Areas Related to Circles

CHAPTER 12

EXAM DRILL

SOLUTIONS

1. (c) : Let the radius of the field be r.

Then, $\frac{\pi r^2}{2} = 3850$ [Given] $\Rightarrow \frac{1}{2}\times\frac{22}{7}\times r^2 = 3850$

$\Rightarrow r^2 = 3850 \times \frac{7}{11} = 2450 \Rightarrow r = 35\sqrt{2}$ m

Thus, perimeter of the field

$= \frac{22}{7}(35\sqrt{2}) + 2(35\sqrt{2}) = \sqrt{2}(110 + 70) = 180\sqrt{2}$ m

2. (a) : Let r be the radius of circle.

Area of circle = 1386 cm^2 [Given]

$\Rightarrow \frac{22}{7}\times r^2 = 1386$

$\Rightarrow r^2 = 441 \Rightarrow r = 21$ cm

3. (b) : Let radius of a circle be r and side of a square be a.

Perimeter of a circle = Perimeter of a square [Given]

$\therefore 2\pi r = 4a \Rightarrow a = \frac{\pi r}{2}$ ...(i)

Now, $\frac{\text{Area of a circle}}{\text{Area of a square}} = \frac{\pi r^2}{(a)^2} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2}$ [From (i)]

$= \frac{4}{\pi} = \frac{4}{22/7} = \frac{28}{22} = \frac{14}{11}$

4. (a) : Circumference of circle = 52.8 cm [Given]

$\Rightarrow 2\pi r = 52.8 \Rightarrow 2\times\frac{22}{7}\times r = 52.8 \Rightarrow r = 8.4$ cm

$\therefore$ Area of a circle $= \left(\frac{22}{7}\times 8.4\times 8.4\right)\text{cm}^2 = 221.76\ \text{cm}^2$

5. (a) : Area of circle, $A = \pi r^2 = \frac{r\times 2\pi r}{2} = \frac{rC}{2}$

$\Rightarrow 2A = rC$

6. (d) : Radius of a wheel = 14 cm [Given]

Circumference of the wheel $= 2\times\frac{22}{7}\times 14 = 88$ cm

Distance covered = Circumference of wheel × Number of rotations

= 88 × 50 = 4400 cm

7. (d) : Let a be the side of the square and R be the radius of the circle. Then, $a^2 = \pi R^2$ [Given]

$\Rightarrow \frac{R^2}{a^2} = \frac{1}{\pi} \Rightarrow \frac{R}{a} = \frac{1}{\sqrt{\pi}}$

Ratio of perimeters of circle and square $= \frac{2\pi R}{4a}$

$= \frac{R}{2}\times\frac{\pi}{a} = \frac{\pi}{2}\times\frac{1}{\sqrt{\pi}} = \frac{\sqrt{\pi}}{2}$

8. (d) : Here, r = 21 m, R = (21 + 7) m = 28 m

Area of circular path = $\pi(R + r)(R - r)$

$= \frac{22}{7}(28 + 21)(28 - 21) = 1078\ \text{m}^2$

9. (c) : Let r be the radius of a circle.

Now, $2\pi r - 2r = 207$ [Given]

$\Rightarrow 2r(\pi - 1) = 207 \Rightarrow 2r\left(\frac{22}{7} - 1\right) = 207$

$\Rightarrow 2r\times\frac{15}{7} = 207 \Rightarrow r = \frac{207\times 7}{15\times 2} = 48.3$ cm

10. (a) : Since the minute hand rotates through 6° in one minute.

$\therefore$ Required area = Area of sector with sector angle 6° and radius 7 cm

$= \frac{6°}{360°}\times\frac{22}{7}\times 7\times 7 = \frac{1}{60}\times 22\times 7 = 2.57\ \text{cm}^2$

11. The distance travelled by a wheel in one revolution is equal to its circumference *i.e.*, πd cm = $\pi(2r)$cm = $2\pi r$ cm [$\because d = 2r$]

12. If $r > 2$, then numerical value of area of a circle is greater than numerical value of its circumference.

13. Perimeter of a semi-circular protractor = Perimeter of a semi-circle = $(2r + \pi r)$ cm

$\therefore 2r + \pi r = 36$ [Given] $\Rightarrow 2r(1 + \pi) = 36$

$\Rightarrow 2r = \frac{36\times 7}{29} = 8.68$ cm = Diameter of protractor

14. Circumference of circle = 44 cm [Given]

$\Rightarrow 2\pi r = 44 \Rightarrow r = \frac{44\times 7}{2\times 22} = 7$ cm

$\therefore$ Area of quadrant of circle $= \frac{1}{4}\times\frac{22}{7}\times 7\times 7 = 38.5\ \text{cm}^2$

15. Here, $\theta = x°$ and radius, $R = 2r$

Area of sector $= \frac{\theta}{360°}\times\pi R^2 = \frac{x°}{360°}\times\pi(2r)^2 = \frac{x°}{90°}\pi r^2$

16. No. Since, diameter of the circle is equal to the breadth of the rectangle.

$\therefore$ The area of the largest circle that can be drawn inside a rectangle is $\pi\left(\frac{b}{2}\right)^2$ cm^2, where $\left(\frac{b}{2}\right)$ is the radius of the circle.

17. Let radius of circle be r and length of arc be l.

Perimeter of a sector of a circle = 24.4 cm [Given]

$\Rightarrow 2r + l = 24.4$

$\Rightarrow 2\times 4.3 + l = 24.4 \Rightarrow l = 24.4 - 8.6 = 15.8$ cm

Area of sector $= \frac{1}{2}\times lr = \frac{1}{2}\times 15.8\times 4.3 = 33.97\ \text{cm}^2$

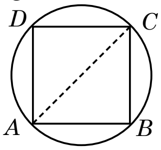
18. Let r be the radius of circle. It is given that

Area of sector of circle = $\frac{3}{18}$ × Area of the same circle

$\Rightarrow \frac{\theta}{360^\circ} \times \pi r^2 = \frac{3}{18} \times \pi r^2 \quad \Rightarrow \quad \theta = \frac{3}{18} \times 360^\circ = 60^\circ$

19. Let $ABCD$ be the square circumscribing a circle.

$AC = 2 \times$ Radius of circle $= 2r$ cm



In right ΔABC, $AB^2 + BC^2 = AC^2$

$\Rightarrow 2AB^2 = (2r)^2 \qquad (\because AB = BC)$

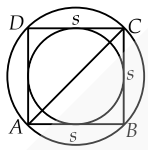
$\Rightarrow AB^2 = \frac{4r^2}{2} = 2r^2 \quad \Rightarrow \quad AB = \sqrt{2}r$ cm

$\therefore$ Perimeter of square $ABCD = 4 \times$ side $= 4\sqrt{2}\,r$ cm

20. Yes. If circumferences of two circles are equal, then their corresponding radii will be equal. So, their areas would be equal.

21. In ΔABC, by Pythagoras theorem

$AC^2 = AB^2 + BC^2 = s^2 + s^2 = 2s^2$



$\Rightarrow AC = s\sqrt{2}$

$\therefore$ Radius of larger circle, $R = \frac{s\sqrt{2}}{2}$

$\therefore$ Area of larger circle $= \pi R^2 = \pi\left(\frac{s\sqrt{2}}{2}\right)^2 = \frac{\pi s^2}{2}$

Radius of smaller circle, $r = \frac{s}{2}$

Area of smaller circle $= \pi r^2 = \pi\left(\frac{s}{2}\right)^2 = \frac{\pi s^2}{4}$

$\therefore$ Ratio of areas $= \frac{\pi s^2/4}{\pi s^2/2} = \frac{1}{2} = 1:2$

22. We know, that number of revolutions made by wheel $= \frac{\text{Distance covered}}{\text{Circumference of wheel}}$

$\Rightarrow 400 = \frac{16.72 \text{ km}}{\text{Circumference of wheel}}$

$\Rightarrow$ Circumference of wheel $= \frac{16.72}{400}$ km

$\Rightarrow 2\pi r = \left(\frac{16.72}{400} \times 1000 \times 100\right)$ cm

$\Rightarrow \frac{2 \times 22}{7} \times r = \frac{1672 \times 10}{4} \quad \Rightarrow 2r = 1330$

$\therefore$ Diameter of wheel = 1330 cm.

23. Let R and r be the radius of outer and inner circle respectively.

Given, $2\pi R = 88$ cm and $2\pi r = 66$ cm

$\Rightarrow R = \frac{88}{2\pi}$ cm and $r = \frac{66}{2\pi}$ cm

Width of the ring $= R - r = \frac{88}{2\pi} - \frac{66}{2\pi}$

$= \frac{1}{2} \times \frac{7}{22} \times 22 = 3.5$ cm

24. Given, $r_1 = 4$ cm and $r_2 = 3$ cm

Let r be the radius of new circle.

Area of first circle, $A_1 = \pi r_1^2 = \pi(4)^2 = 16\,\pi$ cm^2

Area of second circle, $A_2 = \pi r_2^2 = \pi(3)^2 = 9\,\pi$ cm^2

Area of new circle $= A_1 + A_2$ [Given]

$\Rightarrow \pi r^2 = 16\pi + 9\pi \quad \Rightarrow \pi r^2 = 25\pi$

$\Rightarrow r^2 = 25 \quad \Rightarrow r = 5$ cm

[Since radius cannot be negative so, $r \neq -5$]

Hence, the radius of new circle is 5 cm.

25. Here, sector angle, $\theta = 90^\circ$, $r = 14$ m

Area of the field graze by the horse, $A_1 = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times (14)^2$

$= \frac{3.14 \times 196}{4} = 153.86$ cm^2

Now, area of field $= (25)^2 = 625$ m^2

$\therefore$ Area of field not grazed by horse = Area of field $- A_1$

$= 625 - 153.86 = 471.14$ m^2

26. Since, ΔABC is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 60^\circ$ and $AB = BC = CA = 10$ cm

E, F and D are mid-points of the given sides.

$\therefore AE = EC = CD = DB = BF = FA = 5$ cm

Radius of a sector $(r) = 5$ cm

Now, area of sector $CDE = \frac{60^\circ}{360^\circ} \times 3.14 \times (5)^2$

$= \frac{78.5}{6} = 13.0833$ cm^2

$\therefore$ Area of shaded region = 3 × (Area of sector CDE)

$= (3 \times 13.0833)$ cm$^2 = 39.25$ cm^2

27. Let R and r be the outer and inner radius of circle respectively.

Area of circular park = 1386 m^2 [Given]

$\Rightarrow \pi r^2 = 1386 \quad \Rightarrow r^2 = 441 \quad \Rightarrow r = 21$ m

Now, cost to construct a track around a circular park at the rate of ₹ 3.50 per m^2 is ₹ 4440

$\therefore$ Area of track $= \frac{4440}{3.5}$ m^2

$\Rightarrow \pi R^2 - \pi r^2 = \frac{4440}{3.5} \quad \Rightarrow \frac{22}{7} \times R^2 - 1386 = \frac{4440}{3.5}$

$\Rightarrow \frac{22}{7} \times R^2 = \frac{4440}{3.5} + 1386 = \frac{4440 + 4851}{3.5} = \frac{9291}{3.5}$

$\Rightarrow R^2 = \frac{9291}{3.5} \times \frac{7}{22} = \frac{18582}{22} = 844.63$

$\Rightarrow R = 29.06$ cm

$\therefore$ Width of track $= R - r = 29.06 - 21 = 8.06$ cm

28. Let $OB = r = AO$

Perimeter of APB + Perimeter of $AQOB = 40$

$\Rightarrow \pi r + \frac{1}{2}\pi r + r = 40$

$\Rightarrow r\left(\frac{3}{2}\pi + 1\right) = 40 \Rightarrow r\left(\frac{3}{2} \times \frac{22}{7} + 1\right) = 40$

$\Rightarrow r\left(\frac{33}{7} + 1\right) = 40 \quad \Rightarrow 40r = 280 \quad \Rightarrow r = 7$ cm

$\therefore$ Shaded area $= \dfrac{\pi(r/2)^2}{2} + \dfrac{\pi r^2}{2}$

$= \dfrac{1}{2}\times\dfrac{22}{7}\times\dfrac{7}{2}\times\dfrac{7}{2}+\dfrac{1}{2}\times\dfrac{22}{7}\times 7\times 7$

$= \dfrac{77}{4}+77=\dfrac{77+308}{4}=\dfrac{385}{4}=96.25\text{ cm}^2$

29. Area of region $ABDC$ = (Area of sector OAB) – (Area of sector OCD)

$= \dfrac{\pi(OA)^2\times 60^\circ}{360^\circ}-\dfrac{\pi(OC)^2\times 60^\circ}{360^\circ}$

$= \dfrac{\pi}{6}\times(42)^2-\dfrac{\pi}{6}\times(21)^2$ [$\because$ OA = 42 cm and OC = 21 cm]

$= \dfrac{\pi}{6}(1764-441)=\dfrac{1323\pi}{6}\text{ cm}^2$

Area of circular ring = $\pi[(42)^2 - (21)^2]$

= $\pi(1764 - 441) = 1323\,\pi$ cm^2

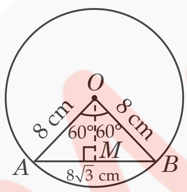
$\therefore$ Area of shaded region = Area of circular ring – Area of region $ABDC$

$= 1323\pi-\dfrac{1323\pi}{6}=1323\pi\left(1-\dfrac{1}{6}\right)$

$= 1323\times\dfrac{22}{7}\times\dfrac{5}{6}=3465\text{ cm}^2$

30. Given diameter of circle = 16 cm

$\therefore$ Radius of circle = $\dfrac{16}{2}=8\text{ cm}$



Draw, $OM \perp AB$

Then, OM bisects the side AB and also bisects $\angle AOB$ ($\because$ $OA = OB$ = 8 cm)

$\Rightarrow AM = BM = \dfrac{1}{2}AB=\dfrac{8\sqrt{3}}{2}=4\sqrt{3}\text{ cm}$

In right ΔAOM, $\sin\angle AOM=\dfrac{AM}{OA}=\dfrac{4\sqrt{3}}{8}=\dfrac{\sqrt{3}}{2}$

$\Rightarrow$ $\angle AOM = 60^\circ$ $\left(\because \sin 60^\circ=\dfrac{\sqrt{3}}{2}\right)$

$\therefore$ $\angle AOB = 2\angle AOM = 2\times 60^\circ = 120^\circ$

and $\cos 60^\circ=\dfrac{OM}{OA}\Rightarrow\dfrac{1}{2}=\dfrac{OM}{8}\Rightarrow OM = 4$ cm

Area of minor segment = Area of sector AOB – Area of ΔAOB

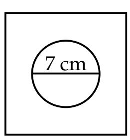
$= \dfrac{\theta}{360^\circ}\times\pi r^2-\dfrac{1}{2}\times AB\times OM$

$= \dfrac{120^\circ}{360^\circ}\times\dfrac{22}{7}\times 8\times 8-\dfrac{1}{2}\times 8\sqrt{3}\times 4$

$= 67.04 - 16\times 1.732 = 39.328$ cm^2

31. Diameter of disc = 7 cm

$\therefore$ Radius of disc, $r=\dfrac{7}{2}$ cm



Area of disc = $\pi r^2=\dfrac{22}{7}\times\dfrac{7}{2}\times\dfrac{7}{2}$

$= \dfrac{11\times 7}{2}=38.5\text{ cm}^2$

Area of aluminium plate = (side)^2 = $(12.5)^2$ = 156.25 cm^2

Area of remaining part = Area of aluminium plate – Area of disc

= 156.25 – 38.5 = 117.75

Now, weight of 1 cm^2 plate = 0.8 g

$\therefore$ Weight of 117.75 cm^2 plate = 0.8 × 117.75 = 94.2 g

32. AB is the diameter of the circle, $\angle ACB = 90^\circ$ [$\because$ Angle in a semi-circle is 90°]

In right ΔACB,

$AC^2 + BC^2 = AB^2$ [By Pythagoras theorem]

$\Rightarrow$ $15^2 + 8^2 = AB^2 \Rightarrow 225 + 64 = AB^2$

$\Rightarrow$ $AB=\sqrt{289}=17$ cm

$\therefore$ Radius of circle, $r=\dfrac{AB}{2}=\dfrac{17}{2}$ cm

Area of $\Delta ACB=\dfrac{1}{2}\times BC\times AC=\dfrac{1}{2}\times 8\times 15=60\text{ cm}^2$

Area of ΔABD= 60 cm^2 ($\because$ ΔACB is congruent to ΔABD)

Area of circle = $\pi r^2 = 3.14\times\dfrac{17}{2}\times\dfrac{17}{2}=226.865\text{ cm}^2$

$\therefore$ Area of shaded region = Area of the circle – (Area of ΔACB + Area of ΔABD)

= 226.865 – 2 × 60 = 106.87 cm^2

33. Here, $\angle AOB = 90^\circ$

$\therefore$ ΔAOB is a right angled triangle.

$OB = OA$ = 10.5 + 3.5 = 14 cm

Area of $\Delta AOB=\dfrac{1}{2}\times OB\times OA=\dfrac{1}{2}\times 14\times 14=98\text{ cm}^2$

Area of sector $ODC=\dfrac{\theta}{360^\circ}\times\pi r^2$

$= \dfrac{90^\circ}{360^\circ}\times\dfrac{22}{7}\times 10.5\times 10.5$

$= \dfrac{1}{4}\times 22\times 1.5\times 10.5 = 86.625\text{ cm}^2$

Area of shaded part = Area of ΔAOB – Area of sector ODC

= 98 – 86.625 = 11.375 cm^2

Cost of silver plated of 1 cm^2 shaded part = ₹ 50

$\therefore$ Cost of silver plated of 11.375 cm^2 shaded part = ₹ (50 × 11.375) = ₹ 568.75

Total cost awarded by school = 15 × 568.75 = ₹ 8531.25

34. Let height of trapezium = h cm

Area of trapezium = $\dfrac{1}{2}$(Sum of parallel sides) × height

$\therefore$ $\dfrac{1}{2}(10+4)\times h=24.5$ [Given]

$\Rightarrow$ $(14)h = 49 \Rightarrow h=\dfrac{49}{14}=3.5$ cm

$\therefore$ Radius of sector ABE = 3.5 cm

Area of sector $ABE=\dfrac{\theta}{360^\circ}\times\pi r^2$

$= \dfrac{90^\circ}{360^\circ}\times\dfrac{22}{7}\times 3.5\times 3.5=\dfrac{1}{4}\times 22\times 0.5\times 3.5=9.625\text{ cm}^2$

$\therefore$ Area of shaded region = Area of trapezium – Area of sector ABE

$= (24.5 - 9.625)\ \text{cm}^2 = 14.875\ \text{cm}^2$

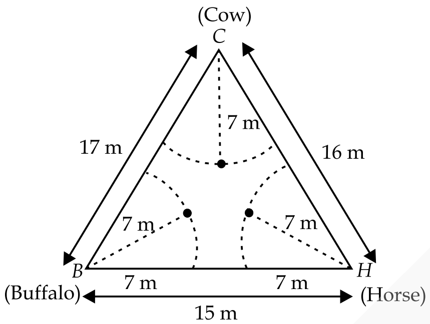
35. Given that, a triangular field CBH with the three corners of the field a cow, a buffalo and a horse are tied respectively. So, each animal grazed the field in each corner of triangular field is forming a sector.

Given, radius of each sector (r) = 7 m

Now, area of sector with $\angle C$

$$= \frac{\angle C}{360^\circ} \times \pi r^2 = \frac{\angle C}{360^\circ} \times \pi \times (7)^2\ \text{m}^2$$

Area of the sector with $\angle B = \frac{\angle B}{360^\circ} \times \pi \times (7)^2\ \text{m}^2$



Area of sector with $\angle H = \frac{\angle H}{360^\circ} \times \pi \times (7)^2\ \text{m}^2$

$\therefore$ Sum of the areas of the three sectors

$$= \frac{\angle C}{360^\circ} \times \pi \times (7)^2 + \frac{\angle B}{360^\circ} \times \pi \times (7)^2 + \frac{\angle H}{360^\circ} \times \pi \times (7)^2$$

$$= \frac{(\angle C + \angle B + \angle H)}{360^\circ} \times \pi \times 49$$

$$= \frac{180^\circ}{360^\circ} \times \frac{22}{7} \times 49 \quad (\because \text{Sum of angles of a triangle is } 180^\circ)$$

$= 11 \times 7 = 77\ \text{m}^2$

Given that, sides of triangle are a = 15 m, b = 16 m and c = 17 m

Now, Semi-perimeter of triangle $(s) = \frac{a+b+c}{2}$

$$= \frac{15+16+17}{2} = \frac{48}{2} = 24\,\text{m}$$

$\therefore$ Area of triangular field CBH

$= \sqrt{24(24-15)(24-16)(24-17)}$ [By Heron's formula]

$= \sqrt{24 \times 9 \times 8 \times 7} = 8 \times 3\sqrt{21} = 24\sqrt{21}\ \text{m}^2$

So, area of the field which cannot be grazed by the three animals = Area of triangular field CBH – Sum of areas of three sectors

$$= (24\sqrt{21} - 77)\ \text{m}^2$$

36. We know that tangent to a circle is perpendicular to radius.

$\therefore$ $OA \perp AP \Rightarrow \angle OAP = 90^\circ$

In right ΔOAP,

$AP^2 = OP^2 - OA^2$ [By Pythagoras theorem]

$\Rightarrow$ $(AP)^2 = (10)^2 - (5)^2 = 100 - 25 = 75$

$\Rightarrow$ $AP = 5\sqrt{3}$ cm

$\therefore$ Area of ΔAOP

$$= \frac{1}{2} \times OA \times AP = \frac{1}{2} \times (5) \times (5\sqrt{3}) = \frac{25\sqrt{3}}{2}\ \text{cm}^2$$

In ΔAOP and ΔBOP,

$AO = BO$ [Radii of circle]

$\angle OAP = \angle OBP$ [Each 90°]

$OP = OP$ [Common]

$\therefore$ $\Delta AOP \cong \Delta BOP$ [By RHS congruency criteria]

$\Rightarrow$ $\angle AOP = \angle BOP$ [By CPCT] ...(i)

$\Rightarrow$ Area of $\Delta BOP = \frac{25\sqrt{3}}{2}\ \text{cm}^2$

In right ΔAOP, $\cos\theta = \frac{OA}{OP}$

$\Rightarrow \cos\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = 60^\circ = \angle BOP$ [From (i)]

$\therefore$ $\angle AOB = 120^\circ$

Hence, area of sector $OACB$

$$= \frac{120^\circ}{360^\circ} \times 3.14 \times 5 \times 5 = 26.17\ \text{cm}^2$$

$\therefore$ Area of shaded region = Area of ($\Delta AOP + \Delta BOP$) – Area of sector $OACB$

$= (25\sqrt{3} - 26.17) = (25 \times 1.73 - 26.17) = 17.08\ \text{cm}^2$

Length of belt which is still in contact with the pulley

$= \frac{240^\circ}{360^\circ} \times 2\pi(5)$ [$\because$ Sector angle = 360° – 120° = 240°]

$= \frac{2}{3} \times 2 \times 3.14 \times 5 = 20.93$ cm

37. Length of arc $PA = \frac{\theta}{360^\circ} \times 2\pi r$

In ΔOAB, $\tan\theta = \frac{AB}{OA} \Rightarrow AB = r\tan\theta$ ...(i) ($\because OA = r$)

Now, $\sec\theta = \frac{BO}{r} \Rightarrow BO = r\sec\theta$

Length of $BP = OB - OP = r\sec\theta - r$

So, perimeter of shaded region = Length of arc $PA + AB + BP$

$$= \frac{\theta}{360^\circ} \times 2\pi r + r\tan\theta + r\sec\theta - r = r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right]$$

Area of sector OPA with sector angle, $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Area of $\Delta OAB = \frac{1}{2} \times OA \times AB$

$$= \frac{1}{2} \times r \times r\tan\theta = \frac{1}{2} r^2 \tan\theta$$

Area of shaded region = Area of ΔOAB – Area of sector OPA

$$= \frac{1}{2} r^2 \tan\theta - \frac{\theta}{360^\circ} \pi r^2$$

$$= \frac{1}{2} r^2 \left(\tan\theta - \frac{\pi\theta}{180^\circ}\right)$$

38. Area of the square lawn $ABCD = (56 \times 56)\ \text{m}^2$ ...(i)

Let $OA = OB = x$ m

In ΔAOB,

$x^2 + x^2 = 56^2$ [By Pythagoras theorem]

$\Rightarrow$ $2x^2 = 56 \times 56 \Rightarrow x^2 = 28 \times 56$ ...(ii)

Now, area of sector OAB = $\frac{90°}{360°}\times\pi x^2=\frac{1}{4}\times\pi x^2$

$=\left(\frac{1}{4}\times\frac{22}{7}\times28\times56\right)$ m^2 [From (ii)] ...(iii)

Also, area of ΔOAB = $\left(\frac{1}{2}\times28\times56\right)$ m^2 ($\because \angle AOB = 90°$) ...(iv)

So, area of flower bed AB

$=\left(\frac{1}{4}\times\frac{22}{7}\times28\times56-\frac{1}{2}\times28\times56\right)$m^2 [From (iii) and (iv)]

$=\frac{1}{4}\times28\times56\left(\frac{22}{7}-2\right)=\left(\frac{1}{4}\times28\times56\times\frac{8}{7}\right)$m^2 ...(v)

Similarly, area of the other flower bed

$=\left(\frac{1}{4}\times28\times56\times\frac{8}{7}\right)$m^2 ...(vi)

Therefore, total area

$=\left(56\times56+\frac{1}{4}\times28\times56\times\frac{8}{7}+\frac{1}{4}\times28\times56\times\frac{8}{7}\right)$m^2

[From (i), (v) and (vi)]

$=\left[28\times56\left(2+\frac{2}{7}+\frac{2}{7}\right)\right]$m^2 $=4032$ m^2

39. Perimeter of the shaded region = Length of $\overset{\frown}{APB}$ + Length of $\overset{\frown}{ARC}$ + Length of $\overset{\frown}{CQD}$ + Length of $\overset{\frown}{DSB}$

Now, perimeter of $\overset{\frown}{APB}$ = Perimeter of $\overset{\frown}{CQD}$

$=\frac{1}{2}\times2\pi\left(\frac{7}{2}\right)=\frac{22}{7}\times\frac{7}{2}=11$ cm

Perimeter of $\overset{\frown}{ARC}$ = Perimeter of $\overset{\frown}{DSB}$

$=\frac{1}{2}\times2\pi(7)=\frac{22}{7}\times7=22$ cm

Thus, perimeter of the shaded region
= 2 × (11) + 2 × (22) = 66 cm

40. In right ΔABC, $AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC = \sqrt{(6)^2+(6)^2}=6\sqrt{2}$ cm

$\therefore$ Radius of the circle, $\frac{6\sqrt{2}}{2}=3\sqrt{2}$ cm

(i) The sum of the areas of the two designed segments made by the chords AB and BC
= (Area of the semi-circle in which ΔABC is inscribed) – (Area of ΔABC)

$=\left(\frac{1}{2}\pi\times(3\sqrt{2})^2\right)-\left(\frac{1}{2}\times6\times6\right)$

$=\left(\frac{1}{2}\times3.14\times18\right)-18$ = 28.26 – 18 = 10.26 cm^2

(ii) The area of the designed segment made by the chord PQ = Area of sector OPQ – Area of ΔOPQ

$=\frac{90°}{360°}\times\pi r^2-\frac{1}{2}\times OP\times OQ$

$=\frac{1}{4}\times3.14\times(3\sqrt{2})^2-\frac{1}{2}\times3\sqrt{2}\times3\sqrt{2}$

= 14.13 – 9 = 5.13 cm^2

(iii) Now, area of the total designed part
= (10.26 + 5.13) cm^2 = 15.39 cm^2

$\therefore$ Total cost of making the designs at the rate of ₹ 10.25 per cm^2 = ₹ (10.25 × 15.39) = ₹ 157.75