$

$= \dfrac{22}{7}\times 7\times 7\times 30 = 4620$ cm^3

17. Here, radius of conical vessel (r) = 5 cm

Height of conical vessel (h) = 24 cm

Radius of the cylindrical vessel (R) = 10 cm

Let H cm be the height to which water rises in cylindrical vessel.

According to the question,

Volume of water in cylinder = Volume of water in cone

$\Rightarrow \quad \pi R^2 H = \dfrac{1}{3}\pi r^2 h \Rightarrow \pi\times 10\times 10\times H = \dfrac{1}{3}\pi\times 5^2\times 24$

$\therefore \quad H = \dfrac{25\times 8}{10\times 10} = 2$ cm

Hence, the height to which the water will rise in the cylindrical vessel is 2 cm.

18. Given that, side of a solid cube (a) = 7 cm

Height of conical cavity (h) = 7 cm

Radius of conical cavity (r) = 3 cm

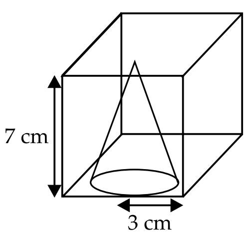
Now, volume of cube

= a^3 = $(7)^3$ = 343 cm^3

Volume of conical cavity

$= \dfrac{1}{3}\pi\times r^2\times h = \dfrac{1}{3}\times\dfrac{22}{7}\times 3\times 3\times 7$

= 66 cm^3



$\therefore$ Volume of remaining solid

= Volume of cube – Volume of conical cavity

= 343 – 66 = 277 cm^3

19. Here, radius of cylindrical portion (r) = Radius of conical portion (r) $= \dfrac{105}{2}$ m

Height of cylindrical portion (h) = 3 m

Slant height of conical portion (l) = 53 m

Total canvas used in making the tent

= Curved surface area of cylindrical portion + Curved surface area of conical portion

$= 2\pi rh + \pi rl = 2\times\dfrac{22}{7}\times\dfrac{105}{2}\times 3 + \dfrac{22}{7}\times\dfrac{105}{2}\times 53$

= 990 + 8745 = 9735 m^2

20. Radius of the hemispherical bowl (r) = 9 cm

$\therefore$ Volume of the water in hemispherical bowl

$= \dfrac{2}{3}\pi r^3 = \dfrac{2}{3}\pi(9)^3$ cm^3

Let height of water in the cylindrical vessel be h cm.

Also, radius of the cylinder (R) = 6 cm

$\therefore$ Volume of water in the cylindrical vessel = $\pi R^2 h$

= $\pi(6)^2 h$ cm^3

$\because$ Volume of water in cylindrical vessel = Volume of the water in hemispherical bowl

$\Rightarrow \quad \pi(6)^2 h = \dfrac{2}{3}\pi(9)^3 \Rightarrow h = \dfrac{2\times(9)^3}{3\times(6)^2}$

$\Rightarrow \quad h = \dfrac{27}{2} \quad \Rightarrow \quad h = 13.5$

Hence, height of water in cylindrical vessel is 13.5 cm

21. Given, radius of sphere (r) = 6 cm

$\therefore$ Volume of the sphere

$= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(6)^3 = 288\,\pi$ cm^3 ...(i)

Let the internal radius of cylinder be r cm.

External radius of cylinder (R) = 5 cm

Height of cylinder (h) = 32 cm

$\therefore$ Volume of hollow cylinder

= $\pi(R^2 - r^2)h = \pi(5^2 - r^2)32$ ...(ii)

According to the question,

Volume of the hollow cylinder = Volume of sphere

$\Rightarrow \quad 32(25 - r^2)\pi = 288\pi$ [Using (i) and (ii)]

$\Rightarrow \quad (25 - r^2) = \dfrac{288}{32} \Rightarrow 25 - r^2 = 9$

$\Rightarrow \quad r^2 = 16 \Rightarrow r = 4$ cm

$\therefore$ Uniform thickness of the cylinder = $R - r$

= 5 – 4 = 1 cm

22. Height of the bucket (h) = 15 cm

Radius of one end of bucket (R) = 14 cm

Radius of the other end of bucket = r cm

Volume of the bucket = 5390 cm^3

$\Rightarrow \quad \dfrac{1}{3}\pi(R^2 + r^2 + Rr)h = 5390$

$\Rightarrow \quad \dfrac{1}{3}\times\dfrac{22}{7}\times(14^2 + r^2 + 14r)\times 15 = 5390$

$\Rightarrow\ 196 + r^2 + 14r = \dfrac{5390 \times 7}{22 \times 5} = 343$

$\Rightarrow\ r^2 + 14r - 147 = 0 \Rightarrow r^2 + 21r - 7r - 147 = 0$

$\Rightarrow\ r(r + 21) - 7(r + 21) = 0$

$\Rightarrow\ (r + 21)(r - 7) = 0 \quad \Rightarrow \quad r = 7, -21$

$\therefore\ r = 7$ ($\because$ Radius can't be negative)

Hence, radius of other end of bucket is 7 cm.

23. Given, diameter of the base and top of frustum are 20 m and 6 m respectively.

$\therefore$ Radius of the base of frustum (r_1) = 10 m and Radius of the top of frustum (r_2) = radius of the base of cone (r_2) = 3 m

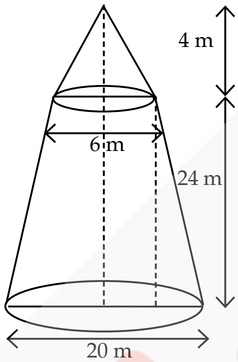
Height of the frustum (h) = 24 cm

Slant height of frustum (l)

$= \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(24^2) + (10-3)^2}$

$= \sqrt{576 + 49} = \sqrt{625} = 25$ cm



Curved surface area of the frustum

$= \pi(r_1 + r_2)l$

$= \dfrac{22}{7}(10 + 3) \times 25$

$= \dfrac{22}{7} \times 13 \times 25 = 1021.43 \text{ m}^2$

Height of the cone (H) = 28 – 24 = 4 m

Slant height of the cone $(l_1) = \sqrt{r_2^2 + H^2}$

$= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ m

Curved surface area of the cone

$= \pi r_2 l_1 = \dfrac{22}{7} \times 3 \times 5 = 47.14 \text{ m}^2$

Area of the canvas required = Curved surface area of frustum + Curved surface area of cone

$= 1021.43 + 47.14 = 1068.57 \text{ m}^2$

24. Diameter of a spherical lead shot = 4.2 cm

$\therefore$ Radius of a spherical lead shot $(r) = \dfrac{4.2}{2} = 2.1$ cm

So, volume of spherical lead shot

$= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3} \times \dfrac{22}{7} \times (2.1)^3 = \dfrac{4}{3} \times \dfrac{22}{7} \times 2.1 \times 2.1 \times 2.1$

$= \dfrac{4 \times 22 \times 21 \times 21 \times 21}{3 \times 7 \times 1000} \text{ cm}^3$

Now, length of solid lead piece (l) = 66 cm

Breadth of solid lead piece (b) = 42 cm

Height of solid lead piece (h) = 21 cm

$\therefore$ Volume of a solid lead piece (cuboid)

$= l \times b \times h = 66 \times 42 \times 21 \text{ cm}^3$

Since, spherical lead shots are made from a solid rectangular lead piece.

$\therefore$ Number of spherical lead shots

$= \dfrac{\text{Volume of solid rectangular lead piece}}{\text{Volume of a spherical lead shot}}$

$= \dfrac{66 \times 42 \times 21}{4 \times 22 \times 21 \times 21 \times 21} \times 3 \times 7 \times 1000$

$= 3 \times 500 = 1500$

Hence, the required number of spherical lead shots is 1500.

25. Given, length of the wall (l) = 24 m,

Thickness of the wall (b) = 0.4 m,

Height of the wall (h) = 6 m

So, volume of the wall constructed = $l \times b \times h$

$= 24 \times 0.4 \times 6 = \dfrac{24 \times 4 \times 6}{10} \text{ m}^3$

Now, $\dfrac{1}{10}^{\text{th}}$ volume of a wall

$= \dfrac{1}{10} \times \dfrac{24 \times 4 \times 6}{10} = \dfrac{24 \times 4 \times 6}{100} \text{ m}^3$

Also, length of a brick $(l_1) = 25 \text{ cm} = \dfrac{25}{100}$ m

Breadth of a brick $(b_1) = 16 \text{ cm} = \dfrac{16}{100}$ m

Height of a brick $(h_1) = 10 \text{ cm} = \dfrac{10}{100}$ m

So, volume of a brick = $l_1 \times b_1 \times h_1$

$= \dfrac{25}{100} \times \dfrac{16}{100} \times \dfrac{10}{100} = \dfrac{25 \times 16}{10^5} \text{ m}^3$

Now, number of bricks

$= \dfrac{(\text{Volume of wall}) - \left(\dfrac{1}{10} \times \text{Volume of wall}\right)}{\text{Volume of a brick}}$

$= \dfrac{\left(\dfrac{24 \times 4 \times 6}{10} - \dfrac{24 \times 4 \times 6}{100}\right)}{\left(\dfrac{25 \times 16}{10^5}\right)} = \dfrac{\dfrac{24 \times 4 \times 6}{100}(10 - 1)}{\dfrac{25 \times 16}{10^5}}$

$= \dfrac{24 \times 4 \times 6}{100} \times 9 \times \dfrac{10^5}{25 \times 16} = \dfrac{24 \times 4 \times 6 \times 9 \times 1000}{25 \times 16}$

$= 24 \times 6 \times 9 \times 10 = 12960$

Hence, the required number of bricks used in constructing the wall is 12960.