Circles

EXAM DRILL

SOLUTIONS

1. **(d) :** Since, tangent is perpendicular to the radius through the point of contact.

$\therefore$ $OA \perp AP$

$\therefore$ By Pythagoras theorem, in right angle ΔAOP

$OA^2 = OP^2 - PA^2 = 10^2 - 8^2 = 36 \Rightarrow OA = 6$ cm

$\therefore$ $OB = OA = 6$ cm [Radii of the same circle]

2. **(c) :** We know that length of tangents drawn from an external point to the circle are equal.

$\therefore$ $BR = BP = 5$ cm, $AR = AQ = 3$ cm

and $QC = PC = 7 - 3 = 4$ cm

So, $BC = BP + PC = 5 + 4 = 9$ cm

3. **(c) :** Since, tangent is perpendicular to the radius through the point of contact.

$\therefore$ $\angle OTP = 90°$

In ΔOTP, $OP^2 = OT^2 + PT^2$ [By Pythagoras theorem]

$\Rightarrow$ $10^2 = 6^2 + PT^2 \Rightarrow PT^2 = 100 - 36 = 64 \Rightarrow PT = 8$ cm

4. **(c) :** $CR = CQ = 3$ cm, $BQ = BP = 5$ cm, $AS = AP = 6$ cm and $DS = DR = 4$ cm

$\therefore$ Perimeter of quadrilateral $ABCD = [(6 + 5) + (5 + 3) + (3 + 4) + (4 + 6)]$ cm $= (11 + 8 + 7 + 10)$ cm $= 36$ cm.

5. We have, $\angle AOB + \angle APB = 180°$

[$\because$ $\angle AOB$ and $\angle APB$ are supplementary]

$\Rightarrow$ $\angle APB = 180° - 107° = 73°$

6. Since, $AB \,||\, PR$ and $QOL \perp AB$ ($\because$ $OQ \perp PR$)

$\therefore$ OL bisects chord AB.

$\therefore$ ΔAQB is isosceles.

$\Rightarrow$ $\angle LQA = \angle LQB$

But, $\angle LQB = 90° - 67° = 23°$

$\therefore$ $\angle AQB = \angle LQA + \angle LQB = 2(23°) = 46°$

7. We have, $AB = 7$ cm, $BC = 9$ cm and $CA = 6$ cm

Now, $AR = AP = r$ (say) [Radii of the same circle]

$BP = BQ = x$ (say)

$CR = CQ = y$ (say)

$\therefore$ $r + x = 7$ ...(i)

$x + y = 9$ ...(ii)

$y + r = 6$ ...(iii)

Subtracting (ii) from (i), we get

$r - y = -2$ ...(iv)

Adding (iii) and (iv), we get

$2r = 4 \Rightarrow r = 2$ cm

8. Since, tangents drawn from an external point are equal.

$\therefore$ $BQ = BR$ [Tangents from B] ...(i)

$CQ = CP$ [Tangents from C] ...(ii)

Now, $BC + BQ = CQ = 11$ [Using (ii)]

$\Rightarrow$ $7 + BQ = 11$

$\Rightarrow$ $BQ = 11 - 7 = 4$ cm

$\therefore$ $BR = 4$ cm [Using (i)]

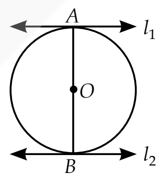
9. We have, $\angle OAT = 90°$ [$\because$ Tangent is perpendicular to the radius through the point of contact.]

In right angle ΔOAT,

$$\frac{AT}{OT} = \cos 30° \Rightarrow \frac{AT}{8} = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $AT = 4\sqrt{3}$ cm

10. Two parallel tangents of a circle can be drawn only at the end points of the diameter.



In figure, $l_1 \,||\, l_2$

$\Rightarrow$ Distance between l_1 and l_2, AB

$=$ Diameter of the circle

$= 2 \times r = 2 \times 9 = 18$ cm

11. Given, $BC = 4.5$ cm

$BC = CP$ [$\because$ Tangents drawn from an external point are equal.]

$\Rightarrow$ $CP = 4.5$ cm

Now, $AC = CP = 4.5$ cm [$\because$ Tangents from an external point are equal.]

$\therefore$ $AB = AC + BC = 4.5 + 4.5 = 9$ cm

12. Since, tangent is perpendicular to the radius through the point of contact.

$\therefore$ $\angle OPT = 90°$

$\therefore$ $\angle OPQ = 90° - 50° = 40°$

Also, $OP = OQ$ [Radii of same circle]

$\Rightarrow$ $\angle OQP = \angle OPQ = 40°$

$\therefore$ $\angle POQ = 180° - (40° + 40°) = 100°$.

13. $\therefore$ PQ is a diameter [Given]

$\therefore$ $\angle QOR + \angle ROP = 180°$ [Linear pair]

$\Rightarrow$ $\angle QOR = 180° - 70° = 110°$

Also, $OQ = OR$ [Radii of same circle]

$\Rightarrow$ $\angle RQO = \angle ORQ$ [$\because$ Angles opposite to equal sides of triangle are equal.]

$$= \frac{180° - 110°}{2} = \frac{70°}{2} = 35° \quad \text{...(i)}$$

Also, $QP \perp PT$ [$\because$ Tangent is perpendicular to the radius through the point of contact]

$\Rightarrow$ $\angle QPT = 90°$ ...(ii)

In ΔQPT, $\angle RQO + \angle QPT + x = 180°$

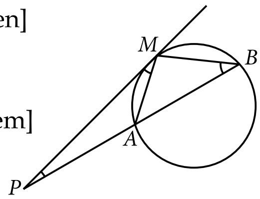
$\therefore \quad x = 180° - 90° - 35°$ [Using (i) and (ii)]
$= 55°$

14. $PA = AM$ [Given]

$\therefore \quad \angle APM = \angle PMA$ ...(i)

Also, $\angle PMA = \angle MBA$ ...(ii)

[By alternate segment theorem]



$\Rightarrow \quad \angle MPB = \angle MBP$

[Using (i) and (ii)]

$\Rightarrow \quad MB = PM$ ...(iii)

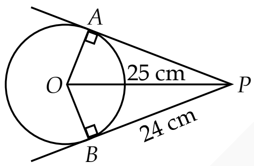
$\therefore \quad \Delta PMB$ is isosceles.

Now, we know that $PM^2 = PA \times PB$,

$\therefore \quad MB^2 = PA \times PB$, [From (iii)]

which means both statements 'A' and 'B' are true.

15. Since, tangents drawn from an external point are equal.



$\therefore \quad PA = PB = 24$ cm.

Also, $\angle OBP = 90°$

[Since, tangent is perpendicular to the radius through the point of contact.]

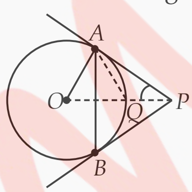
In ΔPOB, we have

$OP^2 = OB^2 + BP^2$ [By Pythagoras theorem]

$\Rightarrow \quad 25^2 = OB^2 + 24^2$

$\Rightarrow \quad OB^2 = 625 - 576 = 49 \Rightarrow OB = 7$ cm

16. Since tangent is perpendicular to the radius through the point of contact.



$\therefore \quad \angle OAP = 90°$

Now, in ΔOAP,

$$\sin(\angle OPA) = \frac{OA}{OP} = \frac{r}{2r} = \frac{1}{2}$$

$\Rightarrow \quad \angle OPA = 30°$

$\therefore \quad \angle APB = 2(\angle OPA) = 2 \times 30° = 60°$ ...(i)

Also, $AP = PB$ [$\therefore$ Tangents drawn from an external point are equal.]

$\therefore \quad \angle PAB = \angle PBA$ ...(ii)

In ΔPAB, $\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow \quad 2\angle PAB = 180° - 60° = 120°$ [Using (i) and (ii)]

$\Rightarrow \quad \angle PAB = 60°$

Hence, $\angle PAB = \angle PBA = \angle APB = 60°$

$\therefore \quad \Delta APB$ is an equilateral triangle.

17. Since angle made by an arc at the centre of a circle is twice the angle subtended by the same arc at any point on the remaining part of the circle.

$\therefore \quad \angle AOQ = 2\angle ABQ$

$\Rightarrow \quad \angle ABQ = \frac{1}{2} \times 78° = 39°$

In ΔABT, $\angle BAT + \angle ABT + \angle ATB = 180°$

$\Rightarrow \quad 90° + 39° + \angle ATB = 180°$

$\Rightarrow \quad \angle ATB = 51°$

$\therefore \quad \angle ATQ = 51°$

18. We have, $\angle APB = 50°$

Now, $PA = PB$ [$\because$ Tangents drawn from an external point are equal]

$\Rightarrow \quad \angle PAB = \angle PBA$

In ΔPAB, $\angle PAB + \angle PBA + \angle PAB = 180°$

$\Rightarrow \quad 2\angle PAB = 180° - 50° \Rightarrow \angle PAB = \frac{130°}{2} = 65°$

Now, $\angle OAB = 90° - \angle PAB$ [$\because OA \perp AP \Rightarrow \angle OAP = 90°$]
$= 90° - 65° = 25°$

19. From the figure, it is clear that O and Q are centres of smaller and bigger circle respectively.

Now, $OT = OQ = \frac{1}{2}(PQ) = \frac{14}{2} = 7$ cm

$\therefore \quad OR = 7 + 14 = 21$ cm

$\angle OTR = 90°$ [$\because$ Tangent is perpendicular to the radius through the point of contact.]

In right ΔOTR,

$OT^2 + RT^2 = OR^2$ [By Pythagoras theorem]

$\Rightarrow \quad (7)^2 + RT^2 = (21)^2 \Rightarrow RT^2 = 441 - 49 = 392$

$\Rightarrow \quad RT^2 = 14 \times 14 \times 2 \Rightarrow RT = 14\sqrt{2}$ cm

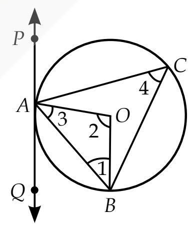
20. We have, $OA = OB$ [Radii of the same circle]

$\Rightarrow \quad \angle 3 = \angle 1 = 35°$

[$\because$ Angles opposite to equal sides of a triangle are equal]

But, $\angle 1 + \angle 2 + \angle 3 = 180°$

[By angle sum property]



$\Rightarrow \quad 35° + 35° + \angle 2 = 180°$

$\Rightarrow \quad \angle 2 = 180° - 70° = 110°$

Also, $\angle 4 = \frac{1}{2}\angle 2$

[Since angle made by an arc at the centre of a circle is twice the angle subtended by the same arc at any point on the remaining part of the circle.]

$$= \frac{1}{2} \times 110° = 55°$$

$\Rightarrow \quad \angle ACB = 55°$

$\therefore \quad$ By alternate segment theorem, $\angle BAQ = \angle ACB = 55°$

21. We have, $OP \perp OQ$

Also, $OP \perp PT$ and $OQ \perp TQ$

[$\because$ Tangent is perpendicular to the radius through the point of contact]

$\therefore \quad$ In quadrilateral $OPTQ$, $\angle P = \angle Q = \angle O = 90°$ ...(i)

Now, $\angle P + \angle Q + \angle O + \angle T = 360°$

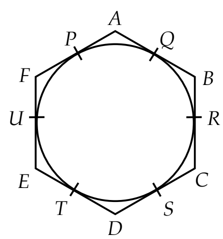
$\Rightarrow \quad \angle T = 360° - (90 + 90° + 90°)$
$= 360° - 270° = 90°$ ...(ii)

$OP = OQ$ [Radii of same circle] ....(iii)

We have, $OPTQ$ is a square

Hence, PQ and OT are right bisectors of each other.

22. Given, a hexagon $ABCDEF$ circumscribes a circle.



Since, tangents from an external point are equal.

$\therefore \quad AQ = AP$, $QB = BR$, $CS = CR$, $DS = DT$, $EU = ET$, $UF = FP$

Now, $AB + CD + EF = (AQ + QB) + (CS + SD) + (EU + UF)$

$= (AP + BR) + (CR + DT) + (ET + FP)$

$= (AP + FP) + (BR + CR) + (DT + ET)$

$= AF + BC + DE$

23. Join OC

Now, $OC \perp CD$ [$\because$ Tangent is perpendicular to the radius through the point of contact]

$\Rightarrow \quad \angle 2 + \angle 3 = 90°$

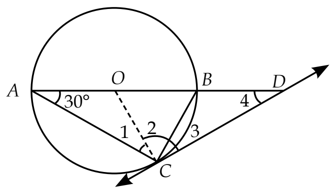
Also, $OC = OA \Rightarrow \angle 1 = 30°$

Now, $\angle 1 + \angle 2 = 90°$

[Angle in a semicircle]

$\therefore \quad \angle 2 = 90° - 30° = 60°$

$\Rightarrow \quad \angle 3 = 30°$



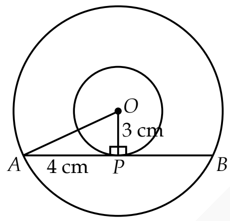
In ΔACD, $\angle ACD + \angle CAD + \angle 4 = 180°$

$\Rightarrow \quad (30° + 60° + 30°) + 30° + \angle 4 = 180° \Rightarrow \angle 4 = 30°$

In ΔBCD, $\angle 3 = \angle 4 \quad \therefore BC = BD$.

24. $OP \perp AB$ [$\because$ Tangent is perpendicular to the radius through the point of contact]

$\therefore \quad AP = BP$ [$\because AB$ is chord to larger circle and $OP \perp AB$]



$\therefore \quad AP = \frac{8}{2} = 4$ cm [$\because AB = 8$ cm]

$OP = \frac{6}{2} = 3$ cm [$\because$ Diameter of smaller circle = 6 cm]

In right ΔOAP, $OA^2 = OP^2 + AP^2$

$= 3^2 + 4^2 = 9 + 16 = 25 \Rightarrow OA = 5$ cm

Thus, diameter of the larger circle is 10 cm.

25. Given, a circle with center O. AB is the diameter of this circle. HK is tangent to the circle at P. AH and BK are perpendicular to HK from A and B at H and K respectively.

Since, AH and HP are tangents from the external point H.

$\therefore \quad AH = HP$ ...(i)

Also, KB and KP are tangents from the external point K.

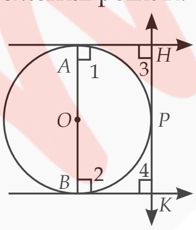
$\therefore \quad BK = KP$ ...(ii)

Adding (i) and (ii), we get

$AH + BK = HP + PK = HK$ ...(iii)

$AB \perp AH$ and $AB \perp BK$

[$\because$ Tangent is perpendicular to the radius through the point of contact]



$\therefore \quad \angle 1 = \angle 2 = 90°$

Also, $AH \perp HK$ [Given]

$\Rightarrow \quad \angle 3 = 90°$

and $BK \perp HK \Rightarrow \angle 4 = 90°$

Thus, $\angle 1 = \angle 2 = \angle 3 = \angle 4 = 90°$

$\therefore \quad AHKB$ is a rectangle.

$\Rightarrow \quad AB = HK$ ...(iv)

[$\because$ Opposite sides of a rectangle are equal]

From (iii) and (iv), $AH + BK = AB$

26. Since, length of tangents drawn form an external point to a circle are equal.

$\therefore \quad QS = QT = 14$ cm,

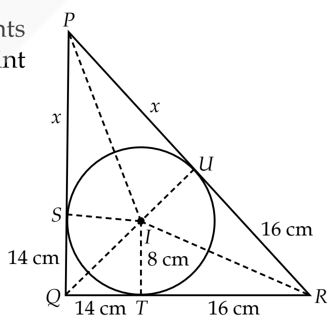
$RU = RT = 16$ cm .

Let, $PS = PU = x$ cm

Thus, $PQ = (x + 14)$ cm

$PR = (x + 16)$ cm

and $QR = 30$ cm



Now, Area of ΔPQR

= Area of ΔIQR + Area of ΔIQP + Area of ΔIPR

$\Rightarrow \quad 336 = \frac{1}{2}(14 + 16) \times 8 + \frac{1}{2}(14 + x) \times 8 + \frac{1}{2}(16 + x) \times 8$

$\Rightarrow \quad 84 = 30 + 14 + x + 16 + x \Rightarrow 24 = 2x \Rightarrow x = 12$

Hence, $PQ = 26$ cm and $PR = 28$ cm

27. We have, $AB = 16$ cm. Therofore, $AL = BL = 8$ cm

In ΔOLB, we have

$OB^2 = OL^2 + LB^2 \Rightarrow 10^2 = OL^2 + 8^2$

$\Rightarrow \quad OL^2 = 100 - 64 = 36 \Rightarrow OL = 6$ cm

Let $PL = x$ and $PB = y$. Then, $OP = (x + 6)$ cm

In Δ's PLB and ΔOBP, we have

$PB^2 = PL^2 + BL^2$ and $OP^2 = OB^2 + PB^2$

$\Rightarrow \quad y^2 = x^2 + 64$ and $(x + 6)^2 = 100 + y^2$

$\Rightarrow \quad (x + 6)^2 = 100 + x^2 + 64$

[Substituting the value of y^2 in second equation]

$\Rightarrow 12x = 128 \Rightarrow x = \frac{32}{3}$ cm

$\therefore \quad y^2 = x^2 + 64 \Rightarrow y^2 = \left(\frac{32}{3}\right)^2 + 64 = \frac{1600}{9} \Rightarrow y = \frac{40}{3}$ cm

Hence, $PA = PB = \frac{40}{3}$ cm

28. $DR = DS = 5$ cm

[$\because$ Tangents drawn from an external point are equal]

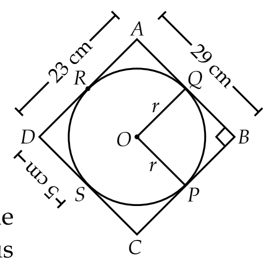
$AR = AD - DR = 23 - 5 = 18$ cm

$AQ = AR = 18$ cm

[$\because$ Tangents drawn from an external point are equal]

$QB = AB - AQ = 29 - 18 = 11$ cm

$QB = BP = 11$ cm



Also, $\angle OQB = \angle OPB = 90°$

[$\because$ Tangent at any point of circle is perpendicular to the radius through the point of contact]

Also, $\angle B = 90°$ [Given]

So, $OQ = OP$ = radius = r [Given]

$\therefore \quad OQBP$ is a square.

$\Rightarrow \quad r = OP = OQ = QB = 11$ cm [Sides of a square]

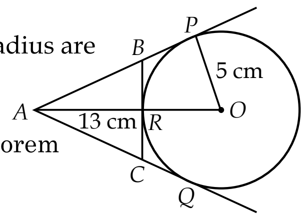
Hence, radius (r) of the circle = 11 cm

29. In ΔAPO,

$\angle P = 90°$ [$\because$ Tangent and radius are perpendicular to each other]

$OP = 5$ cm, $AO = 13$ cm

In ΔAPO, by Pythagoras theorem



$OA^2 = OP^2 + AP^2$

$\Rightarrow \quad 13^2 = 5^2 + AP^2$

$\Rightarrow \quad 169 - 25 = AP^2 \Rightarrow 12 = AP$

Since, tangents from an external point to a circle are equal.

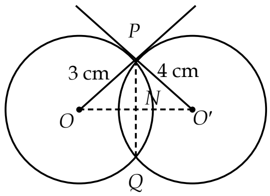
$\therefore \quad AP = AQ, BP = BR, CQ = CR$ ...(i)

Perimeter of $\Delta ABC = AB + BC + AC$

$= AB + (BR + RC) + AC = AB + BP + CQ + AC$ [Using (i)]

$= AP + AQ = AP + AP = 2AP = 2 \times 12 = 24$ cm

30. Here, two circles are of radii $OP = 3$ cm and $O'P = 4$ cm.
These two circles intersect at P and Q.
Here, OP and $O'P$ are two tangents drawn at point P.



$\angle OPO' = 90°$
[$\because$ Tangent at any point of circle is perpendicular to radius through the point of contact]
Join OO' and PQ such that OO' and PQ intersect at point N.
In right angled $\Delta OPO'$,
$(OO')^2 = (OP)^2 + (PO')^2$ [By Pythagoras theorem]
$\Rightarrow \quad (OO')^2 = (3)^2 + (4)^2 = 25$
$\Rightarrow \quad OO' = 5$ cm
Also, $PN \perp OO'$
Let $ON = x$, then $NO' = 5 - x$
In right angled ΔONP,
$(OP)^2 = (ON)^2 + (NP)^2$ [By Pythagoras theorem]
$\Rightarrow \quad (NP)^2 = 3^2 - x^2 = 9 - x^2$ ...(i)
and in right angled $\Delta PNO'$,
$(PO')^2 = (PN)^2 + (NO')^2$ [By Pythagoras theorem]
$\Rightarrow \quad (4)^2 = (PN)^2 + (5 - x)^2$
$\Rightarrow \quad (PN)^2 = 16 - (5 - x)^2$ ...(ii)
From (i) and (ii), we have
$9 - x^2 = 16 - (5 - x)^2$
$\Rightarrow \quad 7 + x^2 - (5 - x)^2 = 0$
$\Rightarrow \quad 7 + x^2 - (25 + x^2 - 10x) = 0$
$\Rightarrow \quad 10x = 18 \Rightarrow x = 1.8$ cm
Again, in right angled ΔOPN,
$OP^2 = (ON)^2 + (NP)^2$ [By Pythagoras theorem]
$\Rightarrow \quad 3^2 = (1.8)^2 + (NP)^2$
$\Rightarrow \quad (NP)^2 = 9 - 3.24 = 5.76$
$\Rightarrow \quad NP = 2.4$ cm
$\therefore$ Length of common chord,
$PQ = 2PN = 2 \times 2.4 = 4.8$ cm